The Use of Filtered Back projection Algorithm for Reconstruction of tomographic Image

Marwa. T. Al Hussani　　　　　　**Mohammed H. Ali Al Hayani**

Electronics and Communication Eng. Dept.,

Al-Nahrain University Baghdad, Iraq

thamer.marwa@yahoo.com

Abstract *:*

Filtered back projection algorithm (FBP) is one of the most common methods that used in to mographic image reconstruction. This algorithm was applied on computerized tomography (CT) scanner. This paper presents the implementation of reconstruction algorithm of CT image 512 x 512 pixels from raw data for the parallel beam projections. This work consist three main parts.

In first part, many numbers of projections are obtained from the input test image called Shepp Logan phantom using the Radon transform. The second part, reconstruct the image from parallel beam projection to reduces the reconstruction time. In the third part, filter the projections using Ram Lak filter and Hann window for image enhancement and then back projection summation to form the reconstructed image.

The computer program has been designed, written and implemented in our work using MATLAB. This made the result very good for reconstruction two dimensions image from its projection in computerized Tomography scanner.

The algorithm was applied on the head phantom image for the purpose of performance improvement.It's important to many applications like medical and industrial application.

Keywords: Filtered Back Projection algorithm (FBP), Computerized Tomography (CT),Data Acquisition System (DAS),Sinogram,Head Phantom.

I-Introduction

IN 1917, Radon, who is an Austrian mathematician, wasthe first to present the mathematics solution of the reconstruction of object from their respective

projections, that is,determining the distribution function and the distribution ofdensity of a region studied based on its projections. In medicalapplications, the first computerized commercial tomographyof X-rays was presented in 1973 by EMI Ltd. [1], and itcaused a great impact on radiological diagnosis. However,an important and decisive contribution to the developmentof tomography was given by Cormack [2],who is a physics Professor which

showed a mathematical technique to reconstruct images by using a finite number of projections. With a basis in experiments and observations, he proposed a matrix of coefficients associated with the slice sections that could be obtained by measuring the transmission of X-rays at various angles across an object and obtaining images of transverse sections [3].

The CT scanner is known to be the tool to view the internal properties of the object. Table 1 shows the generations of the CT scanner [4].The basic system generally consists of a gantry, patient table, control console, and a computer system.The gantry contains the x-ray source, x-ray detectors, and the data-acquisition system (DAS) [5].

In this paper reconstruction process is applied on CT images using filtered backprojection algorithm. CT scanner imaging has been widely used in biological application and diagnostics. This is due to Anatomical details such as bone, organs and soft tissue contrast, high spatial resolution and slice selection at any orientation.

Table 1.The generations of CT scanner

Scanner Generation	Scanner Description
First generation	The 1stgeneration scanner is the combination of linear translation followed by incremental rotation, and is well-known as translate-rotate motion.
Second generation	The 2ndgeneration CT uses multiple narrow beams from a single source and multiple detectors. The scannermovement is calledtranslate-rotate.
Third generation	In3rdgeneration CT scanner, the detectorarray is rigidly linked to the X-ray tube, so that both the tube and the detectors are rotated together around the object (a motion is referred to as a rotation-movement).
Fourth generation	The 4thgeneration CT scanner is incorporated with a large stationary ring of detectors, with the X-ray tube alone rotating around the object.
Fifth generation	The 5thgeneration CT scanner refers to a multi-slice CT scanner that is equipped with a multiple-row detector array.

II-Filtered Back Projection Algorithm

Image reconstruction is the process of estimating an object image slice of *f(x,y)* from a set of projections *p(t, θ)*. Several algorithms with different advantages can accomplish this task. The foundation of the mathematical package for image reconstruction is the reconstruction algorithm [6].The FBP algorithm is often referred as the convolution method using a one-dimensional integral equation for the reconstruction of a two-dimensional image. This method is the most popular reconstruction algorithm used at present in CT application.

Consider a single projection and its Fourier transform. By the Fourier Slice Theorem,this projection gives the values of the object's two-dimensional (2D) Fourier transform along a single line. If the values of the Fourier transform of this projection are inserted into their proper place in the object's two-dimensional Fourier domain then a simple reconstruction can be formed by assuming the other projections to be zero and finding the 2D inverse Fourier transform.

The complete filtered back projection algorithm can be written as:

- Sum for each of the K angles *Θ*, between 0º and 180º,

- Measure the projection, *Pθ (t)*,

- Fourier transform it to find *Sθ (w)*,

- Multiply it by the weighting function *2π|w|/K*.

- Sum over the image plane the inverse Fourier transforms of the filtered projections (the back projection process).

By recalling the formula for 2D of the inverse Fourier transform, the object function*f(x,y)*, can be expressed as[7]:

$$f(x,y) = \int_{-\infty}^{\infty}\int_{-\infty}^{\infty} F(u,v)e^{j2\pi(ux+vy)}dudv \qquad \ldots\ldots (1)$$

Exchanging the rectangular coordinate system in the frequency domainfor a polar coordinate system, by making the substitutions[7]:

$$u = w\cos\theta \qquad \ldots(2)$$
$$v = w\sin\theta$$

and then changing the differentials by using:

$$dudv = wdwd\theta \qquad \ldots..(3)$$

we can write the inverse Fourier transform of a polar function as:

$$f(x,y) = \int_{0}^{2\pi}\int_{0}^{\infty} F(w,\theta)e^{j2\pi w(x\cos\theta + y\sin\theta)}wdwd\theta$$
$$+ \int_{0}^{\pi}\int_{0}^{\infty} F(w,\theta + 180^\circ)e^{j2\pi w[x\cos(\theta+180^\circ)+y\sin(\theta+180^\circ)]}wdwd\theta,) \qquad \ldots..(4)$$

and then using the property:

$$F(w,\theta + 180^\circ) = F(-w,\theta) \qquad \ldots.(5)$$

The above expression for $f(x,y)$ may be written as:

$$f(x,y) = \int_{0}^{\pi}\left[\int_{-\infty}^{\infty} F(w,\theta)|w|e^{j2\pi wt}dw\right]d\theta \qquad \ldots.(6)$$

Here we have simplified the expression by setting

$$t = x\cos\theta + y\sin\theta \qquad \ldots..(7)$$

If we substitute the Fourier transform of the projection at angle *Θ*, for the two-dimensional Fourier transform, we get

$$f(x,y) = \int_{0}^{\pi}\left[\int_{-\infty}^{\infty} S_\theta(w)|w|e^{j2\pi wt}dw\right]d\theta \qquad \ldots(8)$$

This integral in above equation may be expressed as:

$$f(x,y) = \int_{0}^{\pi} Q_\theta(x\cos\theta + y\sin\theta)d\theta \qquad \ldots..(9)$$

Where

$$Q_\theta(t) = \int_{-\infty}^{\infty} S_\theta(w)|w|e^{j2\pi wt}dw) \qquad \ldots.(10)$$

This estimate of*f(x,y)*, given the projection data transform $S_\theta(w)$, has simple form. The latter equation represents a filtering operation, where thefrequency response of the filter is given by $|w|$; therefore $Q_\theta(w)$ is called " filtered projection." The resulting projections for different angles*Θ*are added to form the estimate of*f(x, y)*, and can be represented by discrete approximation to the integral in (9),

$$f(x,y) = \frac{\pi}{K} \sum_{i=1}^{k} Q_{\theta i}(x \cos \theta_i + y \sin \theta_i) \qquad ..(11)$$

Where the K angles θ_i are those for which the projections $P\theta(t)$

III-Description of the Computer Model

The Filtered Back Projection algorithm is used to reconstruct an object from its projections. A simple back projection causes blurring of the reconstructed object. To overcome this effect the projections is filtered using a high pass filter. A proposed computer model to simulate the entire process through the following steps:

i. The acquisition of parallel sinogram (Radon transform) from two-dimensional images 512x512 pixel,

ii. Fast Fourier Transform (FFT) is implemented for the output matrix from Radon transform.

iii. Filtering the projections uses ramp filter in the frequency domain multiplied by Hann window. To avoid the excessive noise amplification at these frequencies, a band-limited filter called Ram-Lak filter is often used in image reconstruction.

iv. The projections are weighted correctly. Nowback projection is implemented as a back smearing,where each radial sample is taken at each angle and smeared along the path that we integrated.

In this paper we have presented a method of image reconstruction for a head phantom to improve the performance of FBP algorithm. The MATLAB package was used as a tool in performing the programming of the proposed reconstruction process. When acquiring the data from the DAS obtained by parallel beam geometry of scanning by using MATLAB functions on image 512x512 pixel presented for reconstruction as a head phantom test image shown in Fig.1 [8]. These raw data will be filtering using Ram-Lak filter multiplied by a window. By using many windows for multiplication, Hann window improved less estimated time for reconstruction. Fig.2 show the block diagram of reconstruction system.

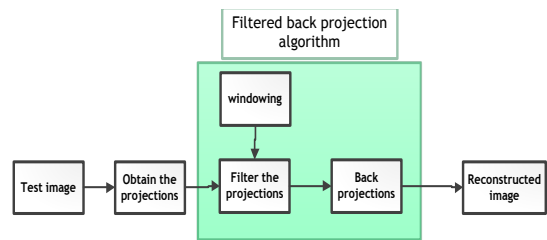**Figure (1):** Head Phantom input image

Figure (2): Block diagram of the reconstruction system

IV-Results and Discussion

After applying the computer simulation of FBP on the image, the performance of algorithm and the quality of the image will be dependent on many considerations such as image size, number of projections and theta incremental.

During the scanning the projection will acquire by X-ray snapshot on the object at theta angle, the data acquired can be represented by 2Dsinogram. A sinogram is the collection of parallel projections of the object taken at equidistant angles, and forming a map of the projection data [9].It's based on the concept of line integral and Radon transform is used for image reconstruction processes. Fig. 3shows the sinogram of the input image:
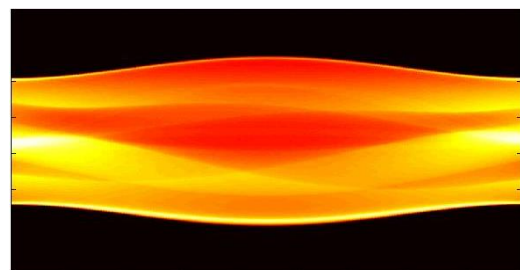

Figure (3): The sinogram of input image

The above sinogram implemented at theta in range (0:179) with step size 1degree, the effect of step size gives an indication about the quality of

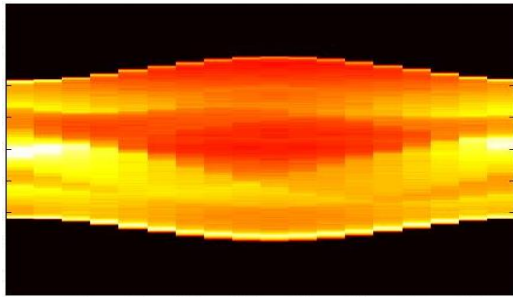the image. Fig. 3shows the Sinogram at step size equal to 10 degree.

Figure (4): The Sinogram of input image

In both case the detectors will sense the X-ray at the same manner in each projection, Fig.5 show the reconstructed images at various Δt:
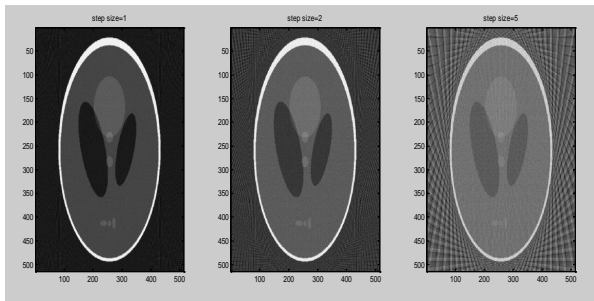

Figure (5):.The Reconstructed images at various Δt

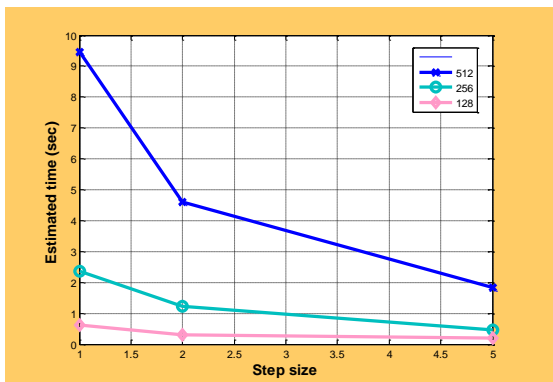The estimated time of reconstruction for each case of Δt is hown in Fig.6.

Figure (6): Plot diagrams for reconstruction estimated time

Each one of these projections will be weighting using Ram-Lakfilter, Ram achandran &Lak shminarayanan (1971) proposed the back projection method to overcome blurring image which is later known as Ram-Lak filter as shown in Eq.(10). The ramp filter will be multiplied by Hanning window to reduce the effect of noise at high frequencies of the image, and the multiplication of both filters. Fig.7. shows the Ram-Lak filter and Hann window:
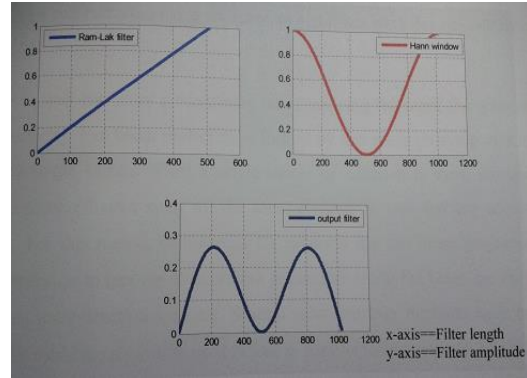

Figure (7): The Ram-Lake filter

Using eq. 11 the object can be reconstructed using these filtered projections. The result of FBP algorithm of an input CT scanner test image for the sinogram Fig. 3 is shown in Fig. 8.

Figure (8): The reconstructed image

The image reconstruction using the Sinogram in Fig.4 will be shown in Fig. 9. Asit's been discussed, the Sinogram will give some indication about the reconstructed image

Figure (9): The reconstructed image

The plot of Fig. 6 shows the relationship of performance with both step size and image size. In this paper we get estimated time less than 10 sec to reconstruct image 512x512 pixel compared with [11], this time will be consider as an estimated value because computer program used high level language.

V- Conclusions

FBP algorithm is seen to be useful and efficient method of reconstruction in CT scanner images. Reconstruction performance is very important part in all the scanning process especially in medical application because the patient must be exposed to radiation at less time as possible, this can be take another slice at any origination after complete first slice reconstruction quickly. This compression of estimated time for reconstruction process belongs to using some filter to reduce the calculations of back projection. The effect of these filters is also important to obtain good image quality after reconstruction. For both issue the results was achieved using Ram-Lak filter and Hann window. It was observed that the Hann filter has consistently given the superior performance for the selected images [12]. This method was reduced the time by three factors from that estimated in other algorithms and which is an essential task to know what the advantage of FBP algorithm is.

VI- References

[1] G. N. Hounsfield, "Computerized transverse axial scanning (tomography): Description of systems," Brit. J. Radio, vol. 46, pp. 1016–1022, 1973.

[2] A. M. Cormack, "Representation of a foundation by its line with some radiological application," J. Appl. Phys., vol. 34, no. 9, pp. 2722–2727,Sep. 1963.

[3] J. Hsieh, "Computed Tomography: Principles, Design, Artifacts, and Recent Advances",2nd ed, 2009.

[4] L. W. Goldman, "Principles of CT and CT Technology," Journal of Nuclear Medicine Technology, vol. 35, pp. 115-128, 2007.

[5] R. C. Gonzales, "Digital Image Processing"3rded,Pearson-Prentice Hill, 2008.

[6] Hendee, W. R. and Ritenour, E. R., "Medical imaging physics," New York: A John Wiley & Sons, Inc., Publication, 2002.

[7] Kak. A. C. and Slaney M. 1987 "Principles of Computerized Tomography" Piscataway, NJ: IEEE.

[8] Gabor. T. Herman, "Fundamentals of Computerized Tomography: Image Reconstruction from Projections" second edition Springer 2009.

[9] Azevedo, D. F. G. d., Helegda, S., Godoy, P. C., Castro, F. d., and Castro, M. C. d., "Tomography simulation and reconstruction tools applied in the evaluation of parameters and techniques," Department of Electrical Engineering, 2001.

[10] Oppenheim, A.V., and R.W. Schafer, Discrete-Time Signal Processing, Prentice-Hall, pp. 447-448, 1989.

[11] S. A. Qureshi, S.M. Mirza, Arif, "Inverse Radon Transform-Based Image Reconstruction Using Various Freuency Domain Filters in Parallel Beam Transmission Tomography", IEEE, 2005

[12] P. Prabhat, S. Arumugam, V. K. Madan, "Filtering in Filtered Back projection Computerized Tomography", Proceedings of the National Conference "NCNTE-2012" at Fr. C.R.I.T., Vashi, Navi Mumbai, Feb. 24-25, 2012.

استخدام خوارزمية اعادة المساقط المرشحة في استرجاع الصورة التوموغرافية

مروه ثامر علي　　　　　　محمد حسين علي

قسم هندسة الالكترونيات والاتصالات – جامعة النهرين

الخلاصة

خوارزمية التسقيط المرشحة هي واحدة من الطرق الاكثر شيوعا التي تستخدم في اعادة تركيب صورة التصوير الشعاعي الطبقي. وقد طبقت هذه الخوارزمية على التصوير المقطعي المحوسب (CT). تعرض هذه المقالة خوارزمية اعادة التركيب صورة ٥١٢x٢١٥ بكسل من البيانات الخام للهندسية المتوازية.

هذا العمل يتضمن ثلاثة اجزاء رئيسية:

في الجزء الاول منها, عدد من المساقط تؤخذ من الصورة المدخلة والتي تسمى شبح الشيب لو كان بأستخدام تحويل الرادون. في الجزء الثاني, استرجاع الصورة من المساقط المتوازية لتقليل الوقت اللازم للاسترجاع. اما في الجزء الثالث, ترشيح المساقط بأستخدام مرشح الرا-لاك واستخدام نافذة الهان لتحسين الصورة واعادة تسقيطها وتجميعها لتشكيل الصورة المسترجعة.

برنامج الحاسبة صمم وكتب ونفذ بأستخدام الماتلاب. هذا مما جعل النتائج جيدة لأسترجاع صورة من بعدين في الماسح التوموغرافي المحوسب.

هذه الخوارزمية طبقت على صورة مقطع رأسي لأغراض تحسين الاداء وهي مهمة لعدد من التطبيقات مثل التطبيقات الطبية والصناعية.