

# Modelling and Development of Linear and Nonlinear Intelligent Controllers for Anti-lock Braking Systems (ABS)

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## Abstract

Antilock braking systems (ABS) are utilized as a part of advanced autos to keep the vehicle's wheels from deadlocking when the brakes are connected. The control performance of ABS utilizing linear and nonlinear controls are cleared up in this research. In order to design the control system of ABS a nonlinear dynamic model of the antilock braking systems is derived relying upon its physical system. The dynamic model contains set of equations valid for simulation and control of the mechanical framework. Two different controllers technique is proposed to control the behaviors of ABS. The first one utilized the PID controller with linearized technique around specific point to control the nonlinear system, while the second one used the nonlinear discrete time controller to control the nonlinear mathematical model directly. This investigation contributes to more additional information for the simulation of the two controllers, and demonstrate a clear and reasonable advantage of the classical PID controller on the nonlinear discrete time controller in control the antilock braking system.

**Keywords:** Anti-lock Braking System, Wheel Slip Ratio, PID Controller, and Discrete Time Controller.

## الخلاصة

نظام منع انغلاق المكابح (ABS) يستخدم كجزء مهم في المركبات الحديثة لمنع الاطار من الغلق بعد تعشيق المكابح. الاداء العام لنظام سيطرة منع انغلاق المكابح مستفيدا من كون النظام خطياً او غير خطي موضعاً في هذا البحث. من اجل تصميم نظام السيطرة، تم اشتقاق نموذج ديناميكي لخطي لمانع الانزلاق استناداً على طبيعته نظامه الفيزيائي. النموذج الديناميكي يتكون من عدة معادلات تحكم عمل النظام الميكانيكي. نظامين سيطرة مختلفين تم استخدامهم للسيطرة على اداء منع انغلاق المكابح، الاول تم الاستفادة من المسيطر الخطي نوع (PID) مع استخدام تقنية تحويل النظام من اللاخطي الى الخطي حول نقطة معينة للسيطرة على النظام اللاخطي. بينما تم استخدام المسيطر اللاخطي الثاني نوع (discrete time) للسيطرة على النظام الديناميكي اللاخطي بشكل مباشر. هذه الدراسة اعطت معلومات اضافية حول كيفية محاكاة هذين المسيطرين، و اعطت افضلية واضحة للمسيطر التقليدي (PID) على المسيطر نوع (discrete time) في السيطرة و التحكم بنظام منع انغلاق المكابح.

**الكلمات المفتاحية:** نظام منع انغلاق المكابح، نسبة انزلاق الاطار، المسيطر نوع (PID)، و المسيطر نوع (discrete time).

## Introduction

Antilock braking system (ABS) is an electronic auxiliary device that prevents the vehicle from skidding under abnormal driving conditions by trying to fix the frictional force between the road and the vehicle's tire as maximum as possible and control the brake oil pressure just below the critical point at which the wheel is locked. Another advantage feature of ABS is to prevent the vehicle from revolves around itself and cause the driver to miss control of it under heavy breaking conditions by its ability to lock and control each wheel on alone, that's leads to stop the vehicle at the straight situation.

The burning and bursting the aircraft's tire problem through landing according to slid the tire and keep a small spot for it always in contact with the road led to introduce the ABS at 1929. Later the ABS was designed to control four wheelers in 1970 and modernist to control two wheelers in 1988, according to an Australian study in Monash University Accident Research Centre in 2014. That's discover make possible to reduce the multiple vehicle crashes by 24% and the risk of run off road crashes is diminished by 41% (Budd Laurie, 2014).

The Authors in (Ayman ,2011). checked on the strategies utilized as a part of the design outline of ABS systems and summarized the last efforts in occurrences control techniques. Modern control systems strategies like fuzzy and neural network controls can be used as an integral of ABS control to reduce the qualitative aspects of human knowledge. While, the nonlinearity and time variety of the ABS dynamic model was described in (Champatiray *et.al.*,2014), the proposed sliding mode controller was developed position on the controlling the slip ratio parameter, which compels the system to become more stable during operation conditions and get better robustness of the sliding control system.

The skidding control model for targets of improving sleep tracking and deviation from the design desired point are formulated in (Nyandoro, 2011). Additionally, the system modelling is performed to build up a braking model incorporating an active suspension. The researchers in (Chankit , 2014) studied the nonlinear dynamic model depending on multi process factors, by design a torque controller to keep most advantageous of wheel slipping ratio, the vehicle linear velocity and the wheel rotation are estimated to represent the slip ratio.

Also the linear velocity is needed to determine the slip ratio during modelling the control of a motorcycle's hydraulic anti-lock brake system in (Shih, 2010.). The final design of a hydraulic control system is simulated and provided on a scooter for multi road tests.

A several experimental tests for a car equipped with a fuzzy logic and ABS interfaced together in a creative way researched in (Subbulakshmi, 2014). The work achieved its aim for designs and develops tuning abilities in spite of the fact that the ABS system is completely nonlinear system and it is so difficult to upgrade the classical controller with a fuzzy logical strategy.

The Authors in (Dragan , 2010) studied the complex issue due to its powerfully nonlinear and doubtful characteristics ABS control by design sliding mode control method using robust control technique. A laboratory experimental setup is utilized to model and simulate the dynamic model nonlinearity behavior of a quarter vehicle's tire.

The researcher in (AL-Mola, 2013) also confess that the ABS system has a nonlinear dynamic model and the brake of a vehicle must be controlled by maintain the piston pressure within a specific allowable point. To overcome these difficulties, an active force control (AFC) used as a fundamental of the study. The AFC methods consist of two stages of fuzzy logic controller and classical PID controller. The paper concluded that the modernist AFC methods give results better than the alone PID controller in simulating the ABS.

The overall aim of this work is to study and analysis the previous works done on ABS systems, focusing on the design methods, main components and the performance of ABS. Furthermore, the governing dynamic model of ABS is derived from the principal physical of quart tire of vehicle in direct contact with road by applying newton's laws. The challenge which is overcome the control issues of the ABS nonlinear dynamic model addressed in this contribution. The relationship between the ground friction coefficient and the slip ratio during different types of road conditions have been taken into account for the mathematical model and the control approach.

The dynamic model of ABS utilizing the Matlab\ Simulation program is designed and simulated. Based on the derived model, two different types of controller algorithms

are suggested to control the operation of the ABS system. Firstly, the ABS model controlled by the linear PID controller using linearization around specific working point. The second controller used the discrete time nonlinear controller to present ABS model's controller. The simulation results of the two controllers are compared to judge which one has better performance. The two systems are classified as multi input multi output systems (MIMO) and the slip ratio is the measuring parameter of the control model.

## Mathematical Modelling

### 1. Quarter Vehicle Dynamics

Essentially, the vehicle dynamic model is too complex and so difficult to drive from the principle physical model, so the quarter-car vehicle dynamic and mathematical model are assumed in the driving the model. The whole partial differential equations that govern the brakes behaviors are differentiated according to the same assumption.

Supposing a wheel of a quarter-car traveling initially with linear velocity  $v(t_0)$  at time  $t = t_0 = 0$  and at this moment the braking was activated led to make the car decelerated until

stopping at time  $t = t_f = 0$  at which the car become at completely stop with final linear velocity  $v(t_f) = 0$ , Apply of Newton's law at a free body diagram shown in Figure (1) gives the dynamic model equations of system's motion in  $x$  and  $y$ ; linear and vertical directions respectively. The vehicle translational dynamics are:

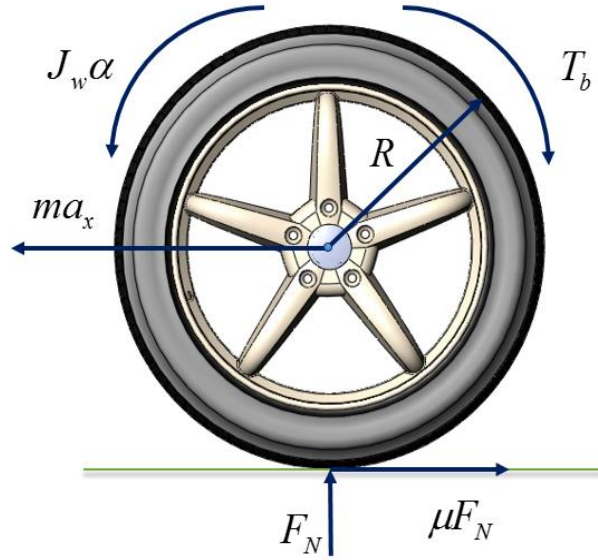


Figure 1: Quarter Vehicle Model.

$$\sum f_x = m \times a_x \Rightarrow m \times \dot{v}_x = -\mu \times F_N \quad \dots\dots\dots (1)$$

$$\dot{v}_x = -\mu \times \frac{F_N}{m} \quad \dots\dots\dots (2)$$

Where:  $m$  is the wheel's mass portion of the total car's mass,  $\mu$  is the coefficient of friction between the road and tire,  $F_N$  is the road reaction force acting normally on the wheel, and  $a_x$  is the vehicle's linear deceleration varying with time. Since,  $g$  is the gravity acceleration, the wheel vertical equilibrium equation can be derived as:

$$\sum f_y = 0 \Rightarrow F_N = m \times g \quad \dots\dots\dots (3)$$

The equation above can be rewritten in the next form by substitute in equation no. (2):

$$\dot{v}_x = -\mu \left( \frac{m \times g}{m} \right) = -\mu \times g \quad \dots\dots\dots (4)$$

Harness Newton's second law for the car deceleration at the amount  $\alpha$  due to brake torque  $T_b$  applied to the wheel of radius  $R$  and moment of inertia  $J_w$ , caused a direct decrease in the wheel angular velocity  $w$ . So, equilibrium equation can be drive as:

$$\sum M = J_w \times \alpha = J_w \times \dot{w} = \mu \times R \times F_N - T_b \quad \dots\dots\dots (5)$$

$$\dot{w} = \frac{\mu \times R \times F_N - T_b}{J_w} \quad \dots\dots\dots (6)$$

Under conventional operating conditions, the vehicle has harmoniously proportional between its linear velocity and the wheel angular velocity. Subsequently, its mess this concord directly after the braking because the brake torque caused the wheel velocity to reduce and may be the wheel to lock some time and create the frictional force between the tire and the road, while the vehicle continuous with its linear velocity according to its inertia. The different ratio between the vehicle linear velocity and the wheel angular velocity called the slip ratio and denoted by  $\lambda$ , it is formed as

$$\lambda = \frac{v_x - w \times R}{v_x} \quad \dots\dots\dots (7)$$

Differentiating the equation above with respect to time (t), get:

$$\dot{\lambda} = \frac{\dot{v}_x(1 - \lambda) - \dot{w} \times R}{v_x} \quad \dots\dots\dots (8)$$

Rewrite the equation above by substituting the equations (2) and (6), so that equation (8) represent in the final form as:

$$\begin{aligned} \dot{\lambda} &= \frac{\frac{-\mu \times F_N}{m}(1 - \lambda) - \left[ \frac{\mu \times R \times F_N - T_b}{J_w} \right] \times R}{v_x} = \frac{\frac{-\mu \times F_N}{m}(1 - \lambda) - \frac{\mu \times R^2 \times F_N}{J_w} + \frac{T_b \times R}{J_w}}{v_x} \\ &= \frac{-\mu \times F_N}{v_x} \left[ \frac{(1 - \lambda)}{m} + \frac{R^2}{J_w} \right] + \frac{R}{J_w \times v_x} \times T_b \quad \dots\dots\dots (9) \end{aligned}$$

Take into consideration that the ABS command the brake operation system by controlling the input torque  $u$  created by the actuator hydraulic system; the main parameters must be controlled and manipulated is the stopping distance, vehicle linear velocity, and the slip ratio.

## 2. Problem Formulation

The studying of relation between the slipping ratio and the road coefficient of friction is too important to understand the main control requirement, and achieve its main aims in steerability, stability, and the shortest stopping distance by keeping the friction between the tire and the road as maximum as possible during different operation and road conditions. The road surface conditions, tire side-slip angle, tire brand (summer tire, winter tire), vehicle speed, and the slip ratio between the tire and the road are the main parameters that effort directly in the value of coefficient of friction.

Frictional coefficient calculation usage in (Champatiray *et.al.*, 2015) was included in this research too. It gives value of coefficient of friction in terms of linear velocity and slip ratio.

$$\mu(\lambda, v_x) = [C_1(1 - e^{-C_2\lambda}) - C_3\lambda]e^{-C_4v_x} \quad \dots\dots\dots (10)$$

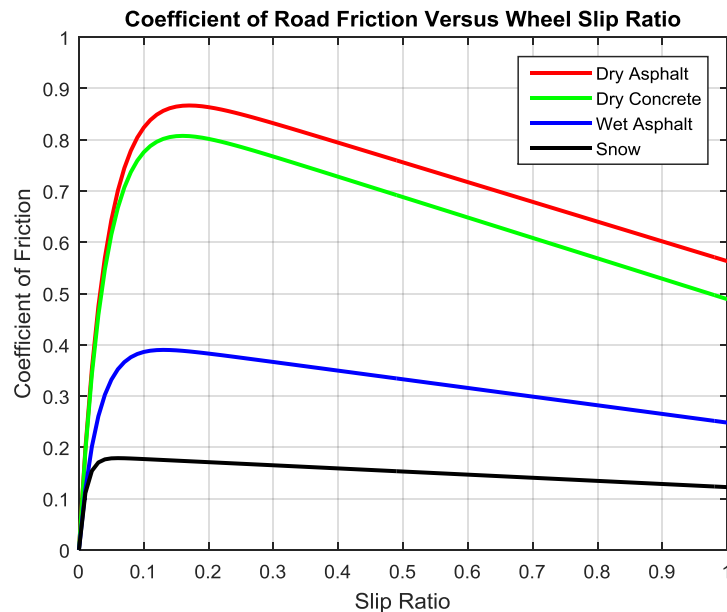
Where:  $C_1$  is the maximum value of friction curve,  $C_2$  is the friction curve shapes,  $C_3$  is the friction curve difference between the maximum value and the value at  $= 1$ , and  $C_4$  is the wetness characteristic value. It lies in the range 0.02–0.04 s/m.

The effective coefficient of friction between the tire and the road has an optimum value at a particular magnitude of the wheel slip ratio, this value contrasts in accordance with the road sort. Utilizing the form information in table (1) and the equation (10), the variance of road coefficient of friction as opposed to a wheel slip ratio can be plotted in figure (2).

**Table 1: Surface parameters for different road conditions (Champatiray et al, 2015).**

<i>Surfaces Types</i>	$C_1$	$C_2$	$C_3$	$C_4$
<i>Dry asphalt</i>	<i>1.2801</i>	<i>23.99</i>	<i>0.52</i>	<i>0.03</i>
<i>Wet asphalt</i>	<i>0.857</i>	<i>33.822</i>	<i>0.347</i>	<i>0.03</i>
<i>Dry concrete</i>	<i>1.1973</i>	<i>25.168</i>	<i>0.5373</i>	<i>0.03</i>
<i>Snow</i>	<i>0.1946</i>	<i>94.129</i>	<i>0.0646</i>	<i>0.03</i>

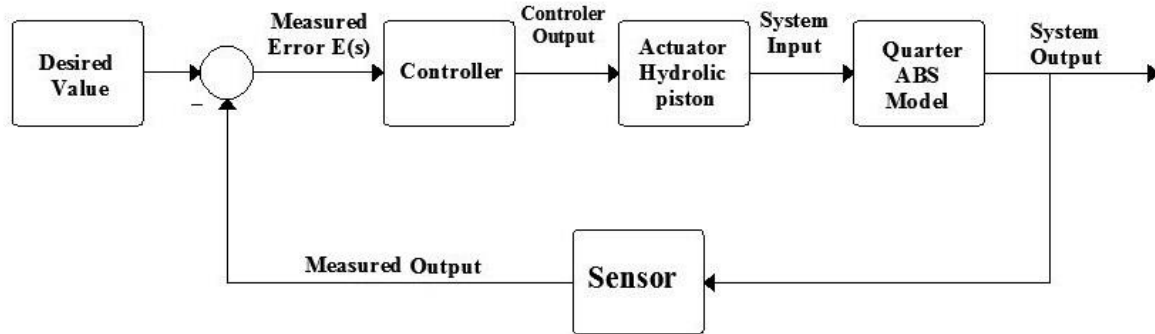
For the Figure (2) it is remarkable that, the magnitude of the road coefficient of friction achieved its perfect value when the slip ratio intermediary between 0.05 - 0.2 and its value has the greatest collapse at 1 slip ratio. In this way, the goal of ABS controller is to control the wheel slip proportion ( $\lambda$ ) to fix on estimated value of 0.1 to realize maximum frictional coefficient ( $\mu$ ) for various road surface and conditions.



**Figure 2: Road coefficient of Frictional VS wheel slip ratio.**

### 3 - Control System

Clearly noted from the above description in a problem formulation section that the ABS control system has major action to keep the slip ratio within a specific value. Figure (3) presents the general block diagram of ABS's feedback control system in which the sensor monitor and measure the slip ratio (system's output) and feedback it for comparison with the slip ratio desired value (system's input) to generate the error signal. This error signal feeds forward to a PID controller or discrete time controller, which assign the output, depending on the sign of the error.



**Figure 3: ABS Feedback Control System Diagram.**

Proportional-Integral-Derivative (PID) controller is the most usual manage control usage in industry and has been wide acceptance in technical control. The output response of the PID controller it's a results as processing of the error signal in three stages as the name of controller denoted; proportional, integral and derivative terms. Consider the control's transfer function in following form:

$$G(s) = K_p + \frac{K_I}{s} + K_D s \quad \dots\dots\dots (11)$$

Where:  $K_p$ ,  $K_I$ , and  $K_D$  all positive, indicate the gain for the proportional, integral, and derivative terms, separately.

A discrete-time controller used in this work to adjust the ABS main parameters by controlling the slip ratio. The modelling of ABS is a nonlinear feedback model, so it is better to control it by nonlinear controller like discrete-time controller. The algorithm is easy to tune and modify to assimilate higher-order mathematical model.

In order to study systems under digital control conditions, the sampled, discrete-time, and the system output  $y_k$  depends on the discrete-time input  $u_k$  must be described. The time discrete part of the time continuous systems are systems presented by difference equations. A first-order discrete-time system is described by the difference equation as in (Tomizuka, 1990):

$$y_{k+1} + ay_k = bu_k \quad \dots\dots\dots (12)$$

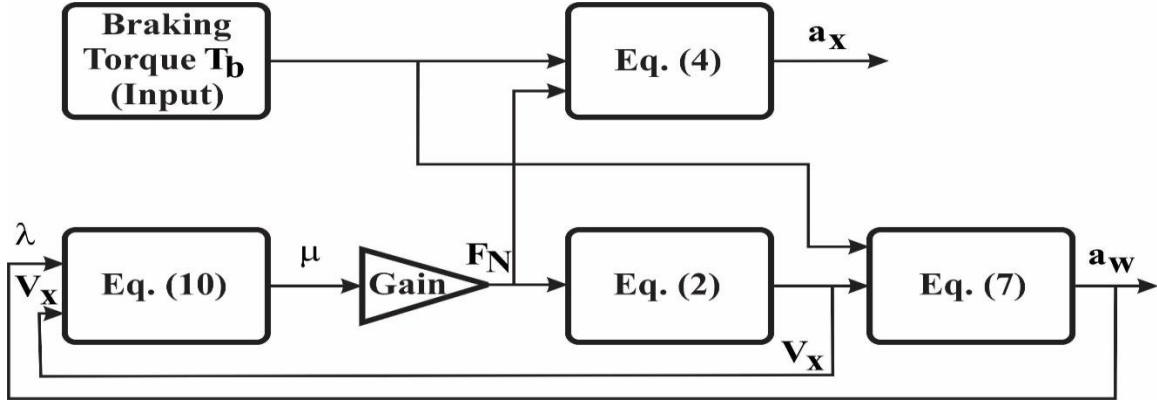
Similarly, a second-order discrete-time system can be characterized as follows:

$$y_{k+2} + a_1 y_{k+1} + a_2 y_k = b_1 u_{k+1} + b_2 u_k \quad \dots\dots\dots (13)$$

Where:  $K$  is constant and  $a_1$ ,  $a_2$ ,  $b_1$ , and  $b_2$  are controller gains. Appropriate values of the gains are determined by the try and error method by taking into account the value of the desired slip ratio using time discrete Simulink blocks tool.

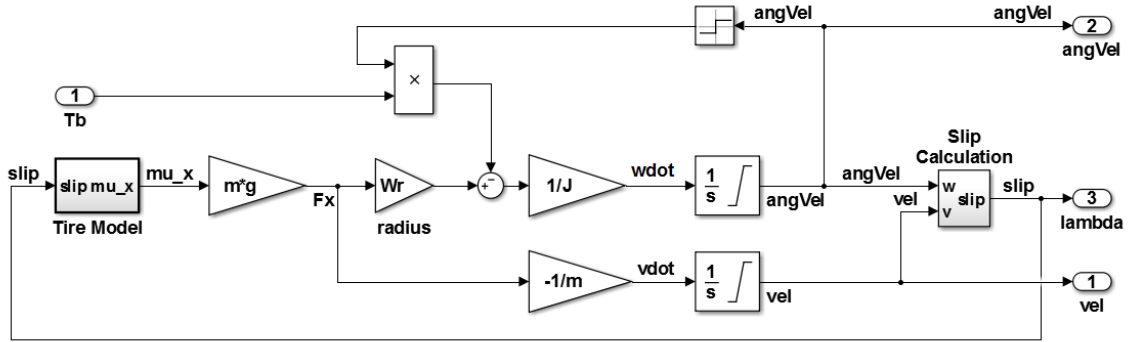
### 3. Simulation model

Keeping in mind that the final research goal to model and control the anti-lock brake with controller systems, including the same dynamic model and mathematical equations, the ABS is designed and simulated as shown in figure (4) according to the dynamic model equations derived in equations (2, 4, 7, and 10).



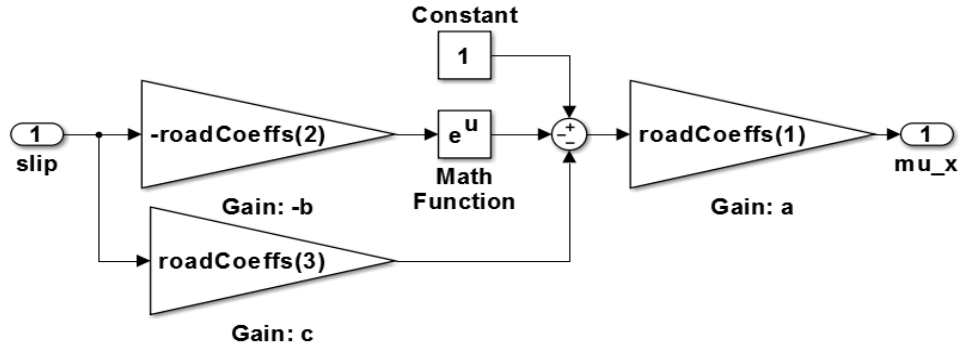
**Figure 4: ABS Governor Equations Representation Diagram.**

To achieve this aim, the model simulated with Matlab\Simulink program, many subgroups are utilized to abolish interference and simplified the simulation. Slip ratio  $\lambda$  determination described in Eq. (7) can be simulated as shown in figure (5).



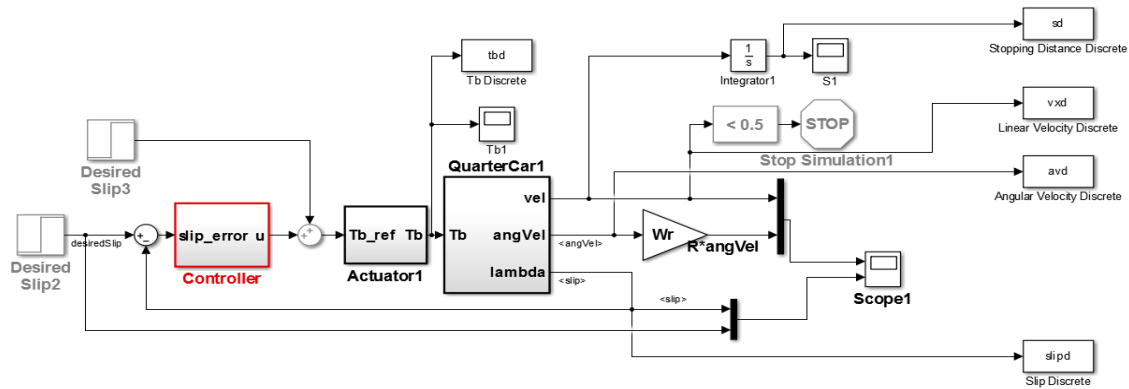
**Figure 5: Slip Ratio Computation Subgroup.**

While the road frictional coefficient  $\mu$  can be calculated and simulated with another subgroup in keeping with equation (10) as described in figure (6).



**Figure 6: Coefficient of friction Computation Subgroup.**

Simulink model shown in figures (7) was modified to describe the feedback term of the ABS model and take in mind that the total effected torque using to stop the wheel rotational is a joint-stock between braking constant torque  $T_b$  and the control manipulating variable  $u$ . Which previous the total input torque  $T$  can be formed as

$$T = u + T_b \quad \dots\dots\dots (14)$$


**Figure 7: ABS Feedback Control system Model.**

## Results & Discussion

In order to simulate the anti-lock brake system model with PID and time- discrete controllers and evaluate the models with Matlab\Simulink program, a real parameter of reference (Sharkawy, 2010) described in table (2) are utilized.

To overcome the issue that the magnitude of the slip ratio approaching to infinity when the vehicle linear and angular velocity closed to zero, the simulation was forced to give up and stopped when the value of linear velocity a crossed to  $0.5m/s$ .

**Table 2: ABS Parameters (Sharkawy, 2010).**

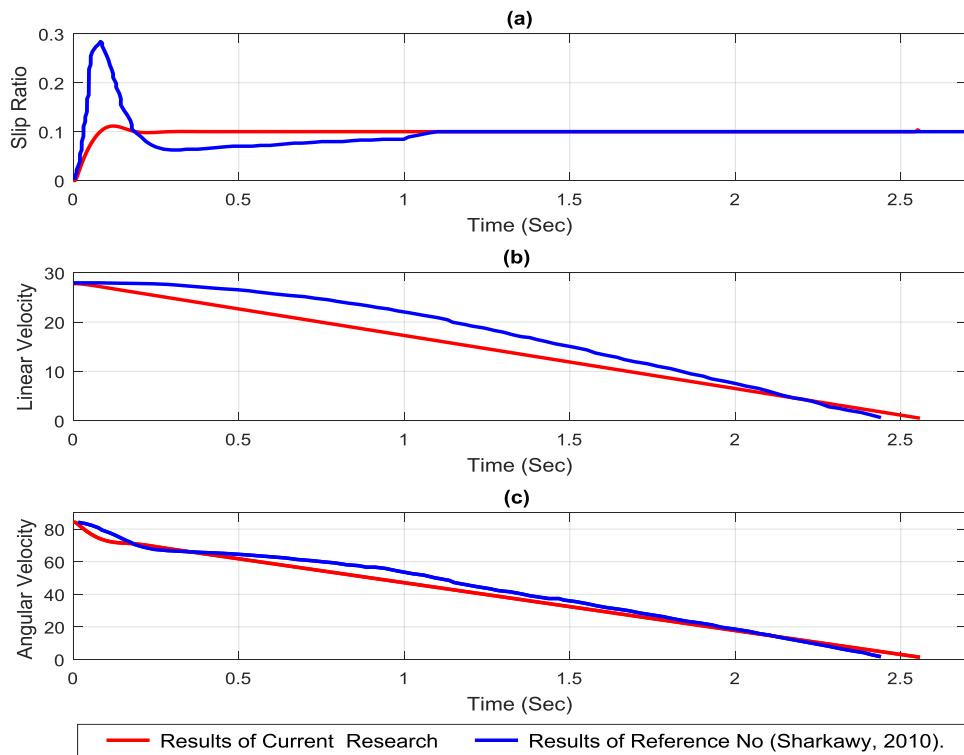
<i>Parameter</i>	<i>Character</i>	<i>Value</i>	<i>Unit</i>
<i>Desired Slip Ratio</i>	$\lambda_d$	0.1	---
<i>Wheel Radius</i>	$R$	0.33	$m$
<i>Initial Velocity</i>	$v_x$	100	$Km/h$
<i>Moment of inertia</i>	$J_w$	1.13	$Kg.m^2$
<i>Vehicle Mass</i>	$m$	342	$Kg$
<i>Gravity</i>	$g$	9.81	$m/s^2$



The discrete time controller model response results (slip ratio, linear velocity, and angular velocity) comparison with the results of reference (Sharkawy, 2010) with the same physical parameters and boundary conditions input to the models. Figure (8.a) shows a acceptant matching between the two models slip ratio, so the two models have the same behavior, except that the slip ratio response of the current work of discrete time controller has the 1.5 % overshoot, it's better than the 18.43 % overshoot of the reference (Sharkawy, 2010) with PID controller. But the response of results (Sharkawy, 2010) has 0.031 sec rising time faster than current results of 0.084 sec rising time, but these nominal differences in rising time accepted comparing with the huge differences in the response overshoot. This difference in the response specifications reflects on the linear velocity and angular velocity response comparisons figure (8.b and 8.c). In general accepted matching between the results refer to that the mathematical model deriving and the control system design have an acceptable result.

It is demonstrated in figure (9.a), that the control model starts to track the desired slip ratio directly after the braking process is beginning for each of the two used controller. The classic PID and the discrete time controllers are observed to have the faster time response to reach the 0.1 slip reference as depicted in figure (2), indicating that the vehicle produces good stability and steerability.

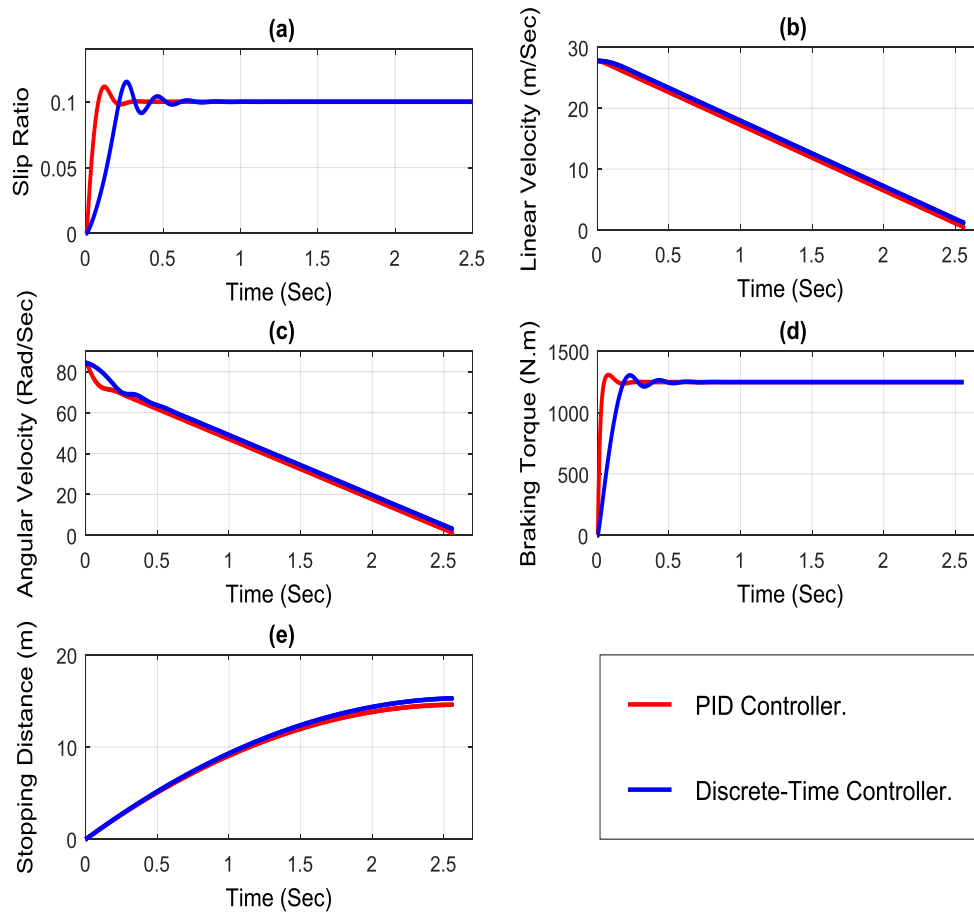
The linear and angular velocity responses of the PID controller and discrete time controller described in figures (9.b) and (9.c) respectively, the response appears clearly the transient and steady state zones, for the PID controller model the transient time equal to 0.285 sec, while the model of discrete time controller completes the transient time during 0.61 sec. The differences in the transient times between the two models due to the differences in the tracking the desired slip ratio described in figure (9.a).



**Figure 8: Comparison results With Reference (Sharkawy, 2010).**

The differences in the time response specification for the two models clearly appear in the braking torque applied to the wheel by the brake pedal and the actuator described in figure (9.d), the overshoot of the two response equal to 0.55% and the response of PID controller faster than the response of discrete time controller. Also the braking torque reach to the  $1304\text{ N.m}$  and still bind lower than its allowable maximum value equal to  $1500\text{ N.m}$ .

Figure (9.e), displays the all distance that the car travels from the moment of the applied brakes to the complete car stopping time, the control model with PID controller needs  $14.6\text{ m}$  to transfer smoothly from the dynamic state to the full stopping state during  $2.55\text{ sec}$ , while the control model with discrete time controller braking along  $15.3\text{ m}$  until final stopping during the same simulation time. These smoothly stopping distance of simulation matching with the ideal stopping distance, because if the car is essentially moving at linear velocity  $v$ , then it's stopping along a distance  $d$  directly after the brake is applying. The stopping distance can be calculated referring to reference (AMSI, 2013) as  $d = \frac{v^2}{2}$ . This formula means that the stopping distance is directly proportional to the square of the vehicle linear velocity at the instant the brakes are applied.



**Figure 9: Comparison results With Reference (Sharkawy, 2010).**

## Conclusions

The design and mathematical modelling of the ABS control system has been proposed in this paper. The dynamic model differential equations of ABS performance have been derived by considering the quarter vehicle dynamic motion. The state space equations are produced from the ABS nonlinear dynamic model and the Matlab\Simulink program is used to solve and model the mathematical equations. The overall ABSs' parameters controlled by forced the slip ratio to still around a specific value, when the road's coefficient of friction achieved its maximum value. The linearized and the nonlinear models have been controlled by using two different controller strategy, the analog classical PID controller and the digital discrete time controller respectively. The results show the effectiveness to use the ABS with analog and digital control system, but the PID controller works in general better than the discrete time controller because due to excellent breaking torque in transient response.

The presented results can be used as an essential element for experimental study and comparison with advanced control strategies like fuzzy and neural network controllers. Also, the further developments are required in order to include various road conditions and environmental effects.

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