

# Finite Element Analysis of Wave Barriers Used to Reduce Train Induced Vibrations

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## Abstract

The finite element method is used to simulate the soil vibration behavior due to the Basrah-Baghdad passenger train and its effect on a targeted building in the Al-Ma'qal quarter, Basrah governorate. Three-dimensional dynamic elastic analyses are performed to calculate the particle velocities for a train speed of 120 km/hr. The effectiveness of screening using active (10 m long) open trench barriers with variable depth (2 m - 5 m) and width (0.4 m - 0.8 m), is being studied. For a given trench width (0.4 m), the results of the parametric study revealed a considerable effect of trench depth where the screening capability near the trench is increased by (10.4 %, 26.1 %, 36.3 %) due to a (50 %, 100 %, 150 %) increase in depth. The results are less sensitive to the variation in trench width. The screening capability of a double open (0.4 m × 10 m × 2 m) trench system was also investigated, where a mitigation improvement of (36.4 %) was achieved. The vibration mitigation using single and double trench systems, filled with (40 %) rubber content mixture, was also analyzed. It is concluded that using the additional passive trench increases the mitigation of the single system by around 19.1 %. An important finding is that the (40 % rubber + 60 % native cohesive soil) mixture proved to be a good filling material, since the infilled-trench systems produced comparable screening ratios to the open systems, where (97.7 %) and (85.4 %) were accomplished for the single and double systems, respectively.

**Keywords:** Train induced vibration, Vibration mitigation, Vibration screening, In-filled trench barriers, Finite element analysis.

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## 1. Introduction

Nowadays, the issue of train-induced vibrations and their effects on human, sensitive equipment, and buildings have received special attention, especially from the environmental standpoint. Four main phases for the transmission of vibrations can be described: generation; transmission; reception; and interception [1]. This research will concentrate on transmission (i.e. wave movement via the underlying ground) and interception (i.e. vibration reduction by implementing wave barriers).

The effectiveness of the wave's barriers is assessed according to how much reduction of the soil particle response amplitude will be achieved. Amplitude reduction factor ( $A_r$ ) is determined by dividing the amplitude of vertical ground motion after trench installing, at a selected point, by its counterpart at the same point before trench placement [2]. The ratio of amplitude reduction is then given by:

$$A_r = \frac{(A_r)_{After}}{(A_r)_{Before}} \quad (1)$$

Many research articles were published regarding the numerical simulation of the problem. Some of them utilized the combined finite element/boundary element method

(e.g. [1], and [3]-[7]), others used the finite element method (e.g. [8]-[24]).

Single/multiple open/infilled trench barriers were analyzed for various train speeds/frequencies. There is a consensus that, the open trench barriers are the most effective way in controlling the ground vibration (e.g. [3]-[6], [11]-[14], [17] and [24]). The multiple trench systems exhibited higher level of efficiency compared to the single trench mitigation system (e.g. [18]-[19] and [22]). The screening efficiency was proportional to the train speed (e.g. [22] and [24]) and it is more pronounced at the critical speed [4].

Extensive parametric studies were conducted regarding trench geometric configurations, trench location, and properties of the filling materials. Although, rectangular circular, triangular [11] and step-shaped [13] trench sections were proposed, the rectangular one proved to be more efficient. The depth of the trench played a significant role in the efficiency of trench barriers (e.g. [3], [6], [18], [20] and [24]-[25]). Chiang and Tsai [5] recommended an optimal trench depth of (0.3 - 0.4) times the Rayleigh wavelength. Trench width had minor effect on vibration mitigation except for extremely shallow trenches (e.g. [3], [6], [17]-[19] and [24]). Active isolation was found to be more efficient in reducing vibration (e.g. [3], [11], [20] and [25]).

Since the open trenches suffer from a stability problem and they constitute obstacles, various materials were used to fill the trenches: soil-bentonite mixture [3], [23], concrete [9]-[10],

[16], [23], rubber [9]-[10], [23], polyurethane [9], geofoam [20], [24]. It was found that, use of softer backfill materials increased the efficiency of in-filled trenches [3], [6], [14] [19], [24], whereas concrete barriers exhibited higher effectiveness in mitigating the vibration at high speeds [9], [10], [16].

The main objective of this work is to examine (numerically) the effectiveness of in-filled trenches, at different positions with respect to the train track and with variable geometric configurations, in mitigating the Basrah-Baghdad train induced vibration regarding a targeted hypothetical building. The study area is shown in Fig. 1.



Fig. 1 The study area in Al-Ma'qal quarter, Basrah province [26].

## 2. Materials and Methods

### 2.1. Train characteristics

A high-speed Chinese (CSR) train of the type (DMU) was delivered on February, 2012. It consists of two-power cars and eight-passenger cars with (343) passengers capacity and ( $\geq 160$  km/hr) speed. The train's entire length, from the first to the last axle is (249.24 m). The single passenger car's load (tare weight plus load) is (51.20 tons) and its length is (25.50 m).

### 2.2. Finite element model

The discretized equation for the time-dependent movement of a volume under the influence of a dynamic load is written as [27]:

$$[M]\{\ddot{d}\} + [C]\{\dot{d}\} + [K]\{d\} = \{F\} \quad (2)$$

Where,

$[M]$  = element mass matrix.

$\{d\}$  = displacement vector.

$\{\dot{d}\}$  = velocity vector.

$\{\ddot{d}\}$  = acceleration vector.

$[C]$  = damping matrix.

$[K]$  = element stiffness matrix.

$\{F\}$  = load vector.

The equation is integrated over time to determine the response at various time steps.

A (30 m  $\times$  90 m  $\times$  30 m) model is discretized using (10-noded tetrahedral) elements. Due to symmetry, only half of the railway embankment is modeled. The model's center-line was locked horizontally in perpendicular to the rail. The track is rested on a (1.8 m) high embankment with widths of (2 m) at the top and (4 m) at the bottom. Soil and ballast are modeled as multilayer elastic solids with the geometric configurations shown in Fig. 2.

The soil profile in the study area is drawn from [28]. The shear wave velocities for various layers are calculated based on the empirical correlation [29]:

$$V_s = 58 N^{0.39} \quad (3)$$

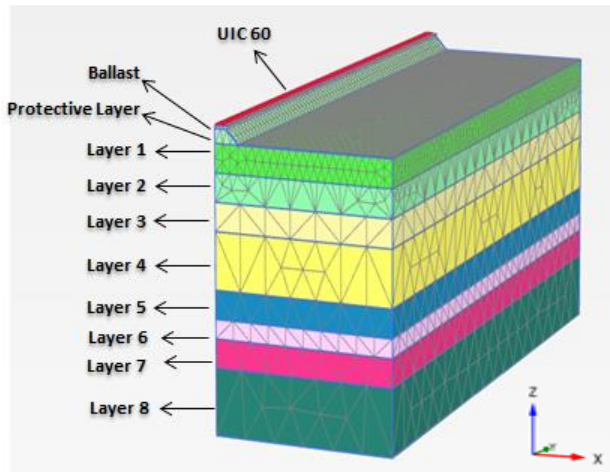


Fig. 2 The finite element model of the case study.

Table 1. Earth layer properties [28].

Layer	Depths Range (m)	Sample Description	SPT ( <i>N</i> )	$\rho$ (kg/m <sup>3</sup> )	$\nu$	$V_s$ (m/s)
Ballast	0.0 - 0.3	Crushed stone	---	1800	0.3	222
Protective Layer	0.3 - 1.8	Very dense sand	---	2200	0.25	236
Layer 1	1.8 - 4.8	Very stiff clay	16	2000	0.3	171
Layer 2	4.8 - 7.8	Stiff clay with silt	10	1900	0.3	142
Layer 3	7.8 - 10.8	Medium stiff clay	5	1750	0.3	109
Layer 4	10.8 - 16.8	Soft clay	4	1700	0.3	100
Layer 5	16.8 - 19.8	Medium stiff clay	6	18	0.3	117
Layer 6	19.8 - 22.8	Stiff clay with silty sand	14	19.5	0.3	163
Layer 7	22.8 - 24.8	Very stiff clay with silty sand	20	21	0.3	187
Layer 8	24.8 - 31.8	Very dense silty sand	> 50	22	0.3	267

Table 2. Mechanical characteristics of the railway track [8].

Parameter	Unit	Rail	Sleepers	Rail Clips
Cross Section Area	m <sup>2</sup>	$7.69 \times 10^{-3}$	$5.13 \times 10^{-2}$	----
Density $\rho$	kg/m <sup>3</sup>	7850	2400	----
Young's Modulus $E$	MPa	$210 \times 10^3$	$30 \times 10^3$	----
Moment of inertia $I_3$	m <sup>4</sup>	$3.055 \times 10^{-5}$	0.0253	----
Moment of inertia $I_2$	m <sup>4</sup>	$5.13 \times 10^{-6}$	$2.45 \times 10^{-4}$	----
$ F_{Max, tension} $	kN	----	----	312
$ F_{Max, compression} $	kN	----	----	1716
Axial Force	kN	----	----	$2 \times 10^6$

### 2.3. Load simulation scheme

The adopted model for analyzing the railway track considers the rail as a beam supported by an elastic base. Depending on train loads and speed, the moving-loads induces reactions below the various track points. The static analyses based on the (beam on the elastic foundation) theory are conducted to determine the effects of moving loads. A value of ( $K_s = 36000$  kN/m<sup>3</sup>) is adopted for the modulus of subgrade reaction as calculated based on the empirical correlation [31]:

$$K_s = 1.17 N + 17.6 \text{ (MN/m}^3\text{)} \quad (4)$$

The carriage axle loads were simulated by four-unit point loads, Fig. 3. The loading length of (32.80 m) includes

The soil profile and earth layer properties are listed in Table 1. The depth of groundwater at the site is (1.1 m) below the ground surface. The rail is modeled by rectangular cross-sectional beam elements, with its properties resembling regular (UIC60) rail. The rail clips are modeled as node to node anchor elements. The (0.6 m) spaced standard (B70) sleepers are modeled as beam elements. The standard railway track parameters listed in Table 2, are adopted in the subsequent analyses. Rayleigh damping with stiffness and proportional mass parameters ( $\alpha, \beta$ ) is used in the incremental finite element analyses [30]. The values of the coefficients adopted in the analysis, which were found based on the characteristics of the soil profile in the study area, are: ( $\alpha = 0.377, \beta = 0.003183$ ).

additional length to allow for farther influence of moving load [8].

Loads due to the moving train are predicted at points spaced at (0.3 m) along the rail beam length. This produces around (110) loaded locations per rail. The value assigned to each point charge is the load of the vertical wheel ( $P = 63$  kN). To include moving load, dynamic signals multipliers are appointed to all of these (110) point loads. The signal for each load location describes how the forces change at that particular point in the rail as the wheel load moves along. For each dynamic time step each point load has been multiplied by the amount of its signal. For each point load in, the shear forces obtained from that preliminary analysis are adopted as the dynamic multipliers.

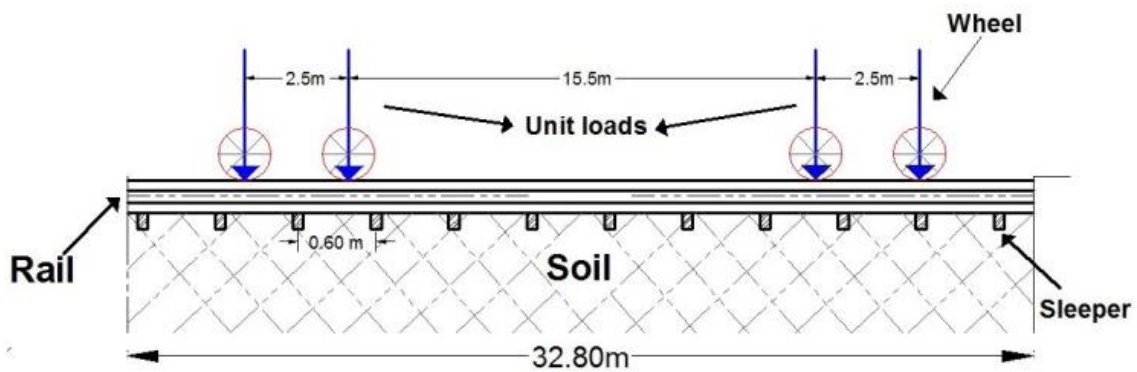


Fig. 3 Track configurations for static analyses.

For a speed of (120 km/hr), the train traverses the distance of (0.3 m) in (0.009 sec). Consequently, the train's new axle takes (0.984 sec) to traverse all of the (110) dynamic points for the loading length of (32.8 m). The distance for a DMU train between the first and last axles is (20.50 m), which takes (0.615 sec) to be traversed. The entire time for crossing the model's length is (1.599 sec). During the aforementioned time, the train's impact before entering and next exiting the model is also considered. An extra time of (0.162 sec) associated with (18) superimposed rows of the multipliers is considered to eliminate any error in the model due to the effect of representing the stress wave in dynamic predictions. Fig. 4 shows the computed time history of the rail shear force due to unit moving loads.

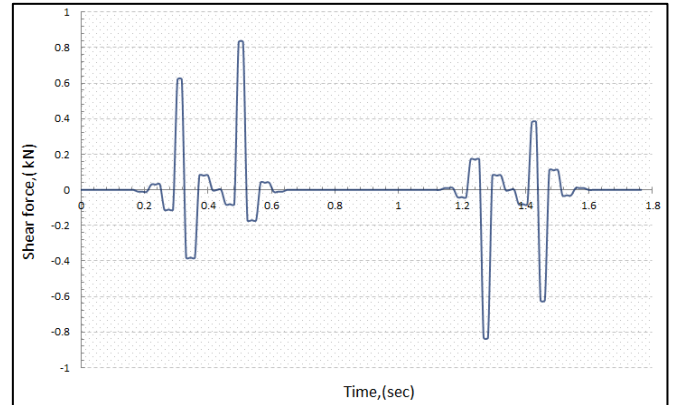


Fig. 4 Distribution of shear force versus dynamic analytical time.

Table 3 shows a partial sequence of multipliers, whereas graphical representations of train locations over time are illustrated in Fig. 5.

Table 3. Distribution of point load multipliers.

Time Steps	Time (s)	Multiplier 1	Multiplier 2	Multiplier 3	Multiplier 4	Multiplier 5	Multiplier 110
1	0.01378	0	0	0	0	0	0
2	0.02756	0	0	0	0	0	0
3	0.04134	0	0	0	0	0	0
4	0.05512	0	0	0	0	0	0
5		0	0	0	0	0	0
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
13	0.16536	0	0	0	0	0	0
14	0.17914	- 0.01	0	0	0	0	0
15	0.19292	- 0.01	- 0.01	0	0	0	0
16	0.2967	- 0.01	- 0.01	- 0.01	0	0	0
17	0.22048	0.03	- 0.01	- 0.01	- 0.01	0	0
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
•	•	•	•	•	•	•	•
125	1.70872	0	0	0	0	0	- 0.01
126	1.72523	0	0	0	0	0	0.03
127	1.74223	0	0	0	0	0	0.03
128	1.764	0	0	0	0	0	0.03



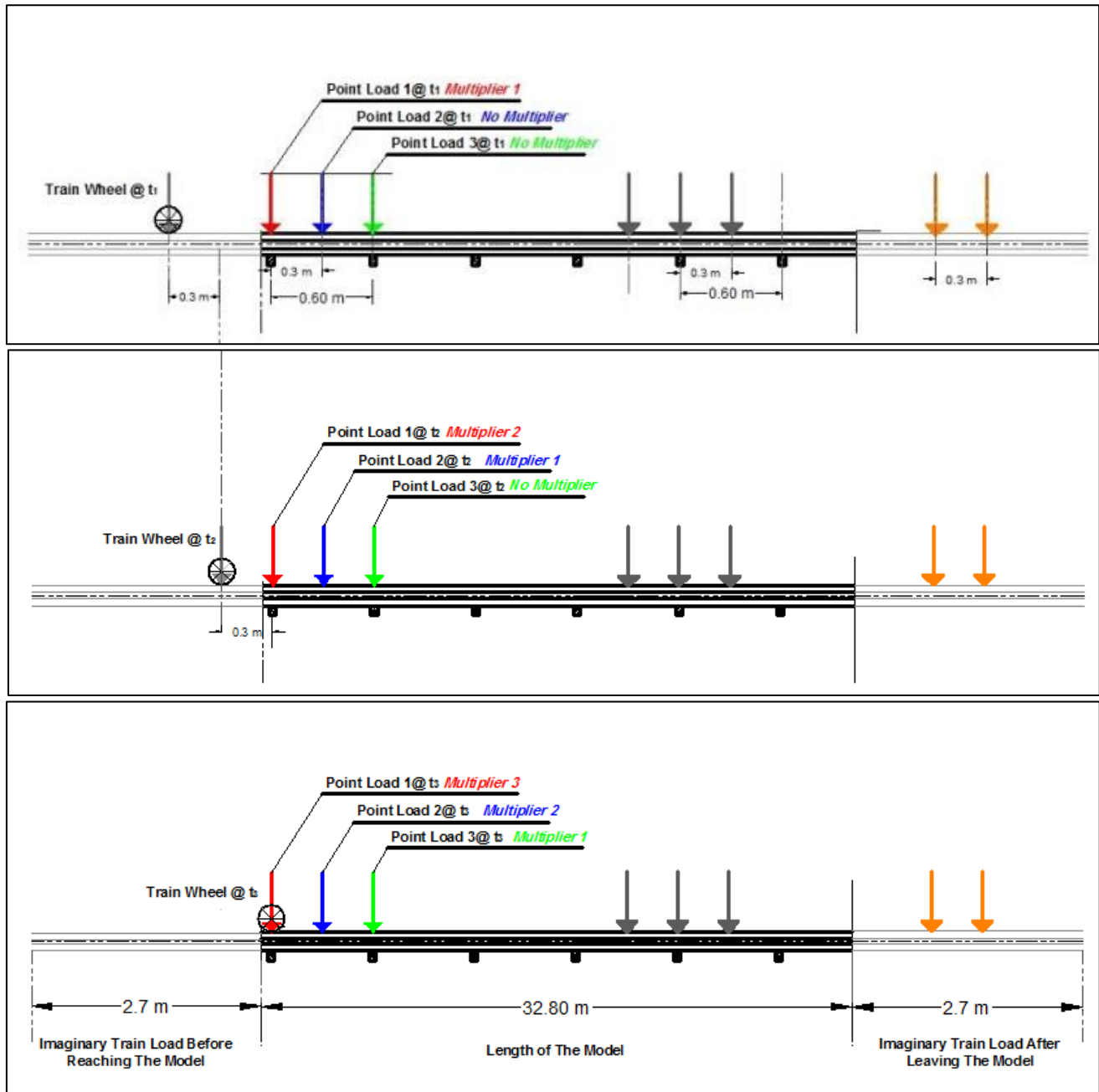


Fig. 5 Graphical definition of a succession of multipliers for (110) point loads in the model.

Figure 6 displays the finite element model at the track and dynamic point loads. Several point loads are deactivated for the sake of clarity.

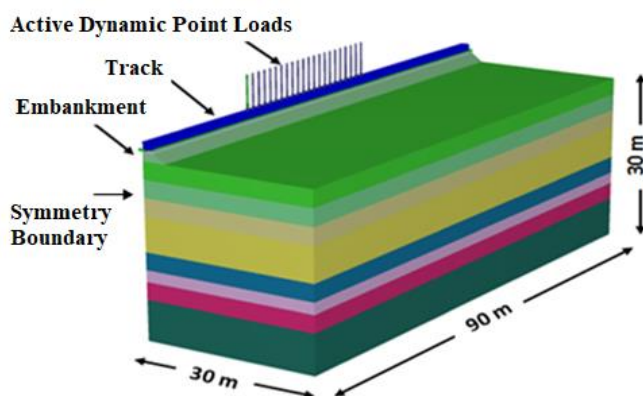


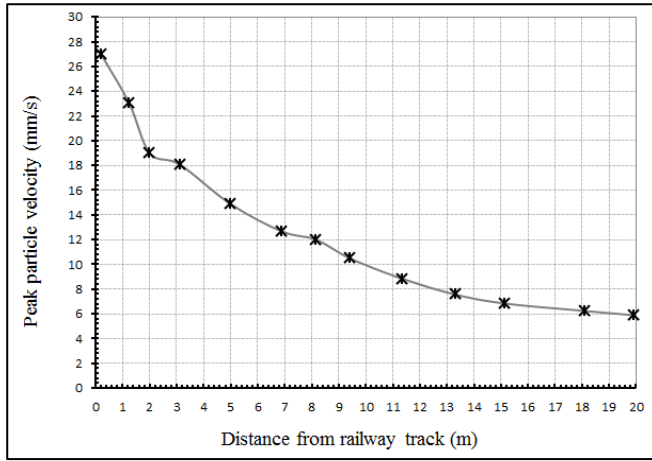
Fig. 6 Numerical model developed for moving load distribution.

### 3. Results and Discussion

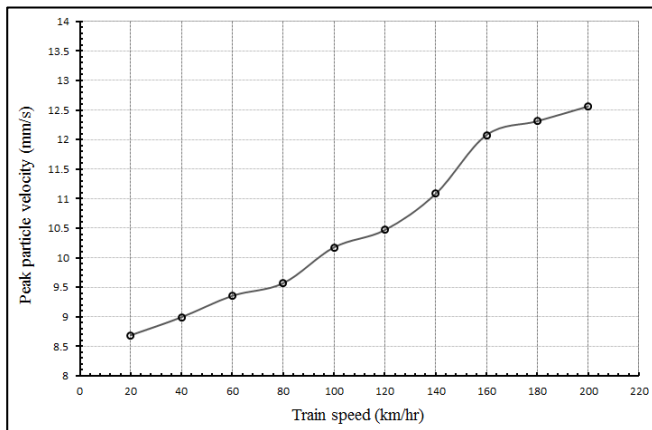
#### 3.1. No barrier

The results of the numerical analyses revealed the attenuation of peak particle vertical velocity at the ground surface, with the distance from the railway track for a specified observation path (perpendicular to the railway track), as shown in Fig. 7. This finding is consistent with the results obtained from previous studies [23].

The effect of changing the speed of a single (DMU) train carriage on the resulting vibration values is also investigated. It is realized from Fig. 8 that, the peak particle velocity at a certain location is proportional to the train velocity. This conclusion also agrees with the previous studies [11].



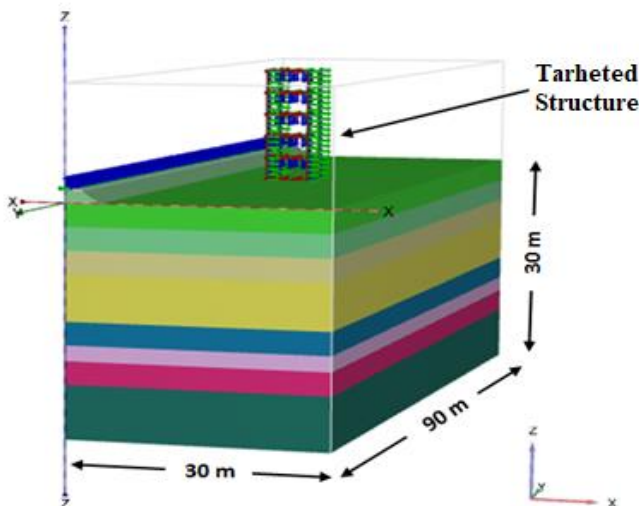
**Fig. 7** Peak particle velocities (PPV) at ground surface vs. distance (train speed = 120 km/hr).



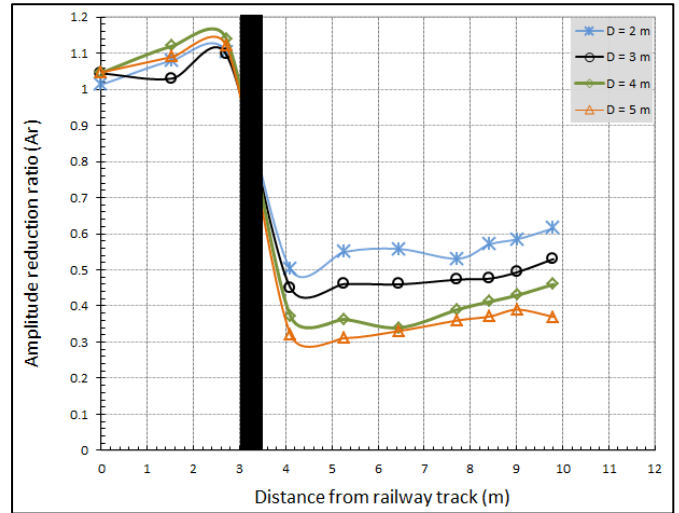
**Fig. 8** Impact of train speed on velocity amplitude of ground surface at (9.43 m) from source of vibration (railway track).

### 3.2. Open-trench barriers

The efficiency of using open-trench barriers, in mitigating the vibrations reaching a targeted hypothetical structure at (10 m) from the railway track is investigated, as shown in Fig. 9. The efficacy of utilizing trenches with plan dimensions of (0.4 m  $\times$  10 m) and various depths (2 m, 3 m, 4 m, and 5 m), located at (3 m) from the railway track (active isolation), are illustrated in Fig. 10.

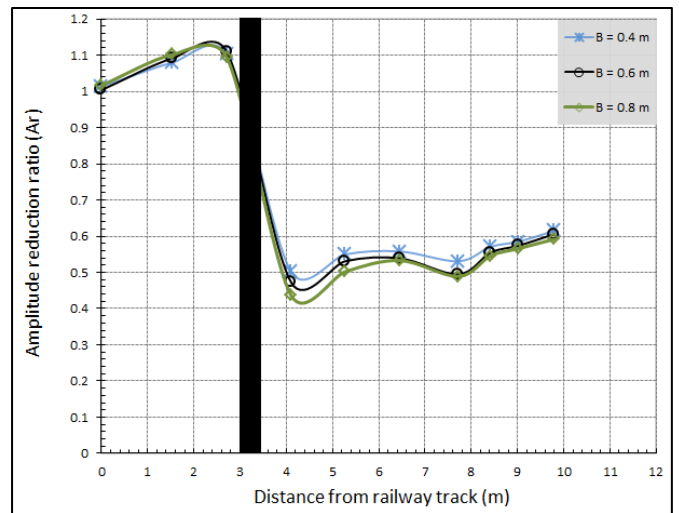


**Fig. 9** A finite element model showing the targeted structure.



**Fig. 10** Efficacy of active isolation using open-trenches with various depths (trench dimensions = 0.4 m  $\times$  10 m, train speed = 120 km/hr).

Although, the presence of trenches increases the amplitudes at the locations before them, where a similar behavior was reported by Sun et al. [7], they achieve great reductions at the locations beyond them. The screening efficacy is decreased as the distance beyond the trench is increased. The active isolation by (2 m) deep trenches with various widths (0.4 m, 0.6 m, and 0.8 m) is also studied. The results are presented in Fig. 11.



**Fig. 11** Efficacy of active isolation using open-trenches with various widths (trench depth = 2 m, train speed = 120 km/hr).

To achieve good screening efficiency without using deep trenches, two (0.4 m  $\times$  10 m  $\times$  2 m) open-trenches are used, active one at (3 m) from the railway track and passive one at (7 m) from the railway track. The results are presented in Fig.12.

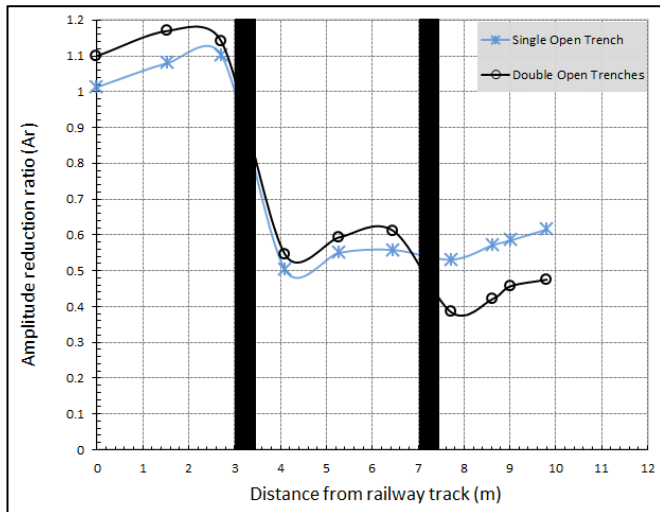


Fig. 12 Efficacy of isolation using double open-trenches (trench width = 0.4 m, depth = 2 m, train speed = 120 km/hr).

### 3.3. In-filled trench barriers

The properties of the filling material are listed in Table 4. The vibration mitigation capability of that material was assessed through an unpublished field experimental program conducted by the authors.

Table 4. The predicted properties of the filling material.

Material	$\gamma$ (kN/m <sup>3</sup> )	$\nu$	$E$ (MPa)	$V_s$ (m/s)
(60 % clay + 40 % rubber) mixture.	10.14	0.31	4	38

Double screening system using infilled-trenches is also analyzed, Fig. 13.

Fig. 14 compares the responses due to using single active and double infilled-trenches as mitigation systems.

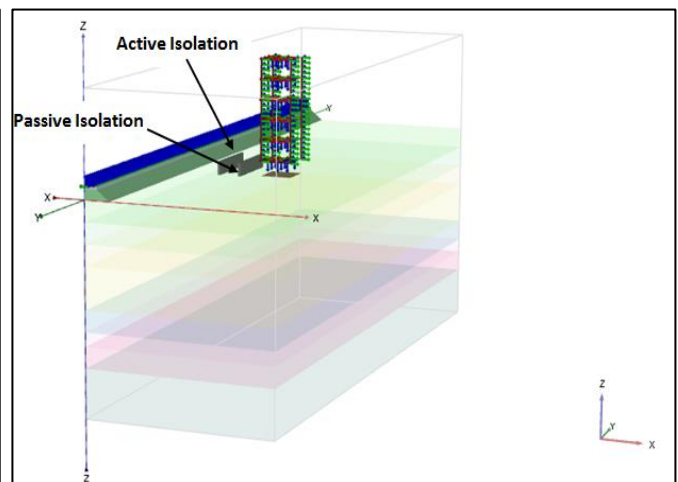
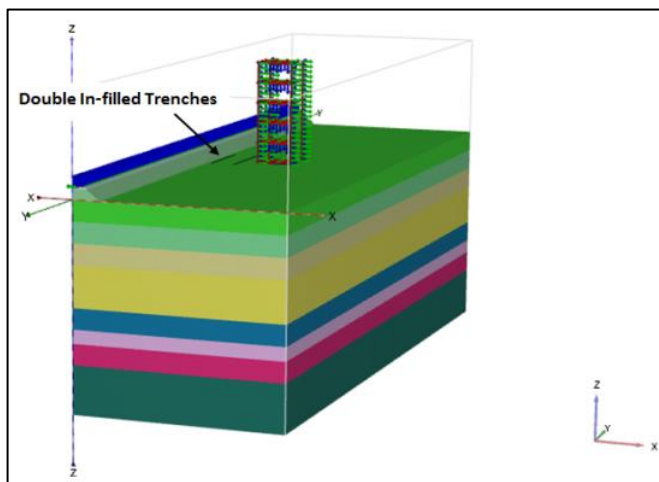


Fig. 13 Finite element models for double in-filled trenches (trench width = 0.4 m, depth = 2 m, train speed = 120 km/hr).

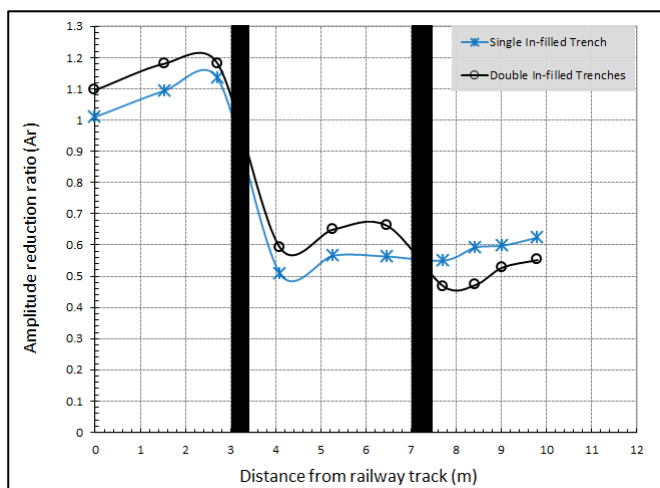


Fig. 14 Efficacy of isolation using single and double in-filled-trenches (trench width = 0.4 m, depth = 2 m, train speed = 120 km/hr).

Fig. 15 shows the comparison between the previous analysis results using an open trench and an infilled trench near the source of the vibration. Fig. 16 shows the comparison among the results of the previous analysis using double open trenches and another infilled, one near the source of vibration

and the other near the point from which the vibrations are to be isolated.

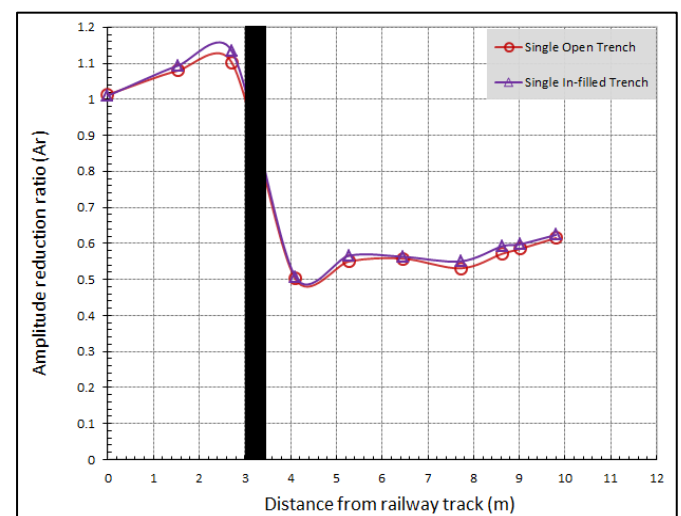
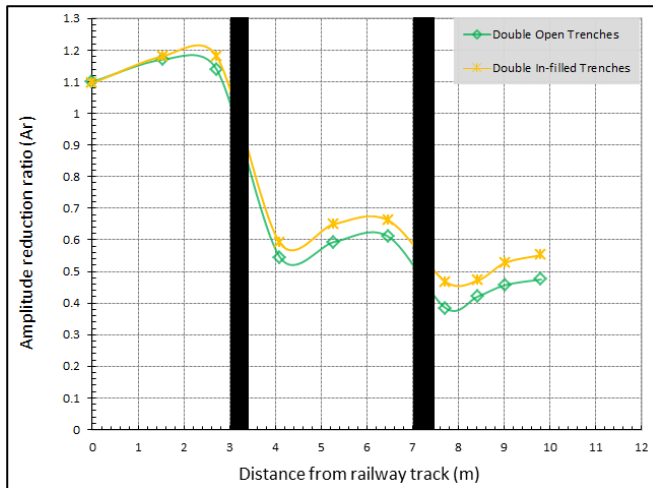


Fig. 15 Comparison of reduction ratios using single open and in-filled-trenches (trench width = 0.4 m, depth = 2 m, train speed = 120 km/hr).



**Fig. 16** Comparison of reduction ratios using double open and in-filled-trenches (trench width = 0.4 m, depth = 2 m, train speed = 120 km/hr).

#### 4. Conclusions

1. The effect of open-trench depth is apparent from Fig. 10, where screening improvements by (10.4%, 26.1%, 36.3%) are achieved adjacent to trench via increasing the depth by (50 %, 100 %, 150 %), whereas the limited effect of trench width on the mitigation ratio is noted from Fig. 11. This is consistent with the findings from previous studies [3], [6], [17]-[19] and [24].
2. Figure 12 reveals the accomplishment of good improvement in isolation efficiency due to using the double open trench screening system.
3. It is realized from Fig. 14 that, around (19.1 %) increase in mitigation is achieved by adding the in-filled passive trench.
4. The results presented in Fig. 15 and Fig. 16, support the feasibility of the infilled-trench systems as they exhibit excellent screening, compared to the open ones, where (97.7 %) is achieved for the single system and (85.4 %) is achieved for the double system.

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