On Generalized Contra Continuous Functions and some Relations with other Kinds of Continuity on Intuitionistic Topological Spaces[1]

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Abstract

We study in this paper the concept of contra continuous functions and generalized them in intuitionistic topological spaces and we studied the relations of each kind of these function by properties, examples and a diagram to summarize these functions. Also we study some relation between almost contra continuous function and some continuous functions.

Introduction

The notion of contra continuity was introduced by (Dontchev, 1996) contra semi continuous function was introduced and investigated by (Dontchev & Noiri, 1999), so contra pre continuous was introduced by (Jafari, & Noiri, 2002), also we are going to generalized them on intuitionistic topological spaces.

In this paper we define some kinds of contra continuous, contra semi continuous, contra α continuous, contra pre continuous, contra β continuous, contra θ continuous, contra generalized continuous, contra generalized semi continuous, contra semi generalized continuous, contra generalized α continuous, contra α generalized continuous, contra generalized pre continuous, contra pre generalized continuous, contra generalized β continuous, and contra θ generalized continuous functions and we give propositions to show the relations among them, some counter examples are given for not implications. We give also a diagram to illustrate these relations. So we study some relations among contra continuous functions and some kinds of continuous functions namely (perfectly continuous, slightly continuous, semi regular-continuous, completely-continuous, regular closed-continuous and B-continuous) by some properties, as well as we give some examples for non-true implications.

Preliminaries

Let X be anon-empty set, an intuitionistic set (IS, for short) A is an object having the form A= $\langle x, A_1, A_2 \rangle$ where A_1 and A_2 are disjoint subset of X. the set A_1 is called a member of A, while A_2 is called non- member of A, an intuitionistic topology (IT, for short) on a non-empty set X, is a family T of IS in X containing $\tilde{\emptyset}, \tilde{X}$ and closed under arbitrary unions and finitely intersections. In this case the pair(X, T) is called an intuitionistic topological space (ITS, for short), any IS in T is known as an intuitionistic open set (IOS, for short) in X. The complement of IOS is called intuitionistic closed set (ICS, for short), so the interior and closure of A are denoted by $\mathrm{int}(A)$ and $\mathrm{cl}(A)$ respectively and defined by $\mathrm{int}(A) = \cup \{G_i : G_i \in T \text{ and } G_i \subseteq A\}$ and $\mathrm{cl}(A) =$ $\cap \{F_i : F_i \text{ is ICS in } X \text{ and } A \subseteq F_i\}$

So $\mathrm{int}(A)$ is the largest IOS contained in A, and $\mathrm{cl}(A)$ is the smallest ICS contain A, a set A is called intuitionistic regular-closed set (IRCS, for short) if $A = \mathrm{clint}A$ intuitionistic α-closed set (IαCS, for short) if $\mathrm{clintcl}A \subseteq A$, intuitionistic semi-closed set (ISCS, for short) if $\mathrm{intcl}A \subseteq A$, intuitionistic pre-closed set (IPCS, for short) if $\mathrm{clint}A \subseteq A$,intuitionistic β-closed set (IβCS, for short) if $\mathrm{intclint}A \subseteq A$. The complement of IRCS (resp. IαCS, ISCS, IPCS and IβCS) is called intuitionistic regular-open set (resp. intuitionistic α-open set, intuitionistic semi-open set, intuitionistic pre-open set and intuitionistic β-open set) in X. (IROS, IαOS, ISOS, IPOS and IβOS, for short), A is saida to be intuitionistic semi-regular set (ISRS, for short) (Dontchev & Noiri, 1998) if A is ISOS and ISCS in X, so A is called intuitionistic B-set (IBS, for short) (Dontchev & Noiri, 1998) if A is the intersection of an IOS and ISCS and A is said to be an intuitionistic θ-closed set (IθCS, for short) if $A = \mathrm{cl}_\theta A$ where $\mathrm{cl}_\theta A = \{x \in X : \mathrm{cl}(U) \cap A \neq \emptyset, U \in T \text{ and } x \in U\}$. A is called intuitionistic θ generalized closed set (Iθg-closed, for short) if $\mathrm{cl}_\theta A \subseteq U$, whenever $A \subseteq U$ and U is IOS.

Generalized contra continuous function on ITS's.

The following definitions of several kinds of contra continuous functions which are given in general topology by (Baker, 2001; Dontchev, 1996 and Dontchev & Maki, 1999). We generalized them into ITS's.

Definition: Let (X, T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then f is said to be:

1. An intuitionistic contra continuous (I contra cont., for short) function if the inverse image of each IOS in Y is ICS in X.

2. An intuitionistic contra semi-continuous (I contra semi-cont., for short) function if the inverse image of each IOS in Y is ISCS in X.
3. An intuitionistic contra α-continuous (I contra α-cont., for short) function if the inverse image of each IOS in Y is IαCS in X.
4. An intuitionistic contra pre-continuous (I contra pre-cont., for short) function if the inverse image of each IOS in Y is IPCS in X.
5. An intuitionistic contra β-continuous (I contra β-cont., for short) function if the inverse image of each IOS in Y is IβCS in X.
6. An intuitionistic contra θ-continuous (I contra θ-cont., for short) function if the inverse image of IOS in Y is IθCS in X.

Next we are going to introduce the definitions of generalized contra continuous functions on ITS's.

Definition: Let (X,T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then f is said to be an intuitionistic contra g-cont. (resp. contra gs-cont., contra sg-cont., contra gp-cont., contra pg-cont., contra gα-cont., contra αg-cont., contra θg-cont. and contra gβ-cont. functions if the inverse image of each IOS in Y is Ig-closed (resp. Igs-closed, Isg-closed, Igp-closed, pg-closed, Igα-closed, Iαg-closed, Iθg-closed and Igβ-closed) set in X.

Proposition: Let(X, T) and(Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then

1- If f is I contra θ-cont. function then f is I contra θg-cont. function.
2- If f is I contra θ-cont. function then f is I contra cont. function.
3- If f is I contra θg-cont. function then f is I contra g-cont. function.
4- If f is I contra cont. function then f is I contra g-cont. function.
5- If f is I contra cont. function then f is I contra α-cont. function.
6- If f is I contra α-cont. function then f is I contra semi-cont. function.
7- If f is I contra α-cont. function then f is I contra pre-cont. function.
8- If f is I contra semi-cont. function then f is I contra β-cont. function.
9- If f is I contra pre-cont. function then f is I contra β-cont. function.
10- If f is I contra α-cont. function then f is I contra gα-cont. function.
11- If f is I contra semi-cont. function then f is I contra sg-cont. function.
12- If f is I contra β-cont. function then f is I contra gβ-cont. function.
13- If f is I contra g-cont. function then f is I contra αg-cont. function.
14- If f is I contra g-cont. function then f is I contra gs-cont. function.
15- If f is I contra gα-cont. function then f is I contra αg-cont. function.
16- If f is I contra sg-cont. function then f is I contra gs-cont. function.
17- If f is I contra pg-cont. function then f is I contra gp-cont. function.
18- If f is I contra αg-cont. function then f is I contra gs-cont. function.

19- If f is I contra αg-cont. function then f is I contra gp-cont. function.

Proof:

1- Let V be IOS in Y then $f^{-1}(V)$ is IθCS in X (since f is I contra θ-cont. function). Now for each IOS A in X and $f^{-1}(V) \subseteq A$ then $cl_\theta(f^{-1}(V)) \subseteq A$ since $cl_\theta\left(f^{-1}(V)\right) = f^{-1}(V)$. There fore, $f^{-1}(V)$ is Iθg-closed set in X. Hence f is I contra θg-cont. function.♦

2- Suppose that V be IOS in Y then $f^{-1}(V)$ is IθCS in X (since f is I contra θ-cont. function). So$f^{-1}(V) = cl_\theta(f^{-1}(V))$. There fore, $f^{-1}(V)$ is ICS in X since every IθCS is ICS by [6]. Hence f is I contra cont. function.♦

3- Let V be IOS in Y then $f^{-1}(V)$ is Iθg-closed set in X (since f is I contra θg-cont. function). So for each IOS A in X and $f^{-1} \subseteq A$ then $cl_\theta(f^{-1}(V)) \subseteq A$. Therefore, $f^{-1}(V)$ is Ig-closed set in X since every Iθg-closed set is Ig-closed set by (Dontchev, J. and Maki, H. (1999)). Hence f is I contra g-cont. function.♦

4- Let V be IOS in Y then $f^{-1}(V)$ is ICS in X (since f is I contra cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $cl(f^{-1}(V)) \subseteq A$ since $f^{-1}(V) = cl(f^{-1}(V))$. Therefore, $f^{-1}(V)$ is Ig-closed set in X and hence f is I contra g-cont. function.♦

5- Suppose that V is IOS in Y then $f^{-1}(V)$ is ICS in X (since f is I contra cont. function) then $cl(f^{-1}(V)) = f^{-1}(V)$ imply $intcl(f^{-1}(V)) \subseteq f^{-1}$ implies $clintcl(f^{-1}(V)) \subseteq cl(f^{-1}(V)) = f^{-1}(A)$. Therefore, $f^{-1}(V)$ is IαCS in X and hence f is I contra α-cont. function.♦

6- Let V be IOS in Y then $f^{-1}(V)$ is IαCS in X (since f is I contra α-cont. function) then $clintcl(f^{-1}(V)) \subseteq f^{-1}(V)$ imply $intcl(f^{-1}(V)) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is ISCS in X and hence f is I contra semi-cont. function.♦

7- Let V be IOS in Y then $f^{-1}(V)$ is IαCS in X (since f is I contra α-cont. function) then $clintcl(f^{-1}(V)) \subseteq f^{-1}(V)$ imply $clint\left(f^{-1}(V)\right) \subseteq f^{-1}(V)$. therefore, $f^{-1}(V)$ is IPCS in X and hence f is I contra pre-cont. function.♦

8- suppose that be IOS in Y then $f^{-1}(V)$ is ISCS in X (since f is I contra semi-cont. function) then $intcl(f^{-1}(V)) \subseteq f^{-1}(V)$ imply $intclint\left(f^{-1}(V)\right) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is IβCS in X and hence f is I contra β-cont. function.♦

9- Let V be IOS in Y then $f^{-1}(V)$ is IPCS in X (since f is I contra pre-cont. function) then $clint(f^{-1}(V)) \subseteq f^{-1}(V)$ imply $intclint\left(f^{-1}(V)\right) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is IβCS in X and hence f is I contra β-cont. function.♦

10- Suppose that V be IOS in Y then $f^{-1}(V)$ is IαCS in X (since f is I contra α-cont. function). Now for each IαOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl(f^{-1}(V)) \subseteq A$ since $f^{-1}(V) = \alpha cl(f^{-1}(V))$ by $\alpha cl(f^{-1}(V)) = f^{-1}(V) \cup clintcl(f^{-1}(V))$ and $clintcl(f^{-1}(V)) \subseteq f^{-1}(V)$. There fore, $f^{-1}(V)$ is I$g\alpha$-closed set in X. Hence f is I contra $g\alpha$-cont. function.♦

11- Let V be IOS in Y then $f^{-1}(V)$ is ISCS in X (since f is I contra semi-cont. function). Now for each ISOS A in X and $f^{-1}(V) \subseteq A$ then $scl(f^{-1}(V)) \subseteq A$ since $f^{-1}(V) = scl(f^{-1}(V))$ by $scl(f^{-1}(V)) = f^{-1}(V) \cup intcl(f^{-1}(V))$ and $intcl(f^{-1}(V)) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is Isg-closed set in X. Hence f is I contra sg-cont. function.♦

12- Suppose that V be IOS in Y then $f^{-1}(V)$ is IβCS in X (since f is I contra β-cont. function). Now for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\beta cl(f^{-1}(V)) \subseteq A$ since $f^{-1}(V) = \beta cl(f^{-1}(V))$ by $\beta cl(f^{-1}(V)) = f^{-1}(V) \cup intclint(f^{-1}(V))$and $intclint(f^{-1}(V)) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is I$g\beta$-closed set in X and hence f is I contra $g\beta$-cont. function.♦

13- Suppose that V be IOS in Y then $f^{-1}(V)$ is Ig-closed set in X (since f is I contra g-cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $cl(f^{-1}(V)) \subseteq A$. Now since every ICS is IαCS then $cl(f^{-1}(V)) = \cap \{F_i : F_i \text{ is } I\alpha CS \text{ and } f^{-1}(V) \subseteq F_i\} \subseteq cl(f^{-1}(V))$. So we have that for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl(f^{-1}(V)) \subseteq A$. Therefore, $f^{-1}(V)$ is Iαg-closed set in X and hence f is I contra αg-cont. function.♦

14- Let V be IOS in Y then $f^{-1}(V)$ is Ig-closed set in X (since f is I contra g-cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $cl(f^{-1}(V)) \subseteq A$. Now since every ICS is ISCS then $scl(f^{-1}(V)) = \cap \{F_i : F_i \text{ is } ISCS \text{ and } f^{-1}(V) \subseteq F_i\} \subseteq cl(f^{-1}(V))$.
so we have that for each IOS A in X and $f^{-1}(V) \subseteq A$ then $scl(f^{-1}(V)) \subseteq A$. therefore, $f^{-1}(V)$ is Igs-closed set in X. Hence f is I contra gs-cont. function.♦

15- Let V be IOS in Y then $f^{-1}(V)$ is I$g\alpha$-closed set in X (since f is I contra $g\alpha$-cont. function). So for each IαOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl(f^{-1}(V)) \subseteq A$. Now since every IOS is IαOS then there exists an IOS U in X such that $f^{-1}(V) \subseteq U$ so $\alpha cl(f^{-1}(V)) \subseteq U$. Therefore, $f^{-1}(V)$ is Iαg-closed set in X. Hence f is I contra αg-cont. function.♦

16- Suppose that V be IOS in Y then $f^{-1}(V)$ is Isg-closed set in X (since f is I contra sg-cont.function). So for each ISOS A in X and $f^{-1}(V) \subseteq A$ then $scl(f^{-1}(V)) \subseteq A$. Now since every IOS is ISOS then there exists an IOS U in X such that $f^{-1}(V) \subseteq U$ then$scl(f^{-1}(V)) \subseteq U$. Therefore, $f^{-1}(V)$ is Igs-closed set in X and hence f is I contra gs-cont. function.♦
17- Let V be IOS in Y then $f^{-1}(V)$ is Ipg-closed set in X (since f is I contra pg-cont. function). So for each IPOS A in X and $f^{-1}(V) \subseteq A$ then $pcl\left(f^{-1}(V)\right) \subseteq A$. Now since every IOS is IPOS then there exists an IOS U in X such that $f^{-1}(V) \subseteq U$ then $pcl(f^{-1}(V)) \subseteq U$. Therefore, $f^{-1}(V)$ is Igp-closed set in X and hence f is I contra gp-cont. function.♦
18- Let V be IOS in Y then $f^{-1}(V)$ is Iαg-closed set in X (since f is I contra αg-cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl\left(f^{-1}(V)\right) \subseteq A$. Now since every I$\alpha$CS is ISCS then $scl\,(f^{-1}(V)) = \cap\{F_i : F_i \text{ is } ISCS \text{ and } f^{-1}(V) \subseteq F_i\} \subseteq \alpha cl\left(f^{-1}(V)\right)$. so we have that for each IOS A in X and $f^{-1}(V) \subseteq A$ then $scl\left(f^{-1}(V)\right) \subseteq A$. Therefore, $f^{-1}(V)$ is Igs-closed set in X and hence f is I contra gs-cont. function.♦
19- Suppose that V be IOS in Y then $f^{-1}(V)$ is Iαg-closed set in X (since f is I contra αg-cont. function). So for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\alpha cl\left(f^{-1}(V)\right) \subseteq A$. Now since every I$\alpha$CS is IPCS then $pcl\left(f^{-1}(V)\right) = \cap\{F_i : F_i \text{ is } IPCS \text{ and } f^{-1}(V) \subseteq F_i\} \subseteq \alpha cl\left(f^{-1}(V)\right)$. So we have that for each IOS A in X and $f^{-1}(V) \subseteq A$ then $pcl(f^{-1}(V)) \subseteq A$. Therefore, $f^{-1}(V)$ is Igp-closed set in X and hence f is I contra gp-cont. function.♦ we start with example which shows I contra θg-cont. does not imply I contra θ-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a, b\}, \{c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \{1,2\}, \{3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 3 \text{ and } f(c) = 2$. Now let $B = f^{-1}(C) = \langle x, \{a, c\}, \{b\}\rangle$ then B is Iθg-closed set in X since the only IOS containing B is X and $cl_\theta B = X \subseteq X$. But B is not I$\theta$CS since $B \neq cl_\theta B = X$. So since the inverse image of each IOS in Y is Iθg-closed set in X then f is I contra θg-cont. function. But f is not I contra θ-cont. function.

The next example shows that:

1- I contra cont. does not imply I contra θ-cont.
2- I contra cont. does not imply I contra θg-cont.
3- I contra g-cont. does not imply I contra θg-cont.
4- I contra g-cont. does not imply I contra θ-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{a, b\}, \{c\}\rangle$ and $C = \langle x, \{a, c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{3\}, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. Now let $G = f^{-1}(D) = \langle x, \{c\}, \{a, b\}\rangle$, then G is ICS and I$g$-closed set in X but G is not IθCS in X since $cl_\theta G = X \nsubseteq G$ so G is not Iθg-closed set since the only IOS containing G in X is C and $cl_\theta G = X \nsubseteq$ C.

The following example shows that I contra g-cont. does not imply I contra cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a, b\}, \{c\}\rangle, B = \langle x, \{b\}, \{a\}\rangle$ and $C = \langle x, \{a, b\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{1,2\}, \emptyset\rangle$. Define a function $f: X \to Y$ by $f(a) = 3, f(b) = 1$ and $f(c) = 2$. Now let $G = f^{-1}(D) = \langle x, \{b, c\}, \emptyset\rangle$ then G is Ig-closed set in X since the only IOS containing G is X and $clG = X \subseteq X$ but G is not ICS in X since $G \neq clG = X$. So the inverse image of each IOS in Y is Ig-closed set in X.

In this example we are going to show I contra α-cont. function does not imply I contra cont. function

Example: Let $X = \{a, b, c, d\}$ and let $T = \{\widetilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where$A = \langle x, \{a, b\}, \{c\}\rangle, B = \langle x, \{b, d\}, \{a\}\rangle, C = \langle x, \{b\}, \{a, c\}\rangle$ and $D = \langle x, \{a, b, c\}, \emptyset\rangle$ and Let $Y = \{1,2,3,4\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, E\}$ where $E = \langle y, \{1\}, \{2,3\}\rangle$. Define a function $f: X \to Y$ by $(a) = 1$, $f(b) = 2, f(c) = 3$ and $f(d) = 4$. So let $G = f^{-1}(E) = \langle x, \{a\}, \{b, c\}\rangle$ then G is IαCS set in X since $clintclG = \emptyset \subseteq G$ but G is not ICS in X since $clG = \bar{C} \neq G$. Then the inverse image of each IOS in Y is IαCS in X.

We are going to show that I contra semi-cont. does not imply I contra α-cont.

Example: Let $X = \{a, b, c, d\}$ and $T = \{\widetilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle$ and $B = \langle x, \emptyset, \{b, c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\widetilde{\emptyset}, \tilde{Y}, C\}$ where $C = \langle y, \emptyset, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = f(d) = 3$ and $f(c) = 2$. Now a set $G = f^{-1}(C) = \langle x, \emptyset, \{c\}\rangle$ is ISCS in X since $intclG = B \subseteq G$ but G is not IαCS in X since $clintclG = \bar{B} \nsubseteq G$. So the inverse image of each IOS in Y is ISCS in X.

The following example shows that:

1. I contra β-cont. does not imply I contra pre-cont.
2. I contra β-cont. does not imply I contra semi-cont.

Example: Let $X = \{a, b, c, d\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{c, d\}, \{a\}\rangle, C = \langle x, \{a, c, d\}, \emptyset\rangle$ and $D = \langle x, \emptyset, \{a, \mathrm{b}, c\rangle$ and let $Y = \{1,2,3,4\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E\}$ where $E = \langle y, \{1,3\}, \{2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2, f(c) = 4$ and $f(d) = 3$. Then a set $G = f^{-1}(E) = \langle x, \{a, d\}, \{b\}\rangle$ is IβCS in X since $intclintG = A \subseteq G$ but G is not IPCS and ISCS in X since $intclG = X \nsubseteq G$ so $clintG = D \nsubseteq G$. Then f is I contra β-cont. function but f is not I contra semi-cont. and not I contra pre-cont function. We are going in the following example to show that:

1. I contra gs-cont. does not imply I contra αg-cont.
2. I contra gs-cont. does not imply I contra g-cont.
3. I contra sg-cont. does not imply I contra g-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a, b\}\rangle, B = \langle x, \{b\}, \{c\}\rangle$ and $C = \langle x, \{b, c\}, \emptyset\rangle$ and let $Y = \{\tilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{1\}, \{3\}\rangle$. Define a function $f: X \to \mathrm{Y}$ by $f(a) = 2, f(b) = 1$ and $f(c) = 3$. SOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, E, F\}$ where $E = \langle x, \{c\}, \{b\}\rangle$ and $F = \langle x, \{a, b\}, \{c\}\rangle$. So αOX= T. We have $B = f^{-1}(D)$ is Igs-closed and Isg-closed in X since the only IOS and ISOS in X that containing B is B,C and F so $sclB = \bar{F} = B$. but B is not Ig-closed set and it's not Iαg-closed set in X since the only IαOS in X containing B is B and C and $clB = \alpha clB = \bar{A} \nsubseteq B$ or C. Therefore, f is I contra gs-cont. (resp. I contra sg-cont.) function but not I contra αg-cont.(resp. I contra g-cont) function.

The following example shows that:

1. I contra gs-cont. does not imply I contra $g\alpha$-cont.
2. I contra g-cont. does not imply I contra $g\alpha$-cont.
3. I contra αg-cont. does not imply I contra $g\alpha$-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{c\}, \{a, b\}\rangle$ and $C = \langle x, \{a, c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{1\}, \{3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$. SOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, E, G, I, L, N\}$ where $E = \langle x, \{a\}, \{c\}\rangle, G = \langle x, \{a, c\}, \emptyset\rangle, I = \langle x, \{a, b\}, \{c\}\rangle, L = \langle x, \{c\}, \{a\}\rangle$ and $N = \langle x, \{b, c\}, \{a\}\rangle$.

So αOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, G\}$. Then we have $E = f^{-1}(D)$ is Ig-closed (resp. Igs-closed, Iαg-closed) set in X since the only IOS containing E is X and $clE = \alpha clE = \bar{B} \subseteq X$ but E is not I$g\alpha$-cosed since $E \subseteq G$ where G is $I\alpha$OS in X but $\alpha clE = \bar{B} \nsubseteq G$. Then the inverse image of each IOS in Y is Ig-closed (resp. Igs-closed and Iαg-closed) set in X.

The next example shows I contra gs-cont. does not imply I contra sg-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a, b\}\rangle, B = \langle x, \{a\}, \{b, c\}\rangle$ and $C = \langle x, \{a, c\}, \{b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, E\}$ where $E = \langle y, \{2\}, \emptyset\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 3$ and $f(c) = 2$. SOX= $\{\tilde{\emptyset}, \tilde{X}, A, B, C, D, F, L, K, M\}$ where $D = \langle x, \{c\}, \{a\}\rangle, F = \langle x, \{a, c\}, \emptyset\rangle, L = \langle x, \{b, c\}, \{a\}\rangle, K = \langle x, \{a\}, \{c\}\rangle \; and \; M = \langle x, \{a, b\}, \{c\}\rangle$.

Now let $G = f^{-1}(E) = \langle x, \{c\}, \emptyset\rangle$, then G is I$g$s-closed set in X since the only IOS containing G is X and $sclG = X \subseteq X$ but G is not Isg-closed set in X since $G \subseteq F$ where F is ISOS in X and $sclG = X \nsubseteq F$. Then the inverse image of each IOS in Y is Igs-closed set in X.

We are going to show I contra $g\alpha$-cont. does not imply I contra α-cont.

Example: Let $X = \{1,2,3\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{b\}, \{a, c\}\rangle \; and \; B = \langle x, \{a\}, \{b, c\}\rangle \; and \; C = \langle x, \{a, b\}, \{c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{2,3\}, \emptyset\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2 \; and \; f(c) = 3$. $\alpha OX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, E\}$ where $E = \langle x, \{a, b\}, \emptyset\rangle$. So a set $G = f^{-1}(D) = \langle x, \{b, c\}, \emptyset\rangle$ is I$g\alpha$-closed set in X since the only IαOS containing G is X and $\alpha clG = X \subseteq X$ but G is not IαCS in X since $clintclG = X \nsubseteq G$ then f is I contra $g\alpha$-cont. function but not I contra α-cont. function.

The following example shows that I contra $g\beta$-cont. does not imply I contra gp-cont.

Example: Let $X = \{a, b, c\}$ and let $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{b\}, \{a, c\}\rangle$ and $C = \langle x, \{a, b\}, \{c\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{3\}, \{1,2\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 3, f(c) = 2$. $\beta OX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, E, H, K, L, I, M, O, N, G, F, J\}$ where $E = \langle x, \{b\}, \{a\}\rangle, H = \langle x, \{b\}, \{c\}\rangle, K = \langle x, \{b\}, \emptyset\rangle, L = \langle x, \{a\}, \{b\}\rangle, I = \langle x, \{a\}, \{c\}\rangle, M = \langle x, \{a\}, \emptyset\rangle, O = \langle x, \{b, c\}, \{a\}\rangle, N = \langle x, \{a, b\}, \emptyset\rangle, G = \langle x, \{b, c\}, \emptyset\rangle, F = \langle x, \{a, c\}, \{b\}\rangle \; and \; J = \langle x, \{a, c\}, \emptyset\rangle$.

$POX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, H, K, I, N, G, J\}$. Now a set B $= f^{-1}(D)$ is I$g\beta$-closed set in X since B is IOS and $\beta clB = B$. But B is not Igp-closed set since

$pclB = O \nsubseteq B$. Then f is I contra $g\beta$-cont. since the inverse image of each IOS in Y is I$g\beta$-closed set in X. so f is not I contra gp-cont. function.

This example shows that:

1. I contra pre-cont. does not imply I contra $g\alpha$-cont.
2. I contra β-cont. does not imply I contra gs-cont.
3. I contra gp-cont. does not imply I contra sg-cont.
4. I contra gp-cont. does not imply I contra αg-cont.
5. I contra $g\beta$-cont. does not imply I contra sg-cont.
6. I contra $g\beta$-cont. does not imply I contra gs-cont.

Example: Let $X = \{a, b, c\}$ and let $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\}\rangle$ and $B = \langle x, \{a, b\}, \emptyset\rangle$ and let $Y = \{1,2,3\}\ and\ \sigma = \{\tilde{\emptyset}, \tilde{Y}, U\}$ where $U = \langle y, \{1\}, \{3\}\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3$.

$\beta OX = POX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D, E, F, G, H, I, J, L, K, M, O, S, W, N, P, Z\}$ where $C = \langle x, \{a\}, \emptyset\rangle, D = \langle x, \{a\}, \{c\}\rangle, E = \langle x, \{a, b\}, \{c\}\rangle, F = \langle x, \{a, c\}, \emptyset\rangle, G = \langle x, \{a, c\}, \{b\}\rangle, H = \langle x, \{a\}, \{b, c\}\rangle, I = \langle x, \{b, c\}, \emptyset\rangle, J = \langle x, \{b, c\}, \{a\}\rangle, K = \langle x, \{c\}, \emptyset\rangle, L = \langle x, \{c\}, \{a\}\rangle$, $M = \langle x, \{c\}, \{b\}\rangle, O = \langle x, \emptyset, \{b, c\}\rangle, S = \langle x, \emptyset, \{b\}\rangle, W = \langle x, \emptyset, \{c\}\rangle, N = \langle x, \{c\}, \{a, b\}\rangle, P = \langle x, \{b\}, \emptyset\rangle$ and $Z = \langle x, \{b\}, \{c\}\rangle$. so $\alpha OX = SOX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, F, G\}$. Then a set $D = f^{-1}(U)$ is IPCS (resp. IβCS, Ip-closed set and I$g\beta$-closed set) in X since $clintD = intclintD = \emptyset \subseteq D$, so the only IOS containing D is B and $pclD = \beta clD = D \subseteq B$ but D is not Igs-closed (resp. Isg-closed, I$g\alpha$-closed, Iαg-closed) set in X since the only IOS, IαOS and ISOS containing D is B and F so $clD = sclD = X \nsubseteq B\ or\ F$.

In the next example we show that I contra sg-cont. does not imply I contra semi-cont.

Example; Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle, B = \langle x, \{c\}, \{a\}\rangle\ and\ C = \langle x, \{a, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, D\}$ where $D = \langle y, \{1,2\}, \emptyset\rangle$. Define a function $f: X \to Y$ by $f(a) = 1, f(b) = 2\ and\ f(c) = 3$.

$SOX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, E, F\}$ where $E = \langle x, \{a\}, \{c\}\rangle, F = \langle x, \{b, c\}, \{a\}\rangle$. So let $G = f^{-1}(D) = \langle x, \{a, b\}, \emptyset\rangle$ then G is Isg-closed set in X since the only ISOS containing G is X and $sclG = X \subseteq X$ but G is not ISCS in X since $intclG = X \nsubseteq G$. So the inverse image of each IOS in Y is Isg-closed set in X.

The following example shows that:

1- I contra g-cont. does not imply I contra sg-cont.
2- I contra gp-cont. does not imply I contra pre-cont.
3- I contra gp-cont. does not imply I contra sg-cont.
4- I contra gp-cont. does not imply I contra pg-cont.
5- I contra $g\beta$-cont. does not imply I contra pre-cont.
6- I contra $g\beta$-cont. does not imply I contra β-cont.
7- I contra $g\beta$-cont. does not imply I contra sg-cont.
8- I contra $g\beta$-cont. does not imply I contra pg-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a, c\}, \emptyset\rangle$ and $B = \langle x, \{a\}, \{c\}\rangle$. and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, H\}$ where $H = \langle y, \{1,3\}, \emptyset\rangle$. Define a function $f: X \to Y$ by
$f(a) = 1, f(b) = 3\ and\ f(c) = 2.$
$POX = \beta OX = POX = \{A, B, C, D, L, I, E, G,\ O, F, K, P, J, S, N, U, Z, M, W\}$
where
C=
$\langle x, \{a\}, \emptyset\rangle, D = \langle x, \{a\}, \{b\}\rangle, L = \langle x, \{b\}, \{a\}\rangle, I = \langle x, \{b, c\}, \emptyset\rangle, E = \langle x, \{a, c\}, \{b\}\rangle, G = \langle x, \{a, b\}, \{c\}\rangle, O = \langle x, \emptyset, \{b, c\}\rangle, F = \langle x, \{a, b\}, \emptyset\rangle, K = \langle x, \{b\}, \emptyset\rangle, P = \langle x, \{c\}, \emptyset\rangle,$
$J = \langle x, \{b, c\}, \{a\}\rangle, S = \langle x, \emptyset, \{c\}\rangle, N = \langle x, \{b\}, \{a, c\}\rangle, U = \langle x, \{a\}, \{b, c\}\rangle, Z = \langle x, \{c\}, \{b\}\rangle, M = \langle x, \{b\}, \{c\}\rangle\ and\ W = \langle x, \emptyset, \{b\}\rangle.$
SOX=$\{\tilde{\emptyset}, \tilde{X}, A, B, C, F, G\}$. Now a set $F = f^{-1}(H)$ is Ig-closed (resp. Igp-closed and I$g\beta$-closed) set in X since the only IOS containing F is X and $clF = pclF = \beta clF = X \subseteq X$ but F is not IPCS (resp, IβCS, Isg-closed set, Ipg-closed) set in X since $clintF = intclintF = X \nsubseteq F$ so $sclF = pclF = X \nsubseteq F$.

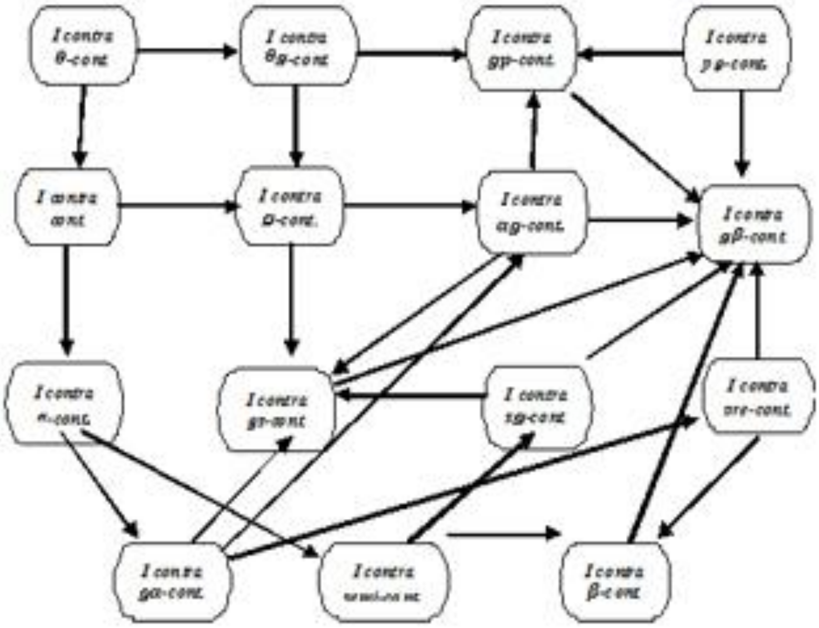
In the last example we show that:

1- I contra αg-cont. does not imply I contra g-cont.
2- I contra gα-cont. does not imply I contra g-cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where$A = \langle x, \{a, c\}, \{b\}\rangle$ and $B = \langle x, \{c\}, \{a, b\}\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, I\}$ where $I = \langle x, \{2\}, \{1,3\}\rangle$. Define a function $f: X \to Y$ by
$f(a) = 2, f(b) = 1 and\ f(c) = 3$. $\alpha OX = \{\tilde{\emptyset}, \tilde{X}, A, B, C, D, E, F, M, H\}$ where $C = \langle x, \{c\}, \{b\}\rangle, D = \langle x, \{c\}, \emptyset\rangle, E = \langle x, \{c\}, \{a\}\rangle, F = \langle x, \{b, c\}. \emptyset\rangle, M = \langle x, \{b, c\}, \{a\}\rangle$
and $H = \langle x, \{a, c\}, \emptyset\rangle$. Now let $G = f^{-1}(I) = \langle x, \{a\}, \{b, c\}\rangle$ then G is Iαg-closed and Igα-closed set in X since the only IOS containing G in X are A and H so $\alpha clG = \overline{M} = G \subseteq A$ and H. but G is not Ig-closed set in X since $clG = \overline{B} \nsubseteq A$. We have the inverse image of each IOS in Y is Igα-closed

and Iαg-closed set in X and hence f is I contra $g\alpha$-cont. function and I contra αg-cont. function but not I contra g-cont. function.We summarized the above result by the following diagram.

Diagram: The following implications are true and not reversed.



Definition: (Al-hawez, 2008) Let (X,T) and (Y,σ) be two ITS's then a function $f: X \rightarrow Y$ is said to be an intuitionistic slightly continuous (I slightly cont., for short) if the inverse image of each clopen (closed and open) set in Y is ICS in X.

Proposition: Let (X,T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be an I contra cont. function then f is I slightly cont. function.
The proof is trivial.In the following example we show that I slightly cont. does not imply I contra cont.

Example: Let $X = \{a, b, c\}$ and $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{c\}\rangle$ and $B = \langle x, \{a, c\}, \emptyset\rangle$ and let $Y = \{1,2,3\}$ and $\sigma = \{\tilde{\emptyset}, \tilde{Y}, H\}$ where H=$\langle y, \{1\}, \{3\}\rangle$. Define a function $f: X \rightarrow Y$ by $f(a) = 1$,
$f(b) = 2$ and $f(c) = 3$. Then f is I slightly cont. function since the inverse image of each clopen set in Y is ICS in X but f is not I contra cont. function since $f^{-1}(H) = A$ is not ICS in X.

Remark: Let (X, T) and (Y,σ) be two ITS's and let $f: X \rightarrow Y$ be a function then I contra cont. function and I slightly cont. function are equivalent if:

1. (X, T) is discrete.
2. (X, T) is indiscrete.
3. (X, T) is disconnected.

The following definitions are given in general topology by (Dontchev & Noiri, 1998), so we generalized them on ITS's.

Definition: Let (X, T) and (Y,σ) be two ITS's then a function $f: X \to Y$ is said to be:

1- An intuitionistic semi-regular continuous (ISR-cont., for short) if the inverse image of each IOS in Y is ISRS in X.
2- An intuitionistic completely continuous (I completely cont., for short) if the inverse image of each IOS in Y is IROS in X.
3- An intuitionistic regular closed continuous (IRC-cont., for short) if the inverse image of each IOS in Y is IRCS in X.
4- An intuitionistic B-continuous (IB-cont., for short) if the inverse image of each IOS in Y is IBS in X.

Proposition: Let (X, T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1- f is ISR-cont. function.
2- f is Iβ-cont. function and I contra semi-cont. function.

Proof: 1⇒2 Suppose that V be any IOS in Y then $f^{-1}(V)$ is ISRS in X (by hypothesis) then $f^{-1}(V)$ is ISOS and ISCS so $f^{-1}(V) \subseteq \mathrm{clint} f^{-1}(V)$ and $\mathrm{intcl} f^{-1}(V) \subseteq f^{-1}(V)$.Now since $f^{-1}(V) \subseteq \mathrm{clint} f^{-1}(V)$ imply $f^{-1}(V) \subseteq \mathrm{clintcl} f^{-1}(V)$. Therefore, $f^{-1}(V)$ is IβOS and ISCS in X. Hence f is Iβ-cont. and I contra semi-cont. function.
2 ⇒1 Suppose that U be IOS in Y then $f^{-1}(U)$ is IβOS and ISCS in X (by hypothesis) then $f^{-1}(U) \subseteq \mathrm{clintcl} f^{-1}(U)$ and $\mathrm{intcl} f^{-1}(U) \subseteq f^{-1}(U)$. Now we have $\mathrm{intcl} f^{-1}(U) \subseteq f^{-1}(U) \subseteq \mathrm{clintcl} f^{-1}(U)$. then $f^{-1}(U)$ is ISOS in X also $f^{-1}(U)$ is ISCS in X. Therefore, $f^{-1}(U)$ is ISRS in X and hence f is ISR-cont. function.♦

Corollary: Every I contra cont. function and Iβ-cont. function is I semi-cont. function.

Proof: Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ an I contra cont. function and Iβ-cont. function, so for any IOS V in Y then $f^{-1}(V)$ is ICS and IβOS in X imply $f^{-1}(V) = \mathrm{cl} f^{-1}(V)$ and $f^{-1}(V) \subseteq \mathrm{clintcl} f^{-1}(V)$ imply $f^{-1}(V) \subseteq \mathrm{clint} f^{-1}(V)$. Therefore, $f^{-1}(V)$ is ISOS in X. hence f is I semi-cont. function.♦

Proposition: Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1- f is I completely cont. function.
2- f is I pre-cont. function and I contra semi-cont. function.

Proof: 1⇒2 Let V be IOS in Y then $f^{-1}(V)$ is IROS in X (since f is I completely cont. function) then $f^{-1}(V) = \mathrm{intcl}f^{-1}(V)$ imply $f^{-1}(V) \subseteq \mathrm{intcl}f^{-1}(V)$ and $\mathrm{intcl}f^{-1}(V) \subseteq f^{-1}(V)$. Therefore, $f^{-1}(V)$ is IPOS and ISCS in X. Hence f is I pre-cont. function and I contra semi-cont. function.

2 ⇒1 Let U be IOS in Y then $f^{-1}(U)$ is IPOS and ISCS in X (by hypothesis) then $f^{-1}(U) \subseteq \mathrm{intcl}f^{-1}(U)$ and $\mathrm{clint}f^{-1}(U) \subseteq f^{-1}(U)$ imply $f^{-1}(U) = \mathrm{intcl}f^{-1}(U)$. Therefore, $f^{-1}(U)$ is IROS in X. Hence f is I completely cont. function.♦

Proposition: Let (X,T) and (Y,σ) be two ITS's and let $f\colon X \to Y$ be a function then the following statements are equivalent:

1. f is IRC-cont. function.
2. f is Iβ-cont. function and I contra cont. function.

Proof: 1⇒2 Let V be IOS in Y then $f^{-1}(V)$ is IRCS in X (since f is IRC-cont. function) then $\mathrm{clint}f^{-1}(V) = f^{-1}(V)$ hence $f^{-1}(V)$ is ICS in X so $\mathrm{clint}f^{-1}(V) \subseteq f^{-1}(V)$ and $f^{-1}(V) \subseteq \mathrm{clint}f^{-1}(V)$ imply $f^{-1}(V) \subseteq \mathrm{clintcl}f^{-1}(V)$. Therefore, $f^{-1}(V)$ is IβCS and hence f is I contra cont. function and Iβ-cont. function.

2 ⇒1 Let U be IOS in Y then $f^{-1}(U)$ is IβOS and ICS in X (by hypothesis) then $f^{-1}(U) \subseteq \mathrm{clintcl}f^{-1}(U)$ and $\mathrm{cl}f^{-1}(U) = f^{-1}(U)$ imply $f^{-1}(U) \subseteq \mathrm{clint}f^{-1}(V)$ and $\mathrm{clint}f^{-1}(U) \subseteq f^{-1}(U)$ imply $f^{-1}(U) = \mathrm{clint}f^{-1}(U)$. Therefore, $f^{-1}(U)$ is IRCS in X. Hence f is IRC-cont. function.♦

The next proposition was proved in (Dontchev, J. and Noiri, T. (1998)) in general topology so we generalized it into ITS's.

Proposition: Let (X,T) and (Y,σ) be two ITS's and let $f\colon X \to Y$ be a function then the following statements are equivalent:

1. f is I contra semi-cont. function.
2. f is IB-cont. function and I contra gs-cont. function.

Proof: 1⇒2 Suppose that V be any IOS in Y then $f^{-1}(V)$ is ISCS in X (by hypothesis). Now let A be IOS in X and $f^{-1}(V) \subseteq A$ then $f^{-1}(V) = A \cap f^{-1}(V)$ imply $f^{-1}(V)$ is IBS, so $f^{-1}(V) = \mathrm{scl}f^{-1}(V)$ since $\mathrm{scl}f^{-1}(V) = f^{-1}(V) \cup \mathrm{intcl}f^{-1}(V)$ and $\mathrm{intcl}f^{-1}(V) \subseteq f^{-1}(V)$. Hence for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\mathrm{scl}f^{-1}(V) \subseteq A$. Therefore, $f^{-1}(V)$ is Igs-closed set and IBS in X, so f is I contra gs-cont. function and IB-cont. function.

2 ⇒1 Suppose that U be any IOS in Y then $f^{-1}(U)$ is IBS and ISCS in X (by hypothesis). Then $f^{-1}(U) = A \cap G$ where A is IOS containing $f^{-1}(U)$

in X and G is ISCS in X. So $\mathrm{sclf}^{-1}(U) \subseteq A$ since $f^{-1}(U)$ is Igs-closed set. Now
$\mathrm{intclf}^{-1}(U) = \mathrm{intcl}\,(A \cap G) \subseteq \mathrm{int}(\mathrm{cl}A \cap \mathrm{cl}G) = \mathrm{intcl}A \cap \mathrm{intcl}G \subseteq \mathrm{intcl}A \cap G$
since G is ISCS. So $\mathrm{intclf}^{-1}(U) \cap A \subseteq \mathrm{intcl}A \cap A \cap G$ since $\mathrm{intclf}^{-1}(U) \cup f^{-1}(U) = \mathrm{sclf}^{-1}(U) \subseteq A$ and $A \subseteq \mathrm{intcl}A$. We have $\mathrm{intclf}^{-1}(U) \subseteq A \cap G = f^{-1}(U)$. Therefore, $f^{-1}(U)$ is ISCS in X and hence f is I contra semi-cont. function.♦

Corollary: Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function then the following statements are equivalent:

1. f is I completely cont. function.
2. f is I pre-cont. function, IB-cont. function and I contra gs-cont. function.

Proof: 1⇒2 Suppose that V is IOS in Y then $f^{-1}(V)$ is IROS in X (by hypothesis). That is $f^{-1}(V) = \mathrm{intclf}^{-1}(V)$ imply $f^{-1}(V) \subseteq \mathrm{intclf}^{-1}(V)$ and $\mathrm{intclf}^{-1}(V) \subseteq f^{-1}(V)$ then $f^{-1}(V)$ is IPOS and ISCS in X. Now let A be IOS in X and $f^{-1}(V) \subseteq A$ then $f^{-1}(V) = A \cap f^{-1}(V)$ imply $f^{-1}(V)$ is IBS. So $f^{-1}(V) = \mathrm{sclf}^{-1}(V)$ since $\mathrm{sclf}^{-1}(V) = f^{-1}(V) \cup \mathrm{intclf}^{-1}(V)$ and $\mathrm{intclf}^{-1}(V) \subseteq f^{-1}(V)$ and hence for each IOS A in X and $f^{-1}(V) \subseteq A$ then $\mathrm{sclf}^{-1}(V) \subseteq A$. Therefore, $f^{-1}(V)$ is Igs-closed set, IBS and IPOS in X. hence f is IB-cont. function, I pre-cont. function and I contra gs-cont. function.
2 ⇒1 Suppose that U is IOS in Y then $f^{-1}(U)$ is IPOS, IBS and Igs-closed set in X (by hypothesis) then $f^{-1}(U) \subseteq \mathrm{intclf}^{-1}(U)$ and $f^{-1}(U) = A \cap G$ where A is IOS containing $f^{-1}(U)$ in X and G is ISCS in X so $\mathrm{sclf}^{-1}(U) \subseteq A$ since $f^{-1}(U)$ is Igs-closed set. Now $\mathrm{intclf}^{-1}(U) = \mathrm{intcl}(A \cap G) \subseteq \mathrm{int}(\mathrm{cl}A \cap \mathrm{cl}G) = \mathrm{intcl}A \cap \mathrm{intcl}G \subseteq \mathrm{intcl}A \cap G$ since G is ISCS so $\mathrm{intclf}^{-1}(U) \cap A \subseteq \mathrm{intcl}A \cap A \cap G$ since $\mathrm{intclf}^{-1}(U) \cup f^{-1}(U) = \mathrm{sclf}^{-1}(U) \subseteq A$ and $A \subseteq \mathrm{intcl}A$ then $\mathrm{intclf}^{-1}(U) \subseteq A \cap G = f^{-1}(U)$. Hence $f^{-1}(U)$ is ISCS in X, then we have $\mathrm{intclf}^{-1}(U) \subseteq f^{-1}(U)$ and $f^{-1}(U) \subseteq \mathrm{intclf}^{-1}(U)$ imply $f^{-1}(U) = \mathrm{intclf}^{-1}(U)$. Therefore, $f^{-1}(U)$ is IROS in X and hence f is I completely cont. function.♦

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حول الدوال المستمرة المعاكسة وعلاقتها مع انواع اخرى من الدوال المستمرة المعممة بين الفضاءات التبولوجية الحدسية

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الخلاصة

سندرس في هذا البحث مفهوم الدوال المستمرة المعاكسة (contra continuous) بكل انواعها (contra semi continuous, contra g-continuous,...) وتعميمها بين الفضاءات التبولوجية الحدسية وكذلك سندرس علاقة هذه الدوال مع بعضها وكذلك سندرس علاقة هذه الدوال مع أنواع أخرى من الدوال المستمرة منها الدوال المستمرة تماماً (perfectly continuous) والدوال المستمرة الواهنة (slightly continuous)...الخ.