Adaptive Control-based synchronization of chaotic systems with uncertain parameters and its application

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Abstract- **This paper is concerned with performance on the widely used control technique: adaptive control for synchronization between two identical chaotic systems embedded in the Master and Slave. It is assumed that the parameters of slave system are unknown. The required stability condition is derived to ensure the stability of error dynamics. Adaptive control laws are designed using appropriate parameters estimation law. The system parameters are asymptotically synchronized; thus the slave parameters can be identified. As an application, the proposed scheme is applied to secure communication system. The information signal is transmitted and recovered on the basis of identification parameters also the system is tested under the consideration of the noisy channel. Finally, through Numerical simulation results, the proposed scheme was success in the communication application.**

Index Terms- **Adaptive control, chaos, parameters identification, secures communications, synchronization.**

I. INTRODUCTION

The first method of the synchronization is introduced by Pecora and Carroll (PC) at 1990s where it deals with two systems one of them is the transmitter (master system) and the other is the receiver (salve system) where both of them are different at the initial conditions [1]. There are many researches are proposed to achieve the synchronization purpose beside the PC method like the OGY, time delay feedback and the adaptive synchronization method [2].

The synchronization failed problem can be solved and satisfied by the aiding of many types of synchronization methods which found lately for solving the failed problem as the observer design method [3]. The main idea of the synchronization is made two chaotic systems -which having a very sensitive behavior to initial conditions change- having the same dynamics. The synchronization is considered as the major idea of the communication which is based on the chaos and it is will be secured [4],[5]. In the adaptive synchronization which is considered as one type of the synchronizations method, the overall states of the master system is sent through the communication channel and the slave is built from the same differential equation The system behaviors which are generated by Rössler system are two types one of them is complex and the other is simple where it is determined with respect to parameters value that are building the master system so they are equal dynamical systems but the parameters of the slave do not known so these systems could be synchronized from deriving the laws of the control adaptive so the states for both systems will be the same and by the aid of the theory of Lyapunov stability [6].
In this paper, dynamics features of chaotic Rössler system has been discussed through the bifurcation diagram, Lapunov exponents spectrum, and phase portrait. The synchronization based on adaptive controller has been introduced. The scheme of synchronization consider the master and slave configuration. The unknown uncertain slave parameters have been estimated during the activation of adaptive controller. Also, as an application of the proposed synchronized chaotic systems, the full communication systems are optimized. Also, for real application, the AWGN is added to the transmitted signals and the assessment of the chaotic communication system with noisy channel has been achieved
The road map of the paper is as follows: section I will provide a general introduction. Section II discusses dynamical analysis of Rössler system. In Section III the synchronization method and the related results have been shown. The proposed scheme for a secure communication system is presented in Section V. Finally, Section VI conclude the paper.

II. RÖSSLER SYSTEM

This system is introduced by Otto Rössler where it can be defined by three first order differential equations and the non-linearity condition satisfied in the 3rd equation [7].

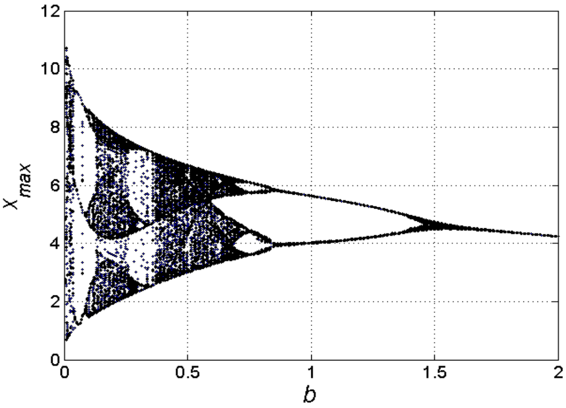
$$\begin{aligned}\dot{x} &= -(y+z)\\ \dot{y} &= x+ay\\ \dot{z} &= b+z(x-c)\end{aligned} \qquad (1)$$

The Rössler system exhibits different dynamics such as: Fixed point, periodic, quasi periodic and chaotic [8]. This system has two fixed points can be determined after equalizing system (1) to zero:

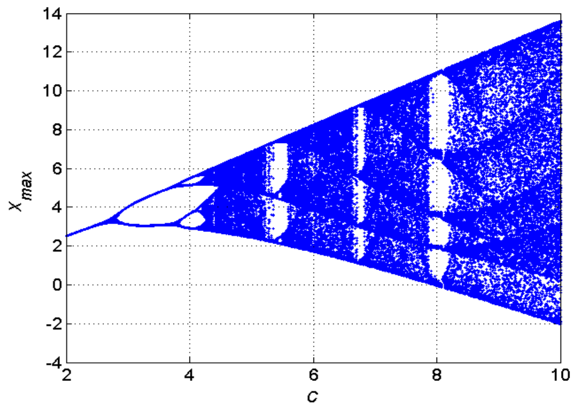
$$\begin{aligned}-(y+z) &= 0\\ x+ay &= 0\\ b+z(x-c) &= 0\end{aligned} \qquad (2)$$

It leads to the fixed points (P^e) equation:

$$P^e = \frac{c\pm\sqrt{c^2-4ab}}{2a}[a,-1,1] \qquad (3)$$



(a)



(b)

Fig. 1 The bifurcation diagram of Rössler system for $a = 0.2$.

Nonlinear characteristic of Rössler system can be observed by the bifurcation diagram, Lyapunov exponents spectrum, or phase portrait. The bifurcation diagram is widely used to describe the transition from the periodic motion to chaotic motion for a dynamic system. Fig.1a shows the system behavior due to change b parameter value. In Fig.1b, the bifurcation diagram is plotted due to changing c parameter value. We can see that the x_{max} keeps stable periodic oscillation for $b < 0.7$ and $c > 4.5$. The Lyapunov exponents measures the exponential rates of divergence and convergence of nearby trajectories in phase space of chaotic system. For convenience, we may order the Lyapunov exponents such that denoted Lyapunov exponents of system(1) are $L_{e_i}(i = 1,2,3)$ satisfying $L_{e_1} \geq L_{e_2} \geq L_{e_3}$. For instance, the Lyapunov exponents spectrum of system(1) when varying the parameter c is depicted in Fig.2.

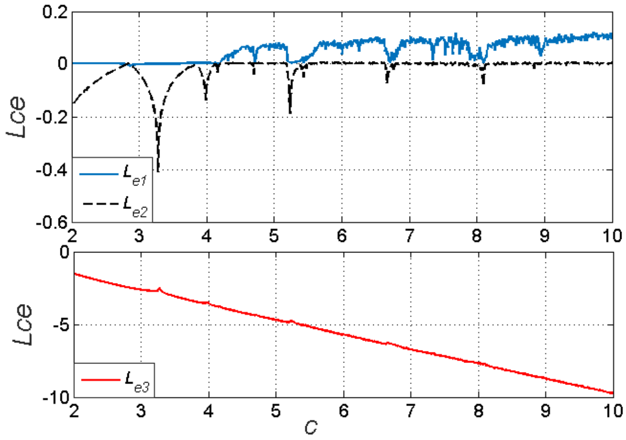
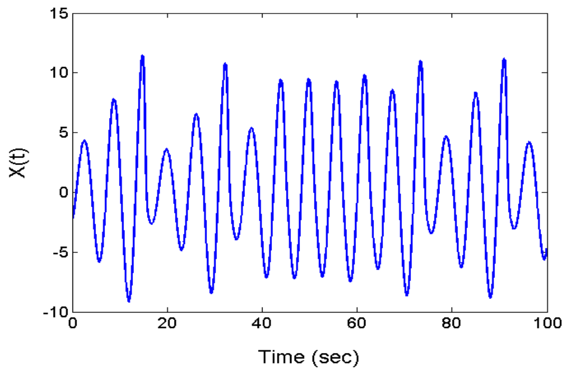


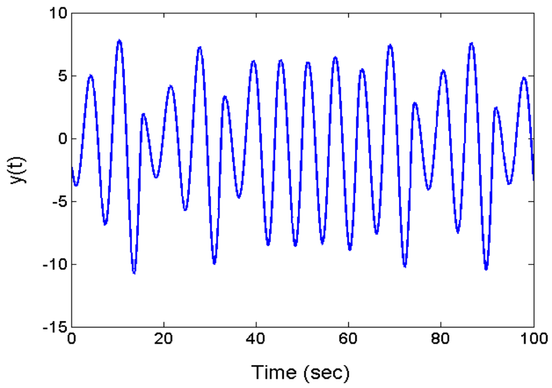
Fig. 2 Lyapunov exponents spectrum of Rössler system. $a = b = 0.2$.

When the selected parameters values are $a = b = 0.2$ and $c = 5.7$, the Lyapunov exponents are $L_{e_1} = 0.1$, $L_{e_2} = 0$, and $L_{e_3} = -5.2$. Its evidence that the system (1) exhibits chaotic dynamics and that is satisfying in the bifurcation diagram (see Fig. 1b).

Figures (3 and 4) show the numerical simulation results from system(1) with respect to the parameters values: $a = b = 0.2$ and $c = 5.7$. We have plotted the state variables $x(t)$, $y(t)$, and $z(t)$. Both the time series evaluation waveforms as shown in Fig.3 and the phase portrait as shown in in Fig.4, indicate a chaotic behavior. The chaotic attractor is produced from two unstable fixed points effect which obtained by substituting the system parameters in (3), one helical trajectory moved in spiral way near the $x - y$ plane around unstable fixed point, then the graphed helical is increased in a certain range and twisted to the z-axis [9]. The initial conditions is setting to $(x0, y0, z0) = (-2, -2,5)$.



(a)



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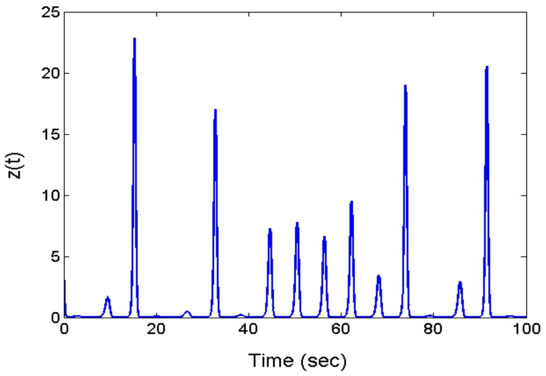
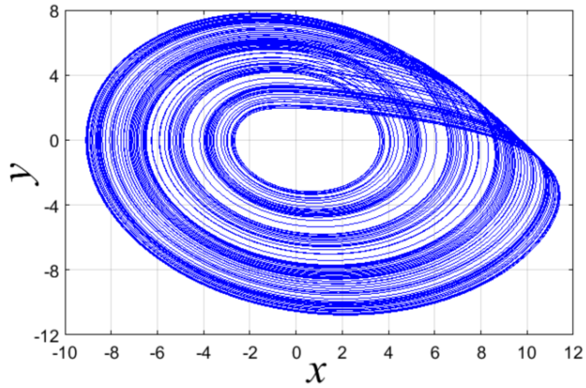
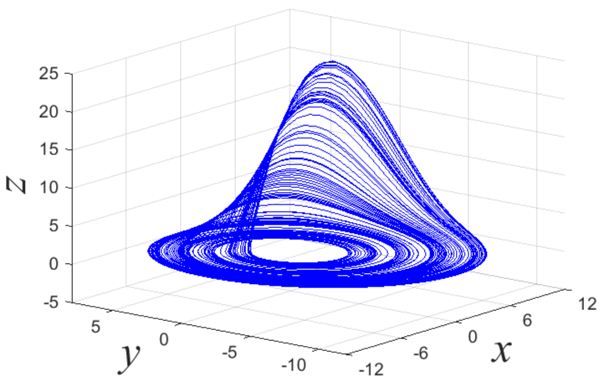


(c)

Fig. 3 Time series waveforms: (a) $x(t)$, (b) $y(t)$, (c) $z(t)$.



(a)



(b)

Fig. 4 (a) $\boldsymbol{x}-\boldsymbol{y}$ phase portrait and (b) 3D view in xyz −space of chaotic Rössler attractor.

III. The synchronization via control with unknown uncertain paramteres

Synchronization scheme used is the master-slave synchronization configuration. A configuration in which considers that the one of the two systems (master) drives the other one (slave) in unidirectional coupling. Both of the two systems are chaotic Rössler system. Synchronization via control technique is required for applications like secure communication in the case of unknown parameters of slave system to achieve both synchronization and parameter identification. In this section, we introduced a method of the adaptive controller to synchronized two identical chaotic Rössler systems [10].

-Problem declaration:

The master system equations are given by:

$$\begin{aligned}\dot{x}_m &= -(y_m + z_m)\\ \dot{y}_m &= x_m + a_1 y_m\\ \dot{z}_m &= a_2 + z_m(x_m - a_3)\end{aligned} \tag{4}$$

the slave system equations are be given by:

$$\begin{aligned}\dot{x}_s &= -(y_s + z_s) + u_1\\ \dot{y}_s &= x_s + b_1 y_s + u_2\\ \dot{z}_s &= b_2 + z_s(x_s - b_3) + u_3\end{aligned} \tag{5}$$

Where the m and s subscript refer to the master and slave systems, respectively, $a_i(i = 1,2,3)$ and $b_i(i = 1,2,3)$ refers to the master and salve parameters, respectively. The $u_i(i = 1,2,3)$ are the controller.

The synchronization errors are given by:

$$\begin{aligned}e_1 &= x_s - x_m\\ e_2 &= y_s - y_m\\ e_3 &= z_s - z_m\end{aligned} \tag{6}$$

The dynamics of errors equations are found according to (6):

$$\begin{aligned}\dot{e}_1 &= -e_3 - e_2 + u_1\\ \dot{e}_2 &= e_1 + e_2 b_1(t) + (b_1(t) - a_1)y_m + u_2\\ \dot{e}_3 &= (b_2(t) - a_2) + e_1 e_3 + e_3 x_m + z_m e_1\\ &\quad -e_3 b_3(t) \;\; - (b_3(t) - a_3) z_m \; + u_3\end{aligned} \tag{7}$$

Let b_1, b_2, and b_3 are unknown uncertain parameters, we can choose Lyapunov function for (7) as follows :

$$V(t) = \frac{1}{2}(e_1^2 + e_2^2 + e_3^2 + \boldsymbol{e}_d^2) \tag{8}$$

Where $\boldsymbol{e}_d$ is the error difference between the master and slave parameters values:

$$\boldsymbol{e}_d = \boldsymbol{b}(t) - \boldsymbol{a} \tag{9}$$

The error dynamics converge to zero according to the condition of the Lyapunov function [11]:

$$\dot{V}(t) \leq -(e_1^2 + e_2^2 + e_3^2) \tag{10}$$

$$\dot{V}(t) = \dot{e}_1 e_1 + \dot{e}_2 e_2 + \dot{e}_3 e_3 + (b_1(t) - a_1)\dot{b}_1(t) + (b_2(t) - a_2)\dot{b}_2(t) + +(b_3(t) - a_3)\dot{b}_3(t) \tag{11}$$

$$\begin{aligned}\dot{V}(t) = (-e_3 - e_2 + u_1)e_1 \; + \; (e_1 + e_2 b_1(t) + \\ (b_1(t) - a_1)y_m + u_2)e_2 + ((b_2(t) - a_2) \\ + \; e_1 e_3 + e_3 x_m \; + z_m e_1 - \; e_3 b_3(t) - \; z_m \\ (b_3(t) - a_3) + u_3)e_3 + (b_1(t) - a_1)\dot{b}_1(t) \\ +(b_2(t) - a_2)\dot{b}_2(t) \quad + (b_3(t) - a_3)\dot{b}_3(t)\end{aligned} \tag{12}$$

We choose the controller as follows :

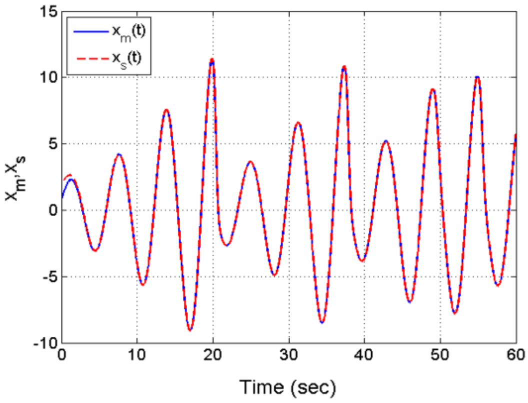
$$\begin{aligned}u_1(t) &= e_2 + e_3 - e_1 \\ u_2(t) &= -e_1 - e_2 b_1 - e_2 \\ u_3(t) &= -e_3 - x_m e_3 - z_m e_1 - e_1 e_3 + e_3 b_3\end{aligned} \tag{13}$$

The updating rules for the slave parameters are given by:

$$\begin{aligned}\dot{b}_1(t) &- y_m e_2 \\ \dot{b}_2(t) &= -e_3 \\ \dot{b}_3(t) &= z_m e_3\end{aligned} \tag{14}$$

IV. THE SIMULATION RESULTS

The confirmation of proposed controller method can be carried out by the numerical simulations. The parameters of chaotic Rössler systems are set to $a = b = 0.2$, and $c = 5.7$. In Fig. 5, the time trends for master and slave systems are shown. Figure(6) illustrates the error of synchronization between the master and controlled slave systems. The estimated parameters b_1, b_2, and b_3 converge to the system parameter values $a = b = 0.2$, and $c = 5.7$ (see Fig.7). The observed results evidence that the effectiveness of proposed method.



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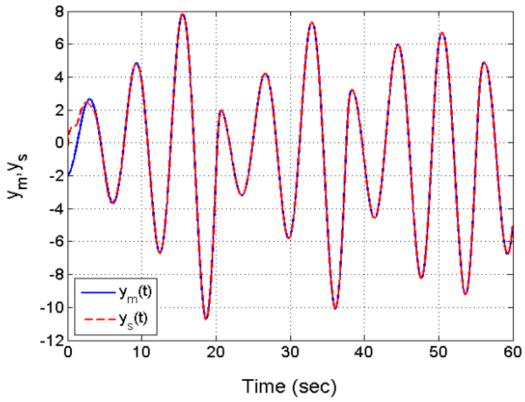

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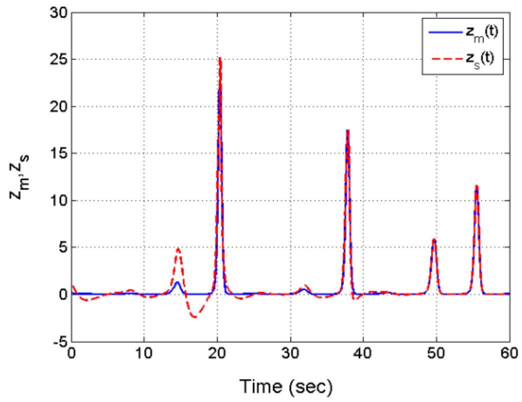

(c)

Fig. 5 The Synchronization between master and slave. The time series of master (blue line) and corresponding slave (red dashed line).

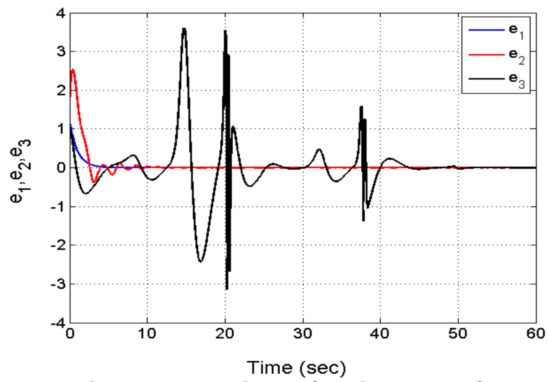

Fig. 6 The time series evolution of synchronization of errors.

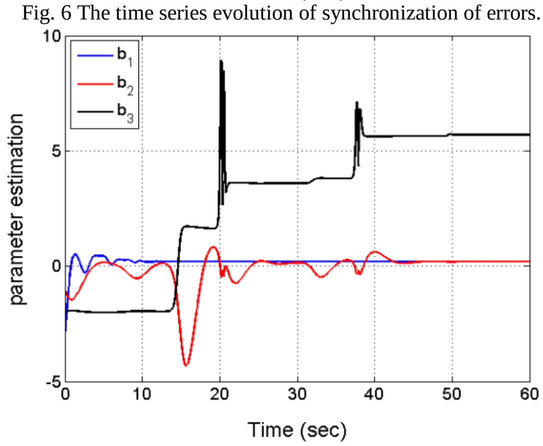

Fig. 7 The system parameters identification.

V. SECURE COMMUNICATION

In this paper, the form of Chaotic Parameter Modulation (CPM) is considered. The parameter c of system(1) used to transmit the useful information. According to the mentioned adaptive synchronization approach, in the following master system:

$$\begin{aligned} \dot{x}_m &= -(y_m + z_m) \\ \dot{y}_m &= x_m + a_1 y_m \\ \dot{z}_m &= a_2 + z_m(x_m - c(t)) \end{aligned} \quad (15)$$

the $c(t)$ refers to the modulated parameter which is contained the information. The parameter varied such that the system (15) will still be chaotic. It is assumed that the signals transmitted in the channel are the states of the system x_m, y_m, and z_m. Since the system (15) is chaotic, it is hard to extract the information from the signals in the channel.

The communication system has two parts: the master (transmitter) and the slave (receiver). The sketch of the proposed communication scheme based on adaptive synchronization as illustrated in Fig.(8). The transmitter consists of chaotic Rössler system and transform function F. The receiver is composed of an identical chaotic Rössler system, an adaptive controller, and a corresponding transform function F^{-1}. It is assumed that the receiver parameters are unknown uncertain and the signals transmitted in the channel are the states of the transmitter systems. The information signal $f(t)$ is modulated into parameter, that is $c(t) = F\big(f(t)\big)$. In Fig.8, $c'(t)$ represents the estimated parameter. It identified when the master system and slave system are synchronized via adaptive controller. The useful information can be recovered by transform function F^{-1} of the estimated parameter $c'(t)$, $\hat{f}(t) = F^{-1}(c'(t))$. We choose the information as a binary message, $f(t)$, and is modulated to the parameter $c(t)$ as follows:

$$c(t) = c_l + (c_h - c_l)f(t) \quad (16)$$

Where c_h and c_l are known scalars. Thus, $c(t) = c_l$ if the digital bit is $''0''$ while $c(t) = c_h$ if the digital bit is $''1''$.

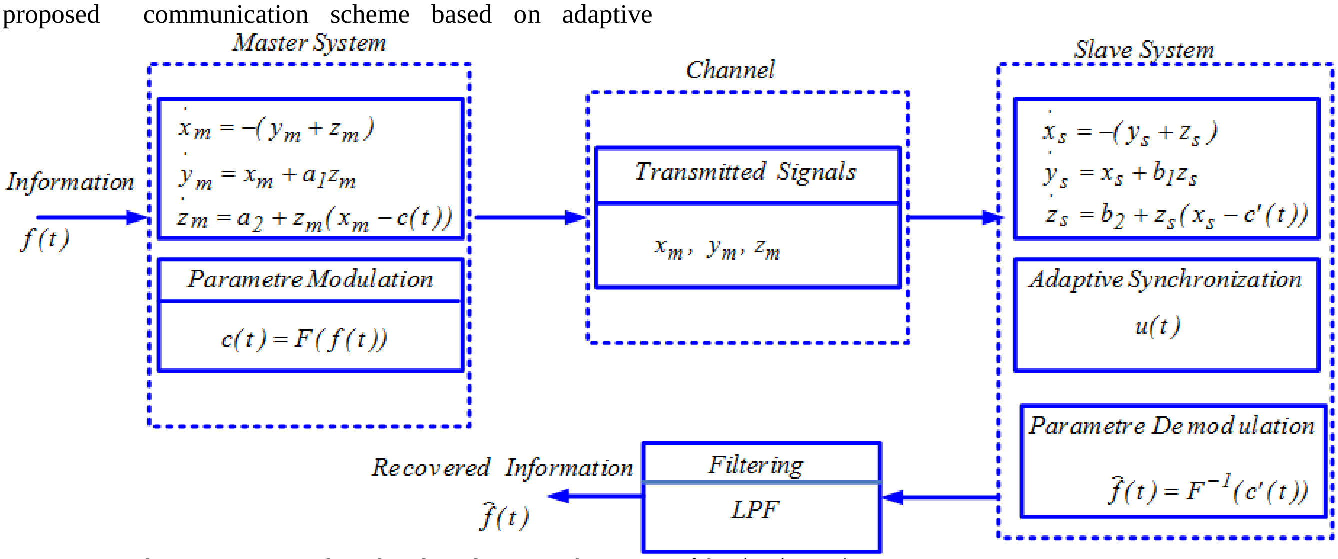

Fig. 8 The communication scheme based on adaptive synchronization of the chaotic Rössler systems.

The transmitter system synchronizes the slave system and the original information can be extracted as the follows:

$$\hat{f}(t) = \frac{c'(t) - c_l}{c_h - c_l} \quad (17)$$

The true values of unknown parameters of the master system as in previous section. The values of c_h and c_l are set to 5 and 5.7, respectively. The binary message transmitted by modulated parameter $c(t)$ was a $2^6 - 1$ (101011) pseudo-random (PN) code, as shown in Fig.9. The dynamical behavior of the resulting transmitter system (15) is shown in Fig.10. The resulting system is still chaotic. Therefore, an eavesdropper cannot be extracted the information from the transmitted signals $x_m(t)$, $y_m(t)$, and $z_m(t)$. The updating parameters rule are achieved successfully via the adaptive control law as depicted in Fig.11.

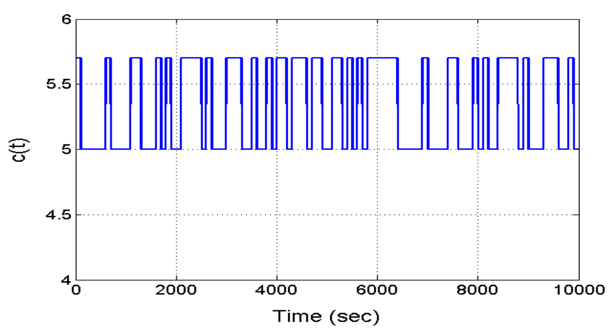

Fig. 9 Digital message by modulating the parameter value $c(t)$.

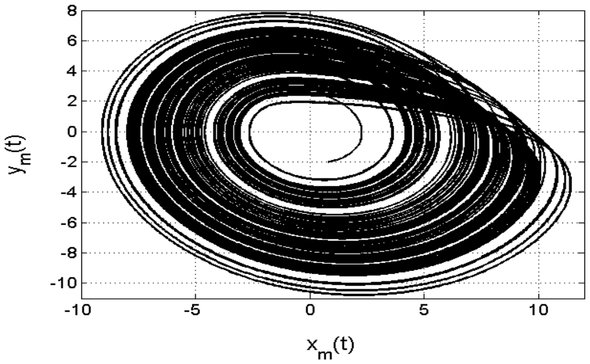

Fig. 10 Dynamic behavior of chaotic Rössler systems. x_m vs. y_m.

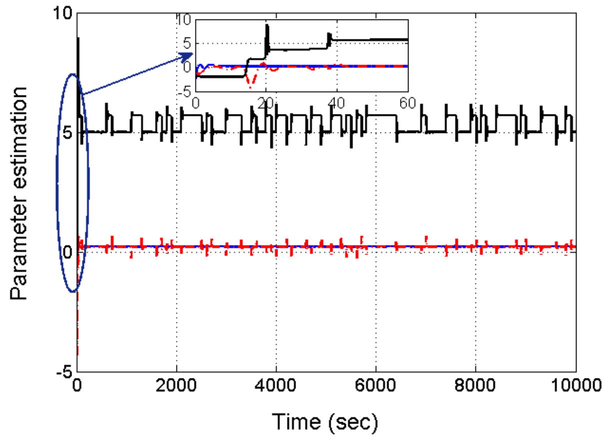

Fig. 11 The parameters estimation results.

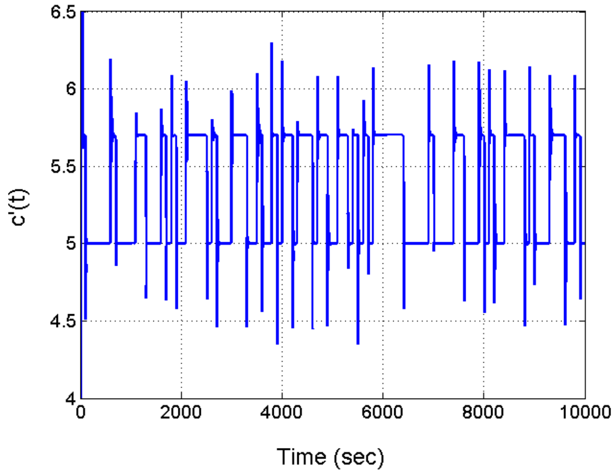

Fig. 12 The message demodulation result.

The estimated modulated parameter $c'(t)$ is shown in Fig.12. It is important to notice that the change of operating parameter causes some slight perturbations in the recovered message. these perturbations do not necessarily affect the correct recovery transmitted message.

In the real channel a specific value of noise which is added to the transmitted signals. In this paper, AWGN with different signal-to-noise ratio (*SNR*) cases are discussed. White Gaussian noise is presented by $n(t)$ as follows:

$$\begin{aligned} \dot{x}_m &= -(y_m + z_m) + n(t) \\ \dot{y}_m &= x_m + a_1 y_m + n(t) \\ \dot{z}_m &= a_2 + z_m(x_m - c(t)) + n(t) \end{aligned} \qquad (18)$$

In Fig.13(a, b, and c), the recovered information are shown with different values of *SNR*. Hence, the secure communication system well performs and recover the message up to signal-to-noise ratio is 35 dB. Further increase of noise level, i.e. decrease of *SNR* results in the distortion observed within the recovered message. Figure(14) shows that the recovering information with case of *SNR*= 45 dB after filtering.

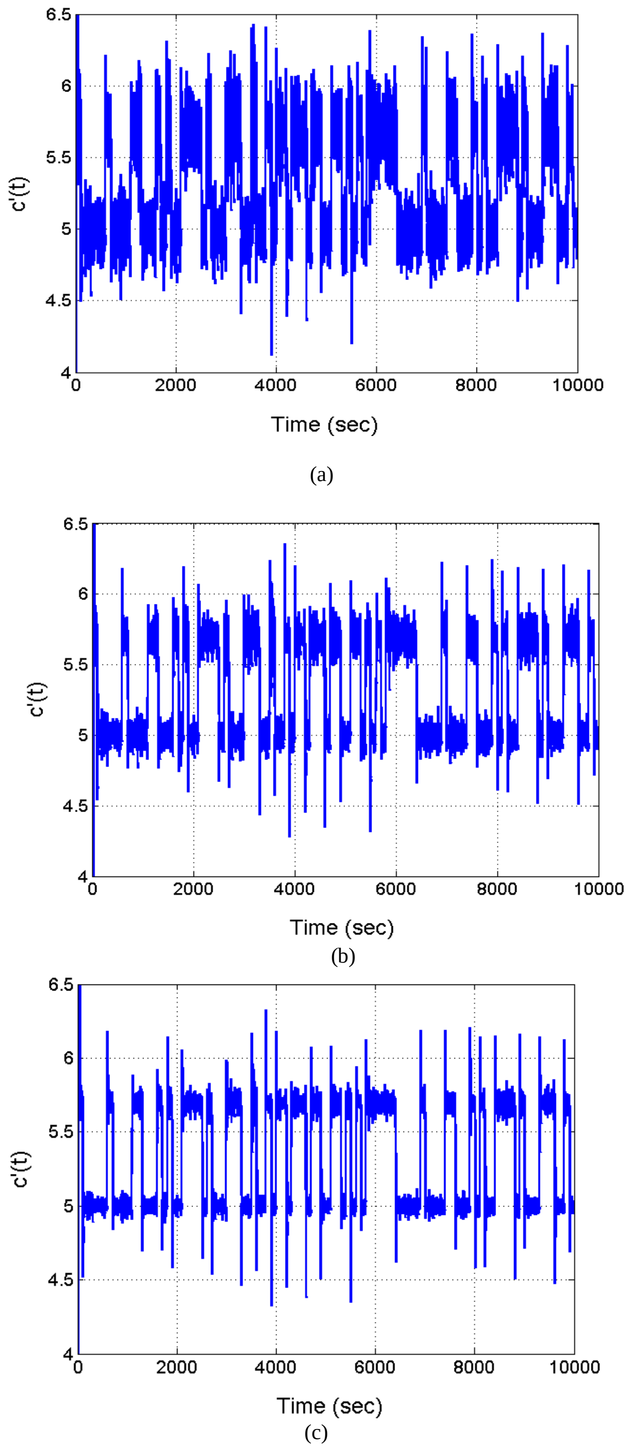

Fig. 13 The estimation of modulated parameter with the AWGN noisy channel where (a) *SNR*=35dB (b) *SNR*=40dB (c) *SNR*=45dB.

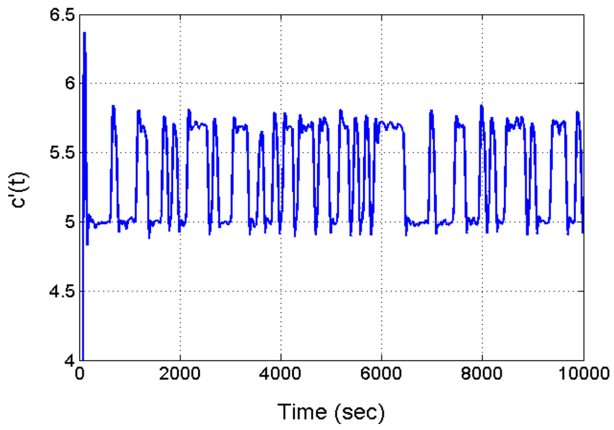

Fig.(14) Estimation of modulated parameter after filtering for $45dB$.

VI. Conclusion

the chaotic Rössler system has been discussed in this paper. The dynamical behavior features have been demonstrated by discussing the bifurcation diagram and Lyapunov exponents spectrum. Moreover, the synchronization scheme based on adaptive controller has been proposed. The approach of synchronization able to identify the eventually unknown slave parameters. Also, used in an application of secure communication in which the recovered information is obtained according to chaotic parameter modulation (CPM) technique. In the last part, the effect caused by channel noise, AWGN, is discussed. The results presented in this paper show potential of proposed adaptive synchronization method for secure chaotic communication system .

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