P- ISSN 1991-8941 E-ISSN 2706-6703 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access
2019,13 (1) :36-40

Some new types connectedness in topological space

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ARTICLE INFO

Received: 26 / 4 /2018
Accepted: 15 / 5 /2019
Available online: 19/7/2022
DOI: 10.37652/juaps.2022.171870

Keywords:
S^*g-separated .
$(S^*g-\alpha)$separated .
S^*g-connected
$S^*g-\alpha)$ Connected.
S^*g-home.
$(S^*g-\alpha)$home .

ABSTRACT

The main purpose of this paper is to introduce new definitions of separation, connectedness in topological spaces namely (S^*g-separation , $(S^*g-\alpha)$separation , S^*g-connected , $(S^*g-\alpha)$connected) by using the definitions $S^*g-(S^*g-\alpha)-$ *open sets* and study the relations among them . Also we study hereditary, topological property and show that $S^*g-(S^*g-\alpha)$connectedness is not - hereditary property but topological property.

Introduction :

Levine, N. [5] introduced and investigated generalized sets , generalized $\alpha-$open sets, Khan, M. and et.al [4] in 2008 introduce and provide the notion of S^*g-open sets in $(X,\mathcal{T})$. Mahmood I.Sabiha and Tareq , S. Jumana [6] introduce new class of sets, namely $S^*g-\alpha-$ open sets and show that the family of all $S^*g-\alpha-$open subset of topological space $(X,\mathcal{T})$ and study new function namely $S^*g-\alpha-$continuous function in topological spaces .

Introduced connected spaces defined as a topological space X is said to be disconnected space if X can be expressed as the union of two disjoint non – empty open subsets of X. Otherwise, X is connected space. Bourbaki, N. [2] , several properties of connected space in [7, 3,1].

In this work ,we introduce a new definition S^*g-separation, $(S^*g-\alpha)-$separation, S^*g-cconnected ,($S^*g-\alpha)$connected spaces using definitions $(S^*g-(S^*g-\alpha)-$ open sets and study the relations among them. At last we show that $(S^*g-(S^*g-\alpha)$ connected is not-hereditary property but topological property.

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Through this paper the topological spaces ($X,\mathcal{T}_x$) and $(Y,\mathcal{T}_y)$ (or simply X and Y) when A is a subset of X , *int(A)* , *cl(A)* which denote the interior and closure of a set A respectively [2] .

1- Preliminaries:

We recall the following definitions.

Definition (1): A subset A *of a space* X is said to be:

1- An S^*g-closed set [4] if $cl(A)\subseteq u$ where $A\subseteq u$ and $u\subseteq cl(int(u))$, the collection of all S^*g-closed subsets in X is denoted by $S^*GC(X)$.

The complement of an S^*g-closed is called S^*g- open set, the collection of all S^*g-open subsets in X is denoted by $S^*GO(X)$.

2- The S^*g-closure of A denoted by $S^*g-cl(A)$ is the intersection of all S^*g- closed subset of X which contains A [4].

3- An $S^*g-\alpha-$open set [6] if $A\subseteq int\,(S^*g-cl(int(A))$, the complement of an $S^*g-\alpha-$open set is defined to be $S^*g-\alpha-$closed, the family of all $S^*g-\alpha-$open subsets of X is denoted by $\tau^{S^*g-\alpha}$. The intersection of all $S^*g-\alpha-$closed sets containing A is denoted by $cl_{S^*g-\alpha}(A)$.

Definition (2): A function $f:X\longrightarrow Y$ is called $S^*g-(S^*g-\alpha)$continuous iff the inverse image of each open set of Y is a $S^*g-(S^*g-\alpha)$open subset of X [6 , 4].

2- On connectedness in a topological spaces.

We introduce the concept of $S^*g-(S^*g-\alpha)-$ connected space and study some of their properties .Also we study that $S^*g-(S^*g-\alpha)-$ connected is not hereditary property but topological property.

Definition 2-1: A topological space X is a S^*g-separation space if and only if there exist two disjoint S^*g-open subsets E and F of X, provided that

$E\cap S^*g-cl(F)=\varphi$ and$F\cap S^*g-cl(E)=\varphi$.

For example: we take $F=\{a,b\}, E=\{c\}$ are S^*g-open subset of $X=\{a,b,c\}$ be defined indiscrete topological spaces, then X is a S^*g-separation space .

Definition 2-2: A topological space X is a $S^*g-\alpha-$separation space if and only if there exists two disjoint $S^*g-\alpha-$open subsets F and E of X, whenever $E\cap cl_{S^*g-\alpha}(F)=\varphi$ and $F\cap cl_{S^*g-\alpha}(E)=\varphi$.

For example: we take $E=\{a,b\}$, $F=\{c,d\}$ are $S^*g-\alpha-$open subset of $X=\{a,b,c,d\}$ be on $\mathcal{T}=\{\varphi,X,\{a,b,c\},\{a,b,d\},\{a,b\}\}$,Then X is a $S^*g-\alpha-$separation space .

Remark 2-3 : in 2014 [6] proved that :

1- Every open set is an $S^*g-(S^*g-\alpha)-$open set but the converse is not true.

Also a separation space is$S^*g(S^*g-\alpha)-$ separation space .

But the converse is not true as in the two examples above .

2- S^*g-open sets and $S^*g-\alpha-$open sets are ingeneral independent , so we'll get that : each S^*g-separation and $(S^*g-\alpha)$separation space are ingeneral independent.

Example 2-4:

1- Every two disjoint a S^*g- $(S^*g-\alpha)-$open subsets of any space, then they are $S^*g-(S^*g-\alpha)$ seperation.

2- Every two disjoint a S^*g- $(S^*g-\alpha)-$closed subsets of any space, then they are $S^*g-(S^*g-\alpha)$ seperation.

Because (let E and F are disjoint $S^*g-(S^*g-\alpha)-$closed subset of X , we have $E\cap cl_{S^*g}(F)=E\cap F=\varphi$ and $F\cap cl_{S^*g}(E)=F\cap E=\varphi$ $(A=cl(A)$ iff A is closed)

By definition we get that E and F are $S^*g-(S^*g-\alpha)$separation.

Definition 2-5: A topological space X is said to be S^*g-**connected** if X can not be expressed as a disjoint union of two non-empty S^*g-open sets,

(i.e. there exists two S^*g-open subsets E and F of X provided that $F\cap E=\varphi$,

$F\cup E\neq X$).

"A topological space X is S^*g-disconnected space if it dose not achieve S^*g-connected space ".

Definition 2-6: A topological space X is said to be $S^*g-\alpha-$**connected** if X can not be expressed as a disjoint union of two non-empty $S^*g-\alpha-$open sets,

(i.e. there exists two $S^*g-\alpha-$open subsets E and F of X provided that

$F\cap E=\varphi$ and $F\cup E\neq X$). So

"A topological space X is $S^*g-\alpha-$disconnected space if it dose not achieve

$S^*g-\alpha-$connected space ".

Remark 2-7 :

1- [6] presented In 2014 that : " Every $S^*g-\alpha-$open set is $\alpha-$open set"

So every $S^*g-\alpha-$ connected space is $\alpha-$connected space.

2- For each S^*g-connected and $S^*g-\alpha-$connected space are in general independent.

As in the example (2-8).

3- A connected space is $S^*g-(S^*g-\alpha)$connectedness space .

4- A subset A of X is said to be $S^*g-(S^*g-\alpha)$ disconnected set if and only if it is the union of two non empty $S^*g-(S^*g-\alpha)$ separated sets. So A is said to be $S^*g-(S^*g-\alpha)$ connected if and only if it is not $S^*g-(S^*g-\alpha)$ disconnected.

Example 2-8:

1- Let $X=\{a,b,c,d\}$ on $\mathcal{T}=\{X,\varphi,\{a,b,c\},\{a,b\}\}$.Then X is S^*g-connected space (because there exists two S^*g-open subsets F and E of X such that $F=\{a\}$ and $E=\{b\}$, whenever $\{a\}\cap\{b\}=\varphi$ and $\{a\}\cup\{b\}\neq X$, but not $S^*g-\alpha-$connected.

2- let $X=\{1,2,3,4\}$ on $\mathcal{T}=\{\varphi,X,\{1\}\}$ .Hence X is $S^*g-\alpha-$connected and X is $\alpha-$connected space, but not S^*g-connected space.

Theorem 2-9:

A subset E of X is $S^*g-(S^*g-\alpha)$disconnected if and only if it is expressed as a union of two non-empty $S^*g-(S^*g-\alpha)$separated subsets of X.

Proof : $\Rightarrow$ suppose that E is S^*g- disconnected , then $E=A\cup B$ where $A \text{ and } B$ are two S^*g-disjoint non empty closed sets ,

Assume that $A \text{ and } B$ are S^*g-separated subsets of X.

$A\cap S^*g-cl(B)=(A\cap E)\cap S^*g-cl(B)$

$=A\cap S^*g-cl_{\mathcal{T}^*{}_E}(B)=A\cap B=\varphi$, So $B\cap S^*g-cl_{\mathcal{T}_E}(A)=B\cap A=\varphi$

$\Leftarrow$ suppose that $E=A\cup B$ where $A \text{ and } B$ are S^*g-open sets disjoint non-empty S^*g-separated subsets of X.

We have $A\cap S^*g-cl(B)=(A\cap E)\cap S^*g-cl(B)=\varphi$ and so that

$B\cap S^*g-cl_{\mathcal{T}_E}(A)=(B\cap E)\cap S^*g-cl(A)=\varphi$, we get that E is the union of non-empty S^*g-separated subsets of E ,Thus E is S^*g-disconnected .

In the same way we demonstrate for $S^*g-\alpha-$ open set .

Corollary 2-10 : If a space X is $S^*g(S^*g-\alpha)$ separation space , then X is the union of two disjoint non-empty $S^*g(S^*g-\alpha)-$closed subsets of X.

Proof : let $X=E\cup F$ where as $E \text{ and } F$ are S^*g-separated sets,

then $S^*g-cl(E)=S^*g-cl(E)\cap(E\cup F)$

$=S^*g-cl(E)\cap E)\cup S^*g-cl(E)\cap F)$

$=S^*g-cl(E)\cap E=E$ (by def. 1-2) .So E is S^*g-closed set .

Similarly F is S^*g-closed set .

We demonstrate the same style for the $S^*g-\alpha-open$ set .

As above noted hence that $\alpha-$ connected is topological property .

Corollary 2-11 :" A space X is a union of two disjoint non-empty"

$S^*g-(S^*g-\alpha)-$open subsets of X, then X is $S^*g-(S^*g-\alpha)$ disconnected .

Proof : suppose that $X=E\cup F$ where $as\ E \text{ and } F$ are disjoint non-empty

S^*g-open sets , then $E=F^c$ is S^*g-closed .So X is S^*g-disconnected .

If P is any property in X , then we call P hereditary if it appears in a relative topological space if we say P is not- hereditary.

Remark 2-12 : The $S^*g-(S^*g-\alpha)$connectedness is not – hereditary property.

As in the example:

Example 2-13:

(1) let $X=\{a,b,c,d\}$ and $\mathcal{T}=\{\varphi,X,\{a\},\{b,c,d\},\{a,c,d\},\{c,d\}\}$

Then X is S^*g-connected space (because $\exists\{a,d\},\{c\}$ are S^*g-open sets such that $\{a,d\}\cap\{c\}=\varphi$ and $\{a,c\}\cup\{a\}=\{a,c,d\}\neq X$).

If $A=\{a,b\}\subseteq X$ and $\mathcal{T}_A=\{\varphi,A,\{a\},\{b\}\}$, Then $(A, \mathcal{T}_A)$ is not $S^*g -$ connected ($\exists \{a\},\{b\}$ are $S^*g -$open sets when ever

$\{a\} \cap \{b\} = \varphi$ and $\{a\} \cup \{b\} = X$) .

(2) let $X = \{a, b, c, e\}$ on $\mathcal{T} = \{\varphi, X, \{b\}, \{c\}, \{a, b\}, \{b, c\}, \{c, e\}, \{a, b, c\}, \{b, c, e\}\}$,
Then X is $(S^*g - \alpha)$connected space , but if $A = \{b, c\} \subseteq X$ and $\mathcal{T}_A = \{\varphi, A, \{b\}, \{c\}\}$ .So A is not $S^*g - \alpha -$connected space (because $\exists \{b\},\{c\}$ are $S^*g - \alpha -$open sets ,

$\{b\} \cap \{c\} = \varphi$, $\{b\} \cup \{c\} = \{b, c\}$) .

Definition 2-14:

A map $f: (X, \mathcal{T}_x) \longrightarrow (Y, \mathcal{T}_y)$ is said to be $S^*g - (S^*g - \alpha)$homeomorphism

($S^*g - (S^*g - \alpha)$home. For short) if

(1) f is bijective map.
(2) f and f^{-1} are $S^*g - (S^*g - \alpha)$continuous .

Let P be any property in $(X, \mathcal{T}_x)$ if P is carried by $(S^*g - (S^*g - \alpha)$ home. to another

space $(Y, \mathcal{T}_y)$ we say P is topological property .

Now, we introduce the main result about a topological property the a $S^*g - (S^*g - \alpha)$ connected.

Theorem 2-15 : A $S^*g -$connected space is a topological property .

Proof : A $f: (X, \mathcal{T}_x) \longrightarrow (Y, \mathcal{T}_y)$ be $S^*g -$home. and space X is $S^*g -$connected space.

So we have to prove that $(Y, \mathcal{T}_y)$ be $S^*g -$connected space .

If $(Y, \mathcal{T}_y)$ be $S^*g -$disconnected space , then there exists two disjoint non- empty $S^*g -$open subsets of Y , E and F are subsets of Y such that $E \cap S^*g - cl(F) = \varphi = F \cap S^*g - cl(E)$ and $E \neq \varphi$, $F \neq \varphi$; as f is $S^*g -$continuous ,

We have $f^{-1}(E) = E_1$ and $f^{-1}(F) = F_1$ where E_1 and F_1 are $S^*g -$ open in X.

$E_1 \cap S^*g - cl(F_1) = \varphi$, $F_1 \cap S^*g - cl(E_1) = \varphi$

Hence X is $S^*g -$disconnected but that is contradiction

Since $F_1 \cup E_1 = f^{-1}(F_1) \cup f^{-1}(E_1) = f^{-1}(F_1 \cup E_1)$

Hence X is $S^*g -$disconnected , $f^{-1}(Y) = X$, We get the assume is not true .

Then $(Y, \mathcal{T}_y)$ is $S^*g -$connected space .

Theorem 2-16: A $S^*g - \alpha -$connected space is a topological property .

Proof : A $f: (X, \mathcal{T}_x) \longrightarrow (Y, \mathcal{T}_y)$ be $S^*g - \alpha -$home. and space X is

$(S^*g - \alpha)$ connected space. So we have to prove that $(Y, \mathcal{T}_y)$ be $(S^*g - \alpha)$connected space. If $(Y, \mathcal{T}_y)$ be ($S^*g - \alpha$)disconnected space , then there exists two disjoint non- empty $(S^*g - \alpha) -$open subsets of Y , E and F are subsets of Y such that $E \cap cl_{S^*g-\alpha}(F) = \varphi = F \cap cl_{S^*g-\alpha}(E)$ and $E \neq \varphi$, $F \neq \varphi$, as F is $S^*g - \alpha -$continuous

We have $f^{-1}(E) = E_1$ and $f^{-1}(F) = F_1$ where E_1 and F_1 are$S^*g - \alpha -$open in X.

$E_1 \cap cl_{S^*g-\alpha}(F_1) = \varphi$, $F_1 \cap cl_{S^*g-\alpha}(E_1) = \varphi$

Hence X is $(S^*g - \alpha)$ disconnected but conditions

Since $F_1 \cup E_1 = f^{-1}(F_1) \cup f^{-1}(E_1) = f^{-1}(F_1 \cup E_1)$

Hence X is ($S^*g - \alpha$)disconnected , $f^{-1}(Y) = X$,We get that the assumption is not true .Then $(Y, \mathcal{T}_y)$ is $(S^*g - \alpha)$connected space .

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بعض الأنواع الجديدة للإتصال في الفضاءات التبولوجية

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الخلاصة:

أن الغرض الرئيسي من هذا البحث هو تقديم تعاريف جديدة للاتصال والتفريق التي اسميناها $(S^*g-$separation, $(S^*g-\alpha)$ separation , S^*g-connected, $(S^*g-\alpha)$connected) باستخدام المجموعات المفتوحة من نمط $S^*g-(S^*g-\alpha)$ ودرسنا العلاقة بينهم. كذلك درسنا الصفات الوراثية و التبولوجية وبرهنا أن الاتصال هو ليس صفة وراثية ولكنه صفة تبولوجية .