Solvability Conditions For A system Of Nonhomogenous Differntial Equations Of The First Order

AKRAM.H.MAHMOOD
LAMYAA.H.SAADOON & RAID.S.KARYAKOS
Department Of Mathematics/College Of Education
University of Mosul

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الملخص

يتضمن هذا البحث دراسة شروط قابلية الحل لنظام من المعادلات التفاضلية غير المتجانسة ومن الرتبة الاولى مع الشروط الحدودية وذلك باستخدام الطريقة المعطاة في [3,4]

Abstract

In this paper we study the solvability conditions for a system of Nonhomogenous ordinary differential equations of the first order with boundary conditions by using the method which is given by [3,4].

1-Introduction :

The author [2] investigated the solvability conditions for certain eigenvalue problems by using perturbation method .

Also [1] investigated the solvability conditions of boundary value problem for partial differential equations of the 2^{nd} order by using the same method which is given by [3] .

Our work is to find the solvability conditions for a system of the first order of differential equations by using the perturbation method which is given by [3] .

We consider the following system of differential equations

$$\frac{dy}{dx} - A(x)y = f(x) \qquad (1.1)$$

with boundary conditions

$$y_i(a) = \beta_i \qquad \text{for} \quad i=1,2,\ldots,m \qquad (1.2)$$

$$y_i(b) = \beta_i \qquad \text{for} \quad i=m+1,m+2,\ldots,n$$

where y and f are column vectors with n components,A is an n×n matrix, and β_i are constants .

2-Main Results:

In this section we formulate the main results by depending on the following theorem :

Theorem :

The necessary conditions for solvability a system of ordinary differential equation (1.1) with conditions (1.2) is :

$$\beta_{m+1} z_{m+1}(b) + \beta_{m+2} z_{m+2}(b) + ... + \beta_n z_n(b) - \beta_1 z_1(a) - \beta_2 z_2(a) - ... - \beta_m z_m(a) = \int_a^b z^T f d(x) \qquad (2.1)$$

Proof :

We consider the solvability conditions for the problem

$$\frac{dy}{dx} - A(x)y = f(x) \qquad (2.2)$$

Is

$$y_i(a) = \beta_i \qquad \text{for} \quad i=1,2,\ldots\ldots\ldots,m \qquad (2.3)$$

$$y_i(b) = \beta_i \qquad \text{for} \quad i=m+1,m+2, \ldots\ldots,n$$

where y and f are column vectors with n components, A is an n×n matrix, and β_i are constants .

We assume that the corresponding homogenous problem of (2.2,2.3) has a nontrivial solution, so that the nonhomogenous problem will have a solution only if solvability conditions are satisfied.

To determine the solvability conditions we multiply (2.2) from the left with z^T where z^T is the transpose of the adjoint column vector z with n components.

Thus ,we have :

$$z^T \frac{dy}{dx} dx - z^T A y = z^T f$$

which ,upon integration from x = a to x = b gives :

$$\int_a^b z^T \frac{dy}{dx} dx - \int_a^b z^T A y dx = \int_a^b z^T f dx \qquad (2.4)$$

Integration by parts the first integral on the left- hand side of (2.4) we find :

$$z^T y\Big|_a^b - \int_a^b \frac{d}{dx} z^T y dx - \int_a^b z^T A y dx = \int_a^b z^T f dx$$

or

$$z^T y\Big|_a^b - \int_a^b \left(\frac{d}{dx} z^T + z^T A\right) y\,dx = \int_a^b z^T f\,dx \qquad (2.5)$$

The adjoint equations are defined by setting the coefficient of y in the integrand in (2.5) equal to zero and obtaining

$$\frac{d}{dx} z^T + z^T A = 0$$

Taking the transpose, we have :

$$\left(\frac{d}{dx} z^T\right)^T + \left(z^T A\right)^T = 0$$

or

$$\frac{dz}{dx} + A^T z = 0 \qquad (2.6)$$

Comparing (2.6) and (2.2) ,we conclude that the differential equations are self- adjoint if $A=-A^T$.

To determine the boundary conditions on z,we consider the corresponding homogenous problem by setting f =0 in (2.5) ,we obtain :

$$z^T y\Big|_a^b = 0 \qquad (2.7)$$

or

$$\left[z_1 y_1 + z_2 y_2 + ... + z_n y_n\right]_a^b = 0 \qquad (2.8)$$

Putting $\beta_i = 0$ in (2.3) and substituting the result into (2.8) ,we have :

$$z_1(b) y_1(b) + z_2(b) y_2(b) + ... + z_m(b) y_m(b) - z_{m+1}(a) y_{m+1}(a) - \\ - z_{m+2}(a) y_{m+2}(a) - .... - z_n(a) y_n(a) = 0 \qquad (2.9)$$

We define the adjoint boundary conditions such that each of the coefficient of the y_i(b) for i=1,2,3,----,m and the $y_i(a)$ for i=m+1,m+2,---,n equal to zero ,so that the result is :

$z_i(a) = 0$ for i=m+1, m+2,…….,n (2.10)
$z_i(b) = 0$ for i=1,2,…………...,m

Returning to the nonhomogenous problem ,we substituting (2.3), (2.6) and (2.10) into (2.5) we find:

$$\beta_{m+1}\mathbf{z}_{m+1}(b)+\beta_{m+2}\mathbf{z}_{m+2}(b)+\ldots+\beta_n\mathbf{z}_n(b)-\beta_1\mathbf{z}_1(a)-\beta_2\mathbf{z}_2(a)-\ldots-$$
$$-\beta_m\mathbf{z}_m(a)=\int_a^b \mathbf{z}^T f\,dx$$

As the desired solvability conditions .

EXAMPLE :

To illustrate the procedure ,we will specify the following system differential equations :

$$\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} - \begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = \begin{pmatrix} 0 \\ x \end{pmatrix}$$

with the boundary conditions

$$y_1(0)=0.25$$
$$y_2(\pi/2)=0.75$$

$$[z_1 \quad z_2]\begin{pmatrix} y_1' \\ y_2' \end{pmatrix} - [z_1 \quad z_2]\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} = [z_1 \quad z_2]\begin{pmatrix} 0 \\ x \end{pmatrix}$$

$$[z_1 \quad z_2]\begin{pmatrix} y_1 \\ y_2 \end{pmatrix}\Bigg|_0^{\pi/2} - \int_0^{\pi/2}\left[z_1' \quad z_2' \right] + [z_1 \quad z_2]\begin{pmatrix} 0 & 1 \\ -1 & 0 \end{pmatrix}\begin{pmatrix} y_1 \\ y_2 \end{pmatrix} dx = \int_0^{\pi/2}[z_1 \quad z_2]\begin{pmatrix} 0 \\ x \end{pmatrix} dx$$

$$\begin{bmatrix} z_1' \\ z_2' \end{bmatrix} + \begin{pmatrix} 0 & -1 \\ 1 & 0 \end{pmatrix}\begin{bmatrix} z_1 \\ z_2 \end{bmatrix} = 0$$

$$z_1 y_1 + z_2 y_2 \Big|_0^{\pi/2} = 0$$

$$z_1(\pi/2)\, y_1(\pi/2) - z_2(0)\, y_2(0) = 0$$

$$z_1(\pi/2) = 0$$

$$z_2(0) = 0$$

$$0.75\, z_2(\pi/2) - 0.25\, z_1(0) = \int_0^{\pi/2} [z_1 \quad z_2] \begin{pmatrix} 0 \\ x \end{pmatrix} dx$$

$$0.75\, z_2(\pi/2) - 0.25\, z_1(0) = \int_0^{\pi/2} x\, z_2(x) dx$$

$$z_1(x) = \cos(x)$$

$$z_2(x) = -\sin(x)$$

$$z_1(0) = 1$$

$$z_2(\pi/2) = -1$$

If we compare with (2.1) ,the desired solvability conditions is

$$-0.75 - 0.25 = -\int_0^{\pi/2} x \sin(x) dx$$

$$-1 = -1$$

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