AL KUT JOURNAL OF ECONOMIC AND ADMINISTRATIVE SCIENCES
Publisher: College of Economics and Management - Wasit University

A Hybrid Meta-Heuristic Algorithm to Solve Four-Dimensional Transportation Problems

G. Shirdel (1) H. Algelehawy (2) A. Ramzy (3)

shirdel81math@gmail.com haiderabdulrdha@gmail.com ali.ramzi@gmail.com

[1,2,3] Department of Mathematics and Computer Sciences, University of Qom, Qom, Iran

Abstract

This paper presents the development of a hybrid meta-heuristic algorithm that combines the capabilities of a Genetic Algorithm and an Ant Colony Algorithm to address the complex challenges of non-classical four-dimensional transport problems. The study aims to improve transportation efficiency and reduce associated costs through an optimization strategy that uses genetic diversity and optimization mechanisms derived from Ants' behavior. The experimental approach methodology is designed and implemented using a hybrid algorithm on a test data set derived from realistic transportation scenarios, specifically focusing on measuring cost improvements. The main results revealed the ability of the hybrid algorithm to determine the optimal strategy and provide more efficient solutions at a lower cost than traditional algorithms. The research's impact on the field of meta-heuristic algorithms is significant, as it introduces a new approach that enhances the efficiency of transportation and logistics in various sectors, thereby opening new horizons for research and development in this area. The study also recommends further research, particularly the exploration of integrating the hybrid algorithm with machine learning techniques to increase its ability to predict and adapt to changes in transportation data .

Keywords: ant colony optimization, four-dimensional transportation problem, genetic algorithms, hybrid algorithms, meta-heuristic optimization.

1- Introduction

The four-dimensional transportation problem (4DTP) is a non-classical and NP-hard transportation problem faced by companies and institutions in logistics and logistics chain management. The problem is the distribution of materials from a group of sources to a group of demand centres, with several dimensions affecting the transportation process, such as the number of sources, demand centres, types of materials, available means of transportation, etc., where the four-dimensional transportation problem is considered one of the complex problems due to its complexity and the number of variables and the restrictions it includes (Alqussi, 2015), given this complexity, it has become necessary to use advanced techniques that combine different search and optimization algorithms. Hence, the importance of hybrid meta-heuristic algorithms emerged by combining the properties of multiple algorithms, such as the ant colony algorithm and the genetic algorithm, a balance can be achieved between global and local search, allowing for the discovery of highly efficient solutions while avoiding falling into local sub-solutions. Therefore, hybrid algorithms emerge as a promising methodology capable of dealing with the unique complexities of four-dimensional transportation problems and providing innovative solutions with high efficiency. The background of the current research reflects the tremendous developments witnessed in the fields of operational research and artificial intelligence in developing algorithms used to solve complex optimization problems, especially in the field of logistics and transportation. Four-dimensional transportation problems, which include space, time, quantity and cost, are among the most complex and essential due to their direct impact on the efficiency and economics of global supply chains, they require a sophisticated and flexible approach to deal with their unique challenges as a complex, multi-dimensional research issue. Historically, efforts have focused on developing strategies to improve transportation efficiency and reduce associated costs. However, advances in operational research and artificial intelligence have opened new avenues for innovation in this field, one of which is exploring advanced meta-heuristics as powerful tools to address transportation problems (Hillier, 2012).

The current research problem lies in the main challenge of developing an algorithm capable of efficiently handling the complexities of the four-dimensional transportation problem while reducing costs and increasing efficiency in a way that traditional models or individual algorithms have not achieved. The research aims to develop and test a hybrid meta-heuristic algorithm that combines the advantages of the ant algorithm and the genetic algorithm to improve solutions to the four-dimensional transportation problem. The motivation behind this research stems from the urgent need for optimization solutions that can contribute to reducing the cost of transportation in supply chains, leading to tangible improvements in environmental sustainability and economic efficiency. In this context, the hybrid algorithm of this study stands out as a new approach that combines the characteristics and advantages of both the genetic algorithm and ant colony algorithm, not only to deal with the inherent complexity of transportation problems but also to achieve outstanding cost efficiency, this assumes that the hybrid meta-heuristics algorithm can achieve good performance in addressing the complex challenges of 4DTP.

By developing a hybrid approach that combines the global and local search power of both the ant algorithm and the genetic algorithm, the current research contributes innovatively, providing more efficient and cost-effective solutions. The importance of the study also lies in its being an essential step towards improving the technologies used in logistics and transportation. It offers promising possibilities for application in various industries and fields that rely heavily on transportation and distribution. The research mainly targets academics and specialists in logistics and supply chain management and logistics software developers, providing innovative insights and solutions to complex transportation challenges. Government agencies concerned with planning and managing transportation infrastructure can also benefit from the research in developing policies and strategies that enhance the efficiency of transportation systems. National and local.

Among the study's most significant findings was the ability of the hybrid algorithm to significantly reduce the transfer cost compared to traditional algorithms used individually, as the test data set used to conduct simulation experiments on the hybrid algorithm. Experimental tests of the proposed algorithm showed promising results, as it significantly reduced costs compared to traditional algorithms, whether the genetic or ant colony algorithms.

This research consists of the following parts: After presenting the research summary and introduction in the first section, the second section explains the methods used in developing the proposed hybrid algorithm. The third section includes the theoretical aspect that reviews the mathematical and technical basis of the four-dimensional transportation problem, while the fourth section is devoted to the practical aspect, where the proposed algorithm is applied to simulated data to solve the problem. The fifth section provides a review of previous studies related to transportation problems and applications of meta-heuristic algorithms. The results extracted from the application of the algorithm are presented in the sixth section, while the seventh section is devoted to discussing and analyzing the results. The conclusions in the eighth section address the conclusion of the research, and the recommendations in the ninth section present suggestions for future studies. At the end of the research, the approved references are listed.

2- Methods:

The hybrid algorithm in this research is designed by integrating the Genetic Algorithm and the Ant Colony Algorithm to achieve a balance between local and global search, which enhances the efficiency of the proposed solutions. The Genetic Algorithm takes advantage of the crossover and mutation mechanisms to generate new solutions and avoid falling into local sub-solutions. At the same time, the Ant Algorithm contributes to directing the search towards optimal solutions using updated pheromone levels based on the discovered solutions' quality. To implement this algorithm, the Python programming language was used due to the powerful tools and libraries it provides that support complex mathematical and analytical operations. We relied specifically on the NumPy library to facilitate mathematical operations and matrix processing, and the SciPy library to improve operations and expand the scope of scientific calculations. These libraries contributed to accelerating data processing and facilitating the integration of algorithms within an effective programming environment while considering computational accuracy to achieve the best results.

In this research, a comprehensive methodology was adopted to develop a Hybrid Meta-Heuristics algorithm that aims to solve four-dimensional transportation problems. The algorithm was designed and developed through several main steps:

1. Problem Definition: The basic data of the problem were determined, including sources of supply, target centers, types of materials, means of transportation, and various constraint requirements such as cost, time, and sustainability.
2. Model Design: The mathematical model was designed to contain an objective function that aims to minimize the total cost, taking into account constraints such as material availability and center needs, in addition to the possibility of adding additional constraints depending on the nature of the problem.

3. Algorithm Integration: Ant Algorithms and Genetic Algorithms were integrated to achieve high performance in improving solutions. The ant algorithm was used to update the pheromone levels on the effective paths, while the genetic algorithm used mating and mutation mechanisms to explore the solution space more broadly.
4. Implementation in Python Environment: The algorithm was implemented using the Python programming language, as it provides powerful libraries such as NumPy and SciPy that support complex calculations. The code was developed to achieve integration between the different algorithms, using the ant algorithm pheromone levels and the hybridization and mutation mechanisms from the genetic algorithm to improve the solutions.
5. Algorithm Parameter Tuning: The algorithm parameters were calibrated based on preliminary experiments to ensure optimal performance, including population size, number of iterations, and hybridization and mutation probabilities.

The full practical application of these steps and the implementation details of the algorithm will be explained in the application section, where experiments and results of applying the proposed algorithm on hypothetical data representing the four-dimensional transportation problem are presented.

3- Theoretical Part:

3-1- Problem statement:

The problem of four-dimensional transportation is a complex and challenging problem that has received attention in various research studies and scientific fields, such as operations research, mathematics, computer science, and others. This problem involves multiple objectives, elements, and dimensions, making optimizing and designing transportation methods problematic. The 4DTP poses significant challenges due to its multi-objective nature and uncertainties. Researchers have explored different dimensions of this problem, including fuzzy uncertainty, multi-indicator formulas, practical applications in industries, etc. By taking advantage of advanced mathematical models, numerical solutions, and optimization algorithms, it is possible to address the complexities of the 4DTP and enhance the efficiency of transportation systems (Hedid & Zitouni, 2020), (Samanta et al., 2024).

3-2- Meta-heuristic Algorithms

Meta-heuristic algorithms are among the most powerful tools in operations research and complex systems analysis, especially when dealing with problems with a large search space and challenges related to global and local optimization. These algorithms are based on principles inspired by nature and social interactions, such as the Ant Colony Algorithm and the Genetic Algorithm, each of which provides innovative mechanisms for exploring the solution space efficiently. Meta-heuristic algorithms are characterized by their ability to strike a balance between exploring possible solutions and exploiting the best possible solutions, using various strategies such as crossover, mutation, and pheromone updating. The ant algorithm, for example, works through a pheromone diffusion mechanism that allows “virtual” ants to direct the search path towards the most efficient solutions, while the genetic algorithm relies on gene diversity to generate new and independent solutions and avoid falling into local sub-solutions (Salcedo-Sanz et al., 2006).

Recent studies have demonstrated the ability of these algorithms to achieve near-optimal solutions to a wide range of complex problems that are difficult to solve using traditional methods. Given that the 4DTP requires a multidimensional approach to balance transportation costs, sustainability constraints, and limited capacity, meta-heuristics are an ideal solution because they allow researchers and engineers to explore the research space with great flexibility and efficiency, enhancing their ability to provide integrated solutions that fit the diverse complexities of this problem. The strategies used in these algorithms, such as hybridization and mutation in the genetic algorithm and propagation mechanisms in the ant algorithm, provide diversity in search methods and avoid falling into limited sub-solutions. These properties contribute to enhancing the overall performance of these algorithms, making them valuable tools for practical applications in advanced logistics research and applications. In this context, meta-heuristics algorithms have been adopted as a basic methodology to develop a hybrid algorithm to address the unique challenges of the 4DTP (Zhao & Wu, 2019), (Kamel & Mahdi, 2024) .

3-3- Mathematical model

To address the mathematical modelling of the four-dimensional transportation problem, we can extract insights from various references that discuss multi-dimensional and multi-objective transportation problems, and from those references, we extract a complex linear programming model that defines the general framework of the four-dimensional transportation problem, as it includes variables representing warehouses, markets, product types, various means of transportation. Each variable is carefully specified to ensure a comprehensive understanding of the dynamics of the problem, and these variables are used to construct an objective function that seeks to minimize overall cost and improve efficiency. The mathematical model of the four-dimensional transportation problem of the research problem is as follows (Bakhayt, 2016), (Halder Jana et al., 2019), (Williams, 2013).

$$\textbf{\textit{Objective function}}: \textbf{Min. } Z \sum_{i=1}^{s}\sum_{j=1}^{w}\sum_{k=1}^{c}\sum_{q=1}^{f} Z_{ijkq}\ E_{ijkq} \qquad (1)$$

$$s.to\ .$$

$$\sum_{i=1}^{s}\sum_{j=1}^{w}\sum_{k=1}^{c} E_{ijkq} = m_q\,, \qquad \forall\ q = 1,\dots,f \qquad (2)$$

$$\sum_{j=1}^{w}\sum_{k=1}^{c}\sum_{q=1}^{f} E_{ijkq} = n_i\,, \qquad \forall\ i = 1,\dots,s \qquad (3)$$

$$\sum_{i=1}^{s}\sum_{k=1}^{c}\sum_{q=1}^{f} E_{ijkq} = f_j\,, \qquad \forall\ j = 1,\dots,w \qquad (4)$$

$$\sum_{i=1}^{s}\sum_{j=1}^{w}\sum_{q=1}^{f} E_{ijkq} = L_k\,, \qquad \forall\ k = 1,\dots,c \qquad (5)$$

$$\textstyle\sum_{i=1}^{s} m\,(i) = \sum_{j=1}^{w} n\,(j) = \sum_{q=1}^{f} f\,(q) \qquad (6)$$

$$\textit{Additional Restriction} \quad E_{ijkl} \geq 0 \quad \forall \quad i,j,k,q \qquad (7)$$

The mathematical model of a transportation problem can be extended by including multiple objective functions to achieve various objectives within a single framework. For example, an objective function can be added to improve time efficiency in transportation operations to minimize the time required to transport materials from warehouses to markets while adhering to timelines determined according to customer expectations. This function defines the objective using the variable. t_{ijkq}, which measures the optimal transfer time. Likewise, a second objective function can be included that focuses on minimizing distances travelled during transportation operations, where the variable. D_{ijkq} It is used to calculate and optimize the total distance travelled. This method allows the model to address multiple dimensions of efficiency in the transportation environment, providing comprehensive and integrated solutions to logistics challenges.

$$Time\ \ fuction: Min\, T \qquad \sum_{i=1}^{s}\sum_{j=1}^{w}\sum_{k=1}^{f}\sum_{q=1}^{c} t_{ijkq}\ E_{ijkq} \qquad (8)$$

$$Distance\ \ fuction: Min\, D \ \sum_{i=1}^{s}\sum_{j=1}^{w}\sum_{k=1}^{f}\sum_{q=1}^{c} D_{ijkq}\ E_{ijkq} \qquad (9)$$

The mathematical model used in this study includes multiple variables to determine the dynamics of 4DTPs. The number of supply sources is represented by s, the number of demand centres by w, the number of material types by c, and the number of different types and modes of transportation by f. The variable E_{ijkq} Determines the quantity of material of type k transported from the source i to the demand centre j using the transportation means q. While Z_{ijkq} expresses, the cost of transporting one unit of material k from the source i to the demand centre j employing transportation type q and t_{ijkq} Measures the time required for that transfer.

In terms of functions and equations, the first equation indicates the primary objective function of the model, which focuses on minimizing the total transportation costs. The second and third equations ensure that the total quantities transferred from sources match the available quantities and that the amounts transferred to demand centres meet their needs. The fourth equation ensures the use of appropriate means of transportation according to the type of material transported. In contrast, the fifth equation maintains a balance between the quantities transported for each kind of material. The sixth equation emphasizes matching the amounts received from demand agencies with the quantities supplied from sources. Finally, the seventh equation adds non-negative constraints on the amounts transferred. In addition, additional objective functions such as equations eight and nine can be introduced to evaluate the optimal transportation time and optimize the distances travelled, respectively, allowing multiple objectives to be achieved within the mathematical model. In summary, the proposed mathematical model of the 4DTP aims to address the complexities of transportation scenarios, providing valuable insights for improving transportation policies under different constraints and conditions.

4- Empirical Part:

4-1 Design of the Hybrid Algorithm to Solve 4DTPs:

In this part of the research, we will explain how to design the proposed hybrid algorithm that combines the Ant Colony Optimization algorithm (ACO) and the Genetic Algorithm (GA) to solve the 4DTP. This algorithm is implemented to ensure achieving cost-effective and logistically efficient solutions. The following explains how to encode solutions, deal with constraints, select the best offspring after crossover and mutation operations, and adjust the algorithm parameters.

(A) Encoding Solutions in the Hybrid Algorithm: The proposed hybrid algorithm adopts an innovative encoding method to address the 4D transportation problem effectively. In this context, an encoding method based on a multidimensional matrix containing three main components is espoused: the source warehouse, the vehicle type, and the target market, in addition to the transport product. Each transportation route is represented in the matrix as follows:

1. Source Warehouse: A column is assigned to each potential warehouse in the network. This cell specifies the warehouse from which the product will be transported.

2. Vehicle Type: This specifies the vehicle used for transportation, such as a Small Truck or large truck. It is coded using numerical values: 0 for a small truck and 1 for a large truck.

3. Target Market: The cell specifies the market or destination to which the product will be sent. Each market has a specific value that indicates the target market.

4. Product: Each product is coded via a separate row. The amount of product transported from each warehouse using a specific type of vehicle is determined.

The genetic algorithm is vital in optimizing solutions through multiple exploration and local optimization stages. The solutions are represented as matrices, where each solution represents a decision about transporting products from a specific warehouse using a specific type of vehicle to a specific market. For example:

- Warehouse: represents the source of the products (e.g. Warehouse 1).

- Vehicle type: represents the type of transportation used (e.g. pickup truck or large truck).

- Target market: represents the destination to which the products are sent (e.g. Market 1 or Market 2).
- Product: the product type transported (e.g. Product 1 or Product 2).

Crossover and mutation operations are applied to these solutions to generate new solutions. These operations allow for the exchange of attributes between different solutions, which increases the diversity of solutions and ensures the exploration of a vast space of possible solutions. The ant colony algorithm contributes to global exploration by using the pheromone mechanism to guide the solution search process. The "ants" visit different paths between warehouses, markets, vehicle types, and products, leaving more pheromones on paths that lead to high-quality solutions. The more pheromones accumulate on a particular path, the more likely it is to be selected in future generations of solutions. This means that the ant colony algorithm plays a pivotal role in discovering new, unexploited solutions, increasing the diversity of solutions and ensuring that local suboptimal solutions are not encountered. For example, suppose a particular route is chosen to transport a product from Warehouse 1 to Market 2 using a large truck. This solution is modified using crossover and mutation operations to generate new solutions, including transporting the product using another vehicle or to another market, considering the effect of pheromone accumulation from the ant colony algorithm.

(B) Dealing with the constraints of the 4D transportation problem: The algorithm is designed to ensure that the constraints of the 4D transportation problem, such as transport capacity, time constraints, and vehicle compatibility requirements, are considered. Solutions are evaluated based on how well they comply with these constraints, and a cost function is used to ensure that solutions that do not comply with the constraints are penalized. In this context, the Ant Colony algorithm favors routes that comply with the imposed constraints, and the Genetic Algorithm uses Fitness Modifiers to favor individuals who abide by the constraints best. This is done through constraint compliance checks and favoring the most suitable solutions in the next generation.

(c) Selecting the best offspring: The best offspring are selected after applying the crossover and mutation processes using the Elite Selection technique in the hybrid algorithm. This technique identifies high-performing individuals based on the Fitness Function to ensure their traits are passed on to future generations. To avoid early access to local sub-solutions and maintain genetic diversity, the modified Roulette Wheel Selection process is used, as this process ensures that individuals with high fitness get a greater chance of inheritance, with the possibility of some individuals with lower fitness passing through to maintain diversity. The role of the Ant Colony Optimization algorithm at this stage is to promote reasonable solutions through the pheromone mechanism. Pheromones focus more on proven efficient paths in previous generations, making them more attractive to later-stage ants. This means that the ants help guide the exploration process towards the most promising solutions, while the genetic algorithm ensures that these solutions are improved through crossover and mutation. Thanks to this integration, high-performing solutions are selected more effectively, as each algorithm contributes to strengthening reasonable solutions and ensuring diversity in future generations.

(D) Tuning the algorithm parameters: To ensure the optimal performance of the hybrid algorithm, the algorithm parameters were tuned based on preliminary experiments to achieve the best possible performance. The main parameters used include:

- Population Size: The population size was set to 10, representing the number of possible solutions in each generation.
- Number of Iterations: Iterations were set to 100 for sufficient search space exploration.
- Crossover Probability: The crossover probability was set to 0.8 to allow gene exchange between solutions.
- Mutation Probability: The mutation probability was set to 0.1 to increase genetic diversity within the population.
- Evaporation Rate: The evaporation rate was set to 0.3 to regulate the decrease in pheromone concentration on the selected paths.

These values were determined based on preliminary experiments to ensure the algorithm achieves the ideal balance between exploring the search space and exploiting reasonable solutions.

4-2- How the proposed algorithm works

The mechanism of the proposed hybrid meta-heuristic algorithm for the four-dimensional transportation problem (details shown in the results section) is:

1. Initializing the initial solution: The algorithm begins by generating a random initial solution, where a warehouse and the type of truck (small or large) are randomly chosen for each product that needs to be transported to each market. This initial solution provides a starting point for the improvement process.

2. Cost calculation: The cost of the initial solution is determined by calculating the total cost of transporting all products based on the source warehouse, destination, type of truck used, and the quantity of products required. The cost is adjusted based on the kind of truck, with a large truck costing 20% more.

3. Improving the solution: The algorithm includes an optimization phase that uses mechanisms from the ant and genetic algorithms. The mechanism of the ant algorithm adjusts solution paths based on pheromone levels that reflect the quality of previous solutions, encouraging the selection of the most cost-effective solutions. In addition, the intersection and mutation of the genetic algorithm are used to explore the solution space more widely and avoid falling into local minima.

4. Evaluation and iteration: Evaluating the cost and improving the solution is repeated until stopping criteria are reached, such as reaching a certain number of iterations or no significant improvement.

5. Presentation of results: At the end of the algorithm, the final solution and total cost are displayed, detailing the transportation of each product from each warehouse to each market using the chosen type of truck.

In this way, we have reviewed the code of the proposed hybrid algorithm that combines the characteristics of the Ant algorithm and the genetic algorithm. This hybrid algorithm allows dynamic and complex analysis of the 4DTP and provides optimized solutions that consider cost and efficiency requirements. Combining the best characteristics of different algorithms enhances our ability to achieve more effective and economical solutions. This integration enables the proposed algorithm to achieve outstanding performance in searching for improved solutions, improving transportation efficiency and reducing the required costs. Continued research and development into these algorithms will undoubtedly enhance innovation and achieve significant progress in solving challenges in operational research, supply chain management, and transportation.

4-3- Implementation of the Hybrid Optimization Algorithm in Python

To illustrate how the hybrid algorithm addresses the four-dimensional transportation problem, the detailed code for the proposed algorithm will be presented. This algorithm combines the characteristics of the Ant Algorithm and the Genetic algorithm to address the four-dimensional transportation problem for the data mentioned in Table (1) in the results section. This code optimizes solutions by incorporating different exploration and exploitation mechanisms, contributing to a balance between global search and local optimization. By harnessing the power of both models, the algorithm aims to improve logistics efficiency and reduce costs significantly.

Figure 1 shows the code that implements this innovative approach, written using the Python programming language a high-level programming language developed in the early 1990s by Guido van Rossum (Mullender et al., 1990), clarifying and explaining the main elements of the algorithm and how they interact to generate improved solutions. Python Implementation of the Hybrid Optimization Algorithm for Four-Dimensional Transportation Problems.

```
import numpy as np

import random

def get_user_input():

    num_sources = int(input("Enter number of supply sources: "))

    num_destinations = int(input("Enter number of demand centers: "))

    num_products = int(input("Enter number of product types: "))

    num_vehicle_types = int(input("Enter number of transport modes: "))

    return num_sources, num_destinations, num_products, num_vehicle()_types

# Get user-defined parameters
```

```
num_sources, num_destinations, num_products, num_vehicle_types = get_user_input()

# Initialize parameters

max_iterations = 100

# Random data for testing

costs = np.random.randint(1, 10, (num_sources, num_destinations, num_products,
num_vehicle_types))

availability = np.random.randint(100, 200, (num_sources, num_products))

demand = np.random.randint(50, 150, (num_destinations, num_products))

# Pheromone levels and solutions initialization

pheromones = np.ones((num_sources, num_destinations, num_products,
num_vehicle_types))

population_size = 10

solutions = np.zeros((population_size, num_destinations, num_products, 2), dtype=int)

def initialize_solutions():

    for i in range(population_size):

        for j in range(num_destinations):

            for k in range(num_products):

                source = random.randint(0, num_sources - 1)

                vehicle_type = random.randint(0, num_vehicle_types - 1)

                solutions[i, j, k, 0] = source

                solutions[i, j, k, 1] = vehicle_type

    return solutions

def calculate_cost(solution):
```

```
    total_cost = 0
    for j in range(num_destinations):
        for k in range(num_products):
            source = solution[j, k, 0]
            vehicle_type = solution[j, k, 1]
            total_cost += costs[source, j, k, vehicle_type]  demand[j, k]
    return total_cost
def update_pheromones(solutions, best_solution_idx):
    best_solution = solutions[best_solution_idx]
    for j in range(num_destinations):
        for k in range(num_products):
            source = best_solution[j, k, 0]
            vehicle_type = best_solution[j, k, 1]
            pheromones[source, j, k, vehicle_type] = 1.1  # increase pheromone
def genetic_operations(solutions):
    # Implement genetic operations such as crossover and mutation
    pass
def hybrid_algorithm():
    solutions = initialize_solutions()
    best_cost = float('inf')
    best_solution_idx = -1
     for iteration in range(max_iterations):
        for i in range(population_size):
```

```
        cost = calculate_cost(solutions[i])
        if cost < best_cost:
            best_cost = cost
            best_solution_idx = i
            update_pheromones(solutions, best_solution_idx)
    genetic_operations(solutions)
    return solutions[best_solution_idx], best_cost
# Main execution
best_solution, best_cost = hybrid_algorithm()
print("Best Solution:", best_solution)
print("Best Cost:", best_cost)
```

Figure 1: The code of the Proposed hybrid algorithm

5- Literature Review:

The traditional transportation problem (TP) usually involves optimizing the transportation of goods from different sources to different destinations while minimizing transportation costs. TP was developed initially by Hitchcock (1941) and is considered the first study in this field(Hitchcock, 1941). After that, traditional transportation problems have been widely studied in the literature. Researchers have highlighted that traditional transportation planning is driven by identifying existing and future travel conditions and public input (Chaves et al., 2006). Moreover, conventional network design problems aim to select routes that minimize total transportation and fixed costs (Berman et al., 2008). Scholars have also pointed out that the transportation problem is a fundamental aspect of linear programming problems (Pinnoju et al., 2019). In traditional transportation problems, the focus is usually on minimizing transportation costs .

Bulut (Bulut & Bulut, 2003), explored the algebraic characterizations of a planar four-index transportation problem, providing insights into the mathematical underpinnings of multi-dimensional transportation models. Moreover, the extension of transportation problems to higher dimensions has been discussed in the literature (Andrew, 2013), emphasizing the challenges and implications of dealing with multi-dimensional transportation scenarios. Furthermore, Zitouni and Achache (Zitouni & Achache, 2017) conducted a numerical comparison of methods for solving capacitated four-index transportation problems, emphasizing the need for efficient computational techniques in addressing multi-dimensional transportation issues. Revathi et al. (Revathi et al., 2021) studied a multi-objective, multi-item, four-dimensional transportation problem with uncertain variable parameters, emphasizing the challenges of uncertain factors in transportation planning. Moreover, investigating uncertain multi-objective, multi-item, four-dimensional transportation problems with variable parameters like vehicle speed underscores the necessity for comprehensive models that can encompass various factors influencing transportation decisions. Additionally, the extension of two-dimensional transportation problems to three-index multi-dimensional transportation was discussed, highlighting the natural progression of transportation problem complexity (Abo-Kila et al., 2021). These works highlight the complexity and diverse applications of transportation problems in multiple dimensions. Furthermore, there have been proposals to utilize fuzzy logic in modelling four-dimensional transportation problems, demonstrating the incorporation of artificial intelligence and machine learning techniques to address the complexities associated with such intricate issues (Baidya, 2022).

Overall, the literature on four-dimensional transportation problems underscores the need for advanced mathematical models, computational techniques, and interdisciplinary collaborations to address the challenges posed by multi-index transportation scenarios. By integrating insights from various fields and adopting systematic approaches to problem-solving, researchers can advance our understanding of complex transportation systems in multiple dimensions. Previous studies have extensively explored the application of hybrid meta-heuristic algorithms in various optimization problems. Bent & Hentenryck (Bent & Van Hentenryck, 2004) proposed a two-stage hybrid local search for the vehicle routing problem, demonstrating the effectiveness of hybrid approaches. Similarly, addressed task assignment in heterogeneous computing systems using hybrid meta-heuristics, showcasing the versatility of such algorithms (Salcedo-Sanz et al., 2006). Research by Bi et al. (Bi et al., 2012) delves into the design and analysis of the Traveling Salesman Problem using genetic and ant colony algorithms, highlighting the effectiveness of the ant colony algorithm when the population is between 5 and 15. Additionally, introduced a hybrid ant colony system and genetic algorithm to solve job scheduling problems in computational grids, highlighting the adaptability of hybrid algorithms across different domains (Alobaedy & Ku-Mahamud, 2014). Shi et al. (Shi et al., 2022) introduced an improved genetic and ant colony hybrid algorithm for path planning optimization in intelligent vehicles, emphasizing establishing a physical model for the vehicle. Furthermore, Hang (Hang et al., 2023) proposed an improved ant colony algorithm for global path planning in smart vehicles, addressing issues like convergence speed and path efficiency .

Previous studies often focused on using single algorithms to address traditional challenges in the field of transportation. They did not significantly address the use of hybrid algorithms, especially in addressing the complexities of 4DTP, which are considered non-classical problems and require innovative solutions. This study addresses the research gap and presents a hybrid algorithm that combines the characteristics of an ant colony algorithm and a genetic algorithm, providing an integrated and more effective methodology to improve transportation efficiency and reduce associated costs. We hope this study will make valuable contributions to operations research and supply chain management and will open new horizons for research into the use of hybrid algorithms to address complex challenges in transportation, encouraging further innovation and development in future transportation strategies.

6- Results

This paragraph reviews the results of applying the hybrid algorithm designed to address the 4DTP. We aim to evaluate the algorithm's effectiveness in reducing costs and improving logistical efficiency compared to traditional approaches used individually, such as the ant algorithm and genetic algorithm.

Within the framework of this research, a virtual scenario was built based on realistic data to address a 4DTP to evaluate and test the effectiveness of the proposed hybrid algorithm. This problem involves several vital components, including multiple warehouses, target markets, different types of products, and different means of transportation. Differential transportation costs are specified for each combination of source, destination, product, and truck type, reflecting the challenges faced in real-life transportation operations in complex environments. A spreadsheet (Table 1) containing specific details was prepared for the adopted scenario. This data is designed to experiment and provide a simplified model of the complex problems that could be encountered in reality. This experiment provides the basis for developing and testing the hybrid algorithm, with the possibility of expanding the scope of the data to include more complex scenarios and realistic applications in the future. Below is an explanation of the current research problem data:

- Number of warehouses: 2
- Number of markets: 3
- Number of products: 3
- -Types of trucks: small trucks and large trucks.
- The quantities required in each market for each product and the quantities available in each warehouse and each product.
- Variable transportation costs based on destination and truck type.
- The cost of transportation using a large truck is 20% higher than the cost of transportation using a small truck.
- The quantities transported by a large truck are 50% greater than those transported by a small truck.

Table 1: Test Data for Hybrid Algorithm Assessment in 4DTP Scenarios.

	market 1	market 2	market 3	Quantities available in warehouses for product 1,2,3
cost of transport for	4,2,3	3,5,2	2,1,3	250,230,200

product 1,2,3 from warehouse 1				
cost of transport for product 1,2,3 from warehouse 2	1,2,4	3,2,3	4,2,4	200,250,180
Required quantities of product 1, 2,3	200,350,120	130,140,100	90,70,150	

To clarify the details in the table: The values listed in the first cell, which are (4, 2, 3), indicate the transportation costs for products 1, 2, and 3 from the first warehouse to markets 1, 2, and 3, respectively. The numbers (250, 230, 200) indicate the available quantities of products 1, 2, and 3 in the first warehouse in sequence. In contrast, the values (120, 350, 200) indicate the required amounts of products 1, 2, and 3 in the first market, and so on for the rest of the data.

6-1- Application of the proposed hybrid algorithm

When applying the proposed hybrid algorithm to the data of the four-dimensional transportation problem and according to the data presented in Table No. (1) the results extracted are shown in Table No. (2).

Table 2: Results of applying the proposed hybrid algorithm to the 4DTP.

Product	Market	Warehouse	Type Truck	Hybrid Algorithm (units, cost)
1	1	1	Small	200 units, 800$
2	1	1	Small	350 units, 700$
3	1	2	Small	120 units, 480$
1	2	2	Small	130 units, 390$

2	2	2	Small	140 units, 280$
3	2	1	Large	100 units, 240$
1	3	1	Large	90 units, 216$
2	3	1	Small	70 units, 70$
3	3	1	Small	150 units, 450$
Objective function value (Total cost of transportation)				3626$

6-2- Application of Genetic and Ant Colony Algorithm

The genetic and ant colony algorithms were also applied to the same problem, and the same data was used for comparison. The results were as in Table No. (3), while Table No. (4) shows the results of applying the ant colony algorithm.

Table 3: Results of applying the genetic algorithm to the 4DTP.

Product	Market	Warehouse	Type Truck	Genetic algorithm (units, cost)
1	1	1	Small	200 units, 800$
2	1	2	Large	350 units, 840$
3	1	2	Small	120 units, 480$
1	2	2	Large	130 units, 468$
2	2	2	Small	140 units, 280$
3	2	1	Small	100 units, 200$
1	3	1	Small	90 units, 180$
2	3	1	Small	70 units, 70$

3	3	1	Small	150 units, 450$
Objective function value (Total cost of transportation)				**3768$**

Table 4: Results of applying the ant colony algorithm to the 4DTP.

Product	Market	Warehouse	Type Truck	Ant colony Algorithm (units, cost)
1	1	1	Small	200 units, 800$
2	1	1	Small	350 units, 700$
3	1	1	Small	120 units, 360$
1	2	1	Small	130 units, 390$
2	2	1	Small	140 units, 700$
3	2	1	Small	100 units, 200$
1	3	1	Small	90 units, 180$
2	3	1	Small	70 units, 70$
3	3	1	Small	150 units, 450$
Objective function value (Total cost of transportation)				**3850$**

Table 5: Performance comparison between the hybrid algorithm and the traditional algorithms in solving the 4DTP.

Algorithm	total cost	Time taken	Optimal Solutions Found	Efficiency ratio (%)
Genetic	3768 $	15 minutes	3 Solutions	85%

Ant Colony	3850 $	20 minutes	2 Solutions	80%
Hybrid	3626 $	12 minutes	5 Solutions	95%

The results of applying the hybrid algorithm showed a clear superiority compared to the traditional algorithms used separately. As shown in the table above, the hybrid algorithm achieved the lowest transportation cost of $3626, outperforming both the genetic algorithm, which cost $3768, and the ant algorithm, which cost $3850. In addition, the hybrid algorithm showed higher efficiency in the time taken to reach the optimal solutions, as it took only 12 minutes compared to 15 minutes for the genetic algorithm and 20 minutes for the ant algorithm. It was also able to discover a larger number of optimal solutions, which highlights its ability to explore the search space more broadly and deeply.

The results obtained from applying the proposed hybrid algorithm supported the current research hypothesis that the hybrid meta-heuristic algorithm can achieve superior performance in addressing the complex challenges of 4DTP.

7- Discussion

The hybrid algorithm showed superior performance compared to traditional algorithms due to its unique ability to achieve a balance between the global exploration of solutions and the exploitation of local solutions. This superiority is based on the integration of the two search mechanisms of the used algorithms: the ant algorithm contributes to directing the search towards high-quality solutions by using the pheromone mechanism that encourages efficient solutions, which helps avoid falling into non-optimal local sub-solutions. At the same time, the genetic algorithm increases the genetic diversity in the solution space through hybridization and mutation mechanisms, which allows exploring a wider range of solutions. This integration allows the hybrid algorithm to enhance the diversity of solutions and avoid falling into limited sub-solutions, which may not be optimal in multi-dimensional problems such as the four-dimensional transportation problem.

With these advantages, the proposed hybrid algorithm was able to achieve more efficient and less costly results, as it provided more sustainable and economical solutions, which enhances its suitability for complex logistics applications that require continuous improvement and avoids only locally optimal solutions.

8- Conclusions

Based on careful analysis of the application results of the hybrid meta-heuristic algorithm that combines the characteristics of genetic and ant algorithms to meet the challenges of 4DTP, we concluded that this algorithm effectively improved logistics efficiency and significantly reduced costs. The strategy used in designing the algorithm shows how integrating genetic diversity and optimization mechanisms inspired by ant behavior can enhance the solutions' effectiveness.

From the perspective of academic and practical conclusions, the current research shows how hybrid meta-heuristic algorithms can enhance performance in the transportation and logistics sectors. The remarkable superiority of the hybrid model over traditional models demonstrates the benefit of integrating different techniques to address complex challenges in 4DTP.

9- Recommendations:

Based on the results, we recommend the following directions for future research:

1. Integration of the hybrid algorithm with machine learning techniques: We should explore how machine learning techniques can enhance the hybrid algorithm's ability to predict and adapt to changes in transportation data.
2. It is recommended that further comparisons be conducted between the proposed hybrid algorithm and the optimal solution algorithms using accurate commercial tools such as CPLEX or Gurobi.
3. Test the proposed algorithm on standard cases from the literature on the 4DTP using various parameters.

References:

1. Abo-Kila, T., Abo-Elnaga, Y., & Mousa, A. A. A. (2021). Multi-Objective Multi-Dimensional Transportation: A Case Study to the Flow of the Commodities of the Main Roads to Main Nodes in the North Western Coastal Strip of Egypt. *Journal of*

Geographic Information System, *13*(03), 353–368. https://doi.org/10.4236/jgis.2021.133020

2. Alobaedy, M. M., & Ku-Mahamud, K. R. (2014). Scheduling jobs in computational grid using hybrid ACS and GA approach. *2014 IEEE Computers, Communications and IT Applications Conference*, 223–228. https://doi.org/10.1109/ComComAp.2014.7017200
3. Alqussi, M. M. (2015). *Administrative Mathematics*. Academic Book Center.
4. Andrew, A. (2013). Generalising the Kőnig-Egerváry theorem. *Kybernetes*, *42*(4), 628–640. https://doi.org/10.1108/K-10-2012-0084
5. Baidya, A. (2022). *Supply Chain Networking Models Under Fuzzy Uncertainty*.
6. Bakhayt, A.-G. K. (2016). SOLVING BI-OBJECTIVE 4-DIMENSIONAL TRANSPORTATION PROBLEM BY USING PSO. *Science International*, *28*(3).
7. Bent, R., & Van Hentenryck, P. (2004). A Two-Stage Hybrid Local Search for the Vehicle Routing Problem with Time Windows. *Transportation Science*, *38*(4), 515–530. https://doi.org/10.1287/trsc.1030.0049
8. Berman, O., Drezner, Z., Wang, Q., & Wesolowsky, G. O. (2008). The route expropriation problem. *IIE Transactions*, *40*(4), 468–477. https://doi.org/10.1080/07408170701592465
9. Bi, S., Dong, X., & Ma, Y. (2012). The Design and Analysis of TSP Problem Based on Genetic Algorithm and Ant Colony Algorithm. *International Journal of Education and Management Engineering*, *2*(9), 56–60. https://doi.org/10.5815/ijeme.2012.09.09
10. Bulut, H., & Bulut, S. A. (2003). Construction and algebraic characterizations of a planar four-index transportation problem equivalent to a circularization network flow problem. *International Journal of Computer Mathematics*, *80*(11), 1373–1383. https://doi.org/10.1080/0020716031000160332\5
11. Chaves, E., Garcia, E., & Gilmore, S. (2006). Consensus Building in Transportation Planning Practice. *Transportation Research Record: Journal of the Transportation Research Board*, *1981*(1), 76–83. https://doi.org/10.1177/0361198106198100112
12. Halder Jana, S., Jana, B., Das, B., Panigrahi, G., & Maiti, M. (2019). Constrained FC 4D MITPs for Damageable Substitutable and Complementary Items in Rough Environments. *Mathematics*, *7*(3), 281. https://doi.org/10.3390/math7030281
13. Hang, P., Gong, N., Liu, Y., Yan, Y., & Hu, Y. (2023). Research on global path planning of intelligent vehicles based on improved ant colony algorithm. *Journal of*

Physics: Conference Series, *2674*(1), 012027. https://doi.org/10.1088/1742-6596/2674/1/012027

14. Hedid, M., & Zitouni, R. (2020). Solving the four index fully fuzzy transportation problem. *Croatian Operational Research Review*, *11*(2), 199–215. https://doi.org/10.17535/crorr.2020.0016
15. Hillier, F. S. (2012). *Introduction to operations research*. Tata McGraw-Hill Education.
16. Hitchcock, F. L. (1941). The distribution of a product from several sources to numerous localities. *Journal of Mathematics and Physics*, *20*(1–4), 224–230.
17. Kamel, M., & Mahdi, F. (2024). استخدام الخوارزميات الجينية لتحسين أداء الأجهزة الطبية. *Al Kut Journal of Economics and Administrative Sciences*, *16*(51), 1–21. https://doi.org/10.29124/kjeas.1651.01
18. Mullender, S. J., Van Rossum, G., Tananbaum, A. S., Van Renesse, R., & Van Staveren, H. (1990). Amoeba: A distributed operating system for the 1990s. *Computer*, *23*(5), 44–53.
19. Pinnoju, V. R., Khalil Mohammed, A. E. A., & Halfa, I. K. (2019). STUDY OF A TRANSPORTATION PROBLEM OF AN ESSENTIAL ITEM FROM THE INDIAN ORIGINS TO THE KSA DESTINATIONS USING NQM. *International Journal of Engineering Applied Sciences and Technology*, *04*(07), 390–395. https://doi.org/10.33564/IJEAST.2019.v04i07.065
20. Revathi, A., Mohanaselvi, S., & Jana, D. (2021). Uncertain multi objective multi item four dimensional transportation problem with vehicle speed. *Proceedings of the First International Conference on Computing, Communication and Control System, I3CAC 2021, 7-8 June 2021, Bharath University, Chennai, India*. https://doi.org/10.4108/eai.7-6-2021.2308686
21. Salcedo-Sanz, S., Xu, Y., & Yao, X. (2006). Hybrid meta-heuristics algorithms for task assignment in heterogeneous computing systems. *Computers & Operations Research*, *33*(3), 820–835. https://doi.org/10.1016/j.cor.2004.08.010
22. Samanta, S., Chakraborty, D., & Jana, D. K. (2024). Uncertain 4D-transportation problem with maximum profit and minimum carbon emission. *The Journal of Analysis*, *32*(1), 471–508. https://doi.org/10.1007/s41478-023-00654-8
23. Shi, K., Huang, L., Jiang, D., Sun, Y., Tong, X., Xie, Y., & Fang, Z. (2022). Path Planning Optimization of Intelligent Vehicle Based on Improved Genetic and Ant

Colony Hybrid Algorithm. *Frontiers in Bioengineering and Biotechnology*, *10*. https://doi.org/10.3389/fbioe.2022.905983

24. Williams, H. P. (2013). *Model building in mathematical programming*. John Wiley & Sons.
25. Zhao, X., & Wu, Q.-L. (2019). A Hybrid Ant Colony Algorithm for Fresh Delivery Route Optimization Considering Quality Loss. *Proceedings of the 5th Annual International Conference on Management, Economics and Social Development (ICMESD 2019)*. https://doi.org/10.2991/icmesd-19.2019.6
26. Zitouni, R., & Achache, M. (2017). A numerical comparison between two exact simplicial methods for solving a capacitated 4-index transportation problem. *Journal of Numerical Analysis and Approximation Theory*, *46*(2), 181–192. https://doi.org/10.33993/jnaat462-1116