Natural Convection Heat Transfer in Arc Shape Wall Porous Cavity Filled with Nano-Fluid

Muneer A. Ismael
Department of Mechanical Engineering
University of Basrah
College of Engineering

Huda A. AL-Mayahi
Department of Mechanical Engineering
University of Basrah
College of Engineering

Ihsan N. Jawad
Department of Mechanical Engineering
University of Basrah
College of Engineering

***Abstract*- Natural convection heat transfer in porous cavity with arc shape wall filled with nanofluid is studied numerically. The right arc shape wall of the cavity is heated at constant temperature (Th) while the left wall is kept cold at constant temperature (Tc), and the other horizontal walls are thermally insulated. The governing equations of the heat transfer and nanofluid flow are solved Flex PDE software. A temperature independent nanofluids properties models are adopted. The investigated parameters are the nanoparticles volume fraction Ø= (0-0.2), Rayleigh number Ra (10-1000) and arc center Ce (1-∞). The results are presented by contour of streamlines, isotherms and the average Nusselt number. The results have showed that the average Nusselt number decreases with increasing Ce and increases with increasing Ra and Ø.**

Key words: Natural convection, nanofluid, porous, wavy wall

1. Introduction

Natural convection heat transfer is an important engineering phenomenon in industry with widespread applications such as cooling of electronic components, grain storage, storage of radioactive waste, solar collectors, etc, many other applications are reviewed by [1,2]. A major limitation against enhancing the heat transfer in such engineering system is the inherently low thermal conductivity of the commonly used fluids, such as, air, water and oil. Nanofluids were introduced in order to circumvent the above limitation. A nanofluid is a dilute suspension of solid nanoparticles with the average size below 100 nm in a base fluid, such as, water, oil and ethylene glycol [3,4]. In fact, convective heat transfer is affected by the thermo-physical properties of the nanofluid such as viscosity and thermal conductivity [5, 6]. Heat and fluid flow in cavities filled with porous media are well-known natural phenomenon and have attracted interest of many researchers due to its many practical situations. Among these: insulation materials, geophysics applications, building heating and cooling operations. Sun and Pop [8] studied numerically free convection heat transfer in a triangle cavity filled with a porous media saturated with nanofluids with flash mounted heater on the left vertical wall. The temperature of the inclined wall is lower than the heater, and the rest of walls are adiabatic. The results showed that, the maximum value of Nusselt number is obtained by decreasing the enclosure aspect ratio and lowering the heater position with the highest value of Rayleigh number and the largest size of heater. Aminossadati et al. [9] conducted a study of free convection heat transfer with localized heat source at bottom of a nanofluid-filled enclosure. The top and vertical walls of the enclosure are maintained at relatively low temperature. The results showed that, the increase of Rayleigh numbers strengthens the natural convection flow. Chamkha and Ismael [10] studied theoretically conjugate heat transfer in porous cavity filled with nanofluids and heated by a triangular thick wall. Three types of nanoparticles were examined under a wide range of nanoparticles volume fraction, triangular wall thickness, triangular wall to base-fluid conductivity ratio and Rayleigh number. The results showed that, the Nusselt number is an increasing function of the Rayleigh number and the heat transfer within nanofluids-saturated porous media may be enhanced or deteriorated with increasing the nanoparticles volume fraction. Ismael and Mahdi [11] studied theoretically conjugate heat transfer in a differentially heated porous cavity filled with nanofluids. The vertical right wall of the solid is in contact with the nanofluid saturated porous medium contained in cavity. The results showed that the natural convection inside the nanofluid-saturated porous medium cavity is enhanced with increasing the solid wall conductivity and decreasing its thickness, and vice versa. Mahmoodi and Hashemi [12] studied numerically natural convection fluid flow and heat transfer inside C-shaped enclosures filled with Cu-water nanofluid by using finite volume method and SIMPLER algorithm. They showed that the mean Nusselt number increased with increase of Rayleigh number and volume fraction of Cu nanoparticles regardless of aspect ratio of the enclosure. Mohmoodi [13] studied numerically free convection fluid flow and heat transfer in a square cavity with an inside thin heater filled with various water based nanofluids by using finite volume method. The results showed that, at low Rayleigh numbers, average Nusselt number is higher for the horizontal positioned heater while at high Rayleigh numbers the position of the heater does not increase heat transfer with increase of Rayleigh number and the volume fraction of the nanoparticles. Oztop and Abu-Nada [14] studied numerically the effect of using different nanofluids on natural convection flow field and temperature distributions in partially heated square enclosure from the left vertical wall using Maxwell-Garnetts (MG) model. Finite volume technique is used to solve the governing equation. The results showed that, both increasing the value of Rayleigh number and heater size enhances the heat transfer and flow strength keeping other parameters fixed, the type of nanofluid is a key factor for heat transfer enhancement and it was found that when increasing the heater size, the difference in heat transfer value increases and depends mainly on the type of nanofluid used.

However, the aim of the present numerical study is to investigate the natural convection in an arc shape wall porous cavity saturated with nanofluid. The authors are not aware of any work that has investigated free convection in arc shape wall porous cavity filled with nanofluids. Therefore, the present work will study this geometry by using two models of thermal conductivity of Cu-nanoparticles. The results are to be presented in terms of streamline, isotherms and average Nusselt number.

2. Mathematical analysis

Figure 1 is a schematic diagram of the cavity. The centre of the arc shape right wall is (xo, yo) and radius (ce) while the other walls are straight with length (L). The right wall is heated at constant temperature (Th) higher than the cold left wall temperature (Tc) while the horizontal walls are insulated. The cavity filled with water-based nanofluid containing Cu-nanoparticles with uniform volume fraction. The flow is assumed to be incompressible, laminar and obeys Darcy model; also, it is assumed that both the nanoparticles and the base fluid are in thermal equilibrium. Moreover, the nanofluid and the matrix forming the pores are in thermal equilibrium too. It is worth to mention that the governing equations of the present model are obtained by eliminating the pressure gradient terms from the two components of the momentum equations. Hence, the governing equations can be written as follow [11]:

Continuity equation:

$$\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}=0 \qquad (1)$$

Momentum equation:

$$\frac{\partial u}{\partial y}-\frac{\partial v}{\partial x}=-\frac{gK(\emptyset\rho_p\beta_p+(1-\emptyset)\rho_f\beta_f)}{\mu_{nf}}\frac{\partial T}{\partial x} \qquad (2)$$

Energy equation:

$$u\frac{\partial T}{\partial x}+v\frac{\partial T}{\partial y}=\alpha_{nf}\left(\frac{\partial^2 T}{\partial x^2}+\frac{\partial^2 T}{\partial y^2}\right) \qquad (3)$$

where β is the thermal expansion coefficient, ρ is the density, K is the permeability of the porous medium, μ is the dynamic viscosity, α is thermal diffusivity of the porous medium and ϕ is nanoparticles volume fraction. The subscripts *p and f* stand for solid nanoparticles and base fluid respectively. Numerous formulations for the thermo-physical properties of nanofluids are proposed in the literature. In the present study, the relations which depend on the nanoparticles volume fraction only and which were proven and used in many previous studies were adopted in the present study and as follows:

Thermal diffusivity (Abu-Nada [5]):

$$\alpha_{nf}=\frac{k_{nf}}{(\rho Cp)_{nf}} \qquad (4)$$

Heat capacity (Khanafer et al. [15]):

$$(\rho Cp)_{nf}=(1-\emptyset)(\rho Cp)_f+\emptyset(\rho Cp)_p \quad (5)$$

Viscosity (Brinkman [16])

$$\mu_{nf}=\frac{\mu_f}{(1-\emptyset)^{2.5}} \qquad (6)$$

where k and (ρCp) represent the thermal conductivity and heat capacity, respectively. Introducing the following dimensionless set: $X=x/L$, $Y=y/L$, $U=uL/\alpha_f$, $V=vL/\alpha_f$, $\theta_{nf}=(T_{nf}-T_c)/(T_h-T_c)$, Ce= ce/L and the dimensionless definition of the stream function as: $U=\partial\Psi/\partial Y$, $V=-\partial\Psi/\partial X$. Equations (2), (3) can be rewritten for the nanofluid-saturated porous medium as

$$\frac{1}{(1-\emptyset)^{2.5}}\nabla^2\Psi=-Ra\left[(1-\emptyset)+\emptyset\left(\frac{\rho_p}{\rho_f}\right)\left(\frac{\beta_p}{\beta_f}\right)\right]\frac{\partial\theta}{\partial X} \qquad (7)$$

$$\frac{\partial\Psi}{\partial Y}\frac{\partial\theta}{\partial X}-\frac{\partial\Psi}{\partial X}\frac{\partial\theta}{\partial Y}=\frac{\alpha_{nf}}{\alpha_f}\nabla^2\theta \qquad (8)$$

Where: $\nabla^2=\partial^2/\partial X^2+\partial^2/\partial Y^2$ is the Laplace operator and Ra is the Rayleigh number of the porous media defined as $Ra=gK\rho_f\beta_f(T_h-T_c)L/(\mu_f\alpha_f)$.

Equations (7) and (8) are subjected to the following boundary conditions:

$\Psi=0$ on the solid boundaries.

$\theta=0$ on the left vertical wall, $X=0$, $0\leq Y\leq 1$

$\theta=1$ on the right arc shape wall, $X=1$, $0\leq Y\leq 1$

$\partial\theta/\partial Y=0$ on the upper and bottom horizontal walls $0\leq X\leq 1$, $Y=1, 0$.

In the present work two models for the thermal conductivity of the nanofluid are to be used:

i. Maxwell model

According to the well-known Maxwell model [17] the effective thermal conductivity of Cu nanofluid (K_{eff}) for spherical Cu nanoparticles is expressed as:

$$K_{eff}=k_f\left[\frac{(k_{np}+2k_f)-2\emptyset(k_f-k_{np})}{(k_{np}+2k_f)+\emptyset(k_f-k_{np})}\right] \qquad (9)$$

Where, K_{np} is the thermal conductivity of dispersed Cu nanoparticles and k_f is the thermal conductivity of pure water. This classical model has been studied by many researchers [18-21].

ii. Modified Maxwell model:

The modified Maxwell model takes into account a nano-layer with a solid-like structure formed by the liquid molecules close to a solid surface [22]. According to this model, the thermal conductivity of the pure water-Cu nanofluid (K_{eff}) having spherical Cu nanoparticles is:

$$K_{eff}=k_f\left[\frac{(k_{eq}+2k_f)+2(k_{eq}-k_{np})(1+\sigma)^3\emptyset}{(k_{eq}+2k_f)-(k_{eq}-k_{np})(1+\sigma)^3\emptyset}\right] \qquad (10)$$

Where: σ is the ratio of the thickness of nano-layer to the original radius of nanoparticles (h_{nL}/r_{np}), k_f is the thermal conductivity of pure fluid, ϕ is the solid volume fraction of the nanofluid and k_{eq} is the equivalent thermal conductivity of nanoparticles and their layer which defines as:

$$K_{eq}=k_{np}\left[\gamma\frac{2(1-\gamma)+2(1+\sigma)^3(1+2\gamma)}{-(1-\gamma)+(1+\sigma)^3(1+\gamma)}\right] \qquad (11)$$

Where, γ is the ratio of thermal conductivity of nano-layers upon the thermal conductivity of the nanoparticles ($\gamma = k_{nL} / k_{np}$). In this work, it is assumed that h_{nL} = 2nm, r_{np} = 3nm and $k_{nL} = 100\ k_f$. Yu and Choi [22] agreed that for these conditions.
The average Nusselt number of the vertical left wall:

$$Nu_{av} = -\frac{k_{nf}}{k_f}\int_0^1 \frac{\partial \theta}{\partial X}\, dY \qquad (12)$$

A comparison study is presented between the results obtained from the original Maxwell and modified Maxwell models. The thermo physical properties of Cu- nanofluid are presented in Table 1.

3. Numerical Solution and Validation:

In the present study, Flex PDE software which implements the finite element (Galarkin) method [23] is applied in the solution of nonlinear system of equations (7) & (8). In order to determine a proper grid for numerical solution, five different grids namely, 1143, 1251, 2400, 5595 and 25346 for different error 0.1, 0.01, 0.001, 0.0001 and 0.00001 are employed for the numerical simulation. The average Nusselt number of cold side wall for these grids are shown in Fig. 2. A computational model also is validated for (Natural convection in a square porous cavity) performed by Baytas and Pop [24]. Figure 3 shows that, the comparison of the flow and thermal fields between the present work and Baytas &Pop [24]. The results show a good agreement and from these comparisons it can be decided that the current code can be relied on to predict the flow and thermal fields for the present work.

4. Results and discussion

This section presents selective results for the contour maps of stream line, isotherms and the average Nusselt number. The results are presented graphically to show the effect of the curvature of the right wall (Ce), the nanoparticles volume fraction (ϕ), and Rayleigh number (Ra) on the steady natural convection inside a porous cavity. The studied parameters ranges are: Ce =1-∞, ϕ =0-20%, and Ra =10-1000. The thermal conductivity of the nanofluid was expressed by two models, namely: Maxwell model and Yu and Choi model. Therefore, figures 4 to 6 are presented for Maxwell model while the next other figures 7 to 10 are presented for both Maxwell and Yu and Choi model.

Figure 4 presents the effect of right wall curvature, for Ra =1000 and ϕ =0.2, on the flow and thermal fields. The fluid movement formed in positive (counter-clockwise) stream lines with central core-vortex. The direction of vortex rotation is widely discussed in literature, that is the fluid adjacent to the hot curved right wall will be warmed up and moves vertically, when it blocked by the adiabatic top horizontal wall, it will return to the left cold vertical wall and from there, the fluid will fall down towards the adiabatic bottom horizontal wall, hence forming counter-clockwise (positive) vortex. The main vortex changes from vertically elongated at higher Ce to horizontally expanded at lower (Ce) value. Moreover, the vortex strength of the higher (maximum) curvature (Ce =1) is greater than the other two lesser curvatures. This refers to the large amount of the heated fluid due to the available larger heated fluid gained by higher curvature. However, a general insight into Fig. 4 discovers a stagnant sector zone formed in the right upper corner of the cavity. This stagnant zone is larger with bigger curvature of right wall. This stagnant zone may be attributed to that the hot fluid above the hot wall will be squeezed and still there. The isotherms presented in the right hand side of Fig. 4 emphasize this attribution. They (the isotherms) reveal an isothermal zone within these stagnant zones.

Figure 5 shows the effect of nanoparticle volume fraction ϕ on the flow and thermal fields, for Ce =1 and Ra = 1000. The main aspect of increasing ϕ from ϕ = 0 to ϕ = 0.2 is the weakened of streamlines strength and the change of the central vortex from horizontally to vertically elongation. This behavior can be elucidated as follow: nevertheless increasing of ϕ leads to increase of thermal conductivity of the nanofluid (Eq. (9)), and the dynamics viscosity (Eq. (6)) are increased also. Therefore, the bit reduction in the strength of the vortex is due to the dominated increase of inertia and viscous forces which restrict the nanofluid motion. No significant different can be distinguished between the isotherms of different ϕ (see the right column of Fig. 5). This because of the dominance convection available already with Ra = 1000.

Figure 6 shows the effect of Rayleigh number Ra, for ϕ = 0 and Ce =1, on the flow and thermal fields. When Ra is very low (Ra = 10) as shown in Fig. 6a, the vortex is relatively weak and vertically elongated with two stagnant zones localized at the right upper and lower corners. This is an indication of conduction dominance as can be demonstrated by the mostly vertical isotherms of Fig. 6a (right). Increasing of Ra leads to convection dominance which can be distinguished by the change of the vortex from vertically elongation to horizontally expansion and by the mostly horizontal isotherms as shown in Fig. 6c. On the other hand, the lower stagnant zone tends to be diminished with increasing Ra and the corresponding isotherms show that the lower isothermal zone is occupied by a thermal boundary layer.

Now, turning to examine the model of Yu and Choi [22] of the nanofluid thermal conductivity, and how it affects the natural convection within the cavity. Keeping in mind that the volumetric fraction of nanofluid should not exceeds ϕ 5% [22]. However, Fig. 7 shows a sample of contours maps for both fluid flow and thermal fields for Ra =1000, ϕ = 0.05 and Ce = 2. From this figure it can be concluded that there are no significant difference between the contours of the two models except that the vortex strength of Yu and Choi model is larger than that of Maxwell model. Indeed, this result is expected because the Yu and Choi model gives rise to the nanofluid thermal conductivity because it takes into account the formation of solid-liquid layer adjacent to the nanoparticle surface. Therefore, and for brevity, the other contours maps will not be displayed for Yu and Choi model.

Figure 8 depicts the variation of the average Nusselt number with Ra for both models. As it well known, in the literature, Nu increases monotonically with Ra. This figure reveals that the increased thermal conductivity gained by Yu and Choi model enhances the convection dominance. The increase of Nu with Ra is faster in Yu and Choi modal [22]. This effect can be seen in Fig. 7, where the vortex core becomes stronger in Yu and Choi model.

Figure 9 depicts the effect of right wall curvature on Nu for both models. The maximum convention could be obtained by maximum possible curvature of the right wall (Ce =1). The Nu values becomes insensitive to Ce beyond Ce =3. However, the percentage increment in Nu when Ce is changed from Ce = 3 to 1 is: 3.6% for Maxwell model and 6% for Yu and Choi model.

Figure 10 is prepared to obtain a comprehensive information about the behavior of both models with ϕ and for different ranges of Ra. Regarding Yu and Choi model [22], increasing of ϕ leads to increase of Nu and for all Ra ranges, this means that the effect of thermal conductivity gained by increasing ϕ overcomes the effect of inertia and viscous forces. Meanwhile, the Maxwell model offers some different behavior, which can be distinguished by the adverse effect of ϕ on Nu at high Ra (Ra =1000). This adverse effect i.e. decreasing of Nu with increasing of ϕ tells us that at high Ra, the effect of increasing of both the inertia and viscous forces overcome the effect of thermal conductivity. As previously discussed, for low Ra values (Ra < 100), the increase of ϕ leads to an increase of Nu.

5. Conclusion

The natural convection of nanofluid-filled in a porous cavity with arc shape right wall is studied numerically using finite element method implemented in Flexpde software package. Two models for nanofluid thermal conductivity are studied with different ranges of Rayleigh number Ra, radius of right wall curvature Ce and nanoparticles volume fraction ϕ. The obtained results led us to the following conclusions:

1- Due to the curvature of the right hot wall, the maximum Nu is obtained with maximum curvature.
2- The percentage increase in Nu gained by changing Ce is larger with respect to Yu and Choi model than Maxwell model.
3- Nusselt number is an increasing function of Rayleigh number and is faster with taking into account the solid-fluid layer effect of nanofluid (Yu and Choi models).
4- For all studied ranges of Ra, Yu and Choi model offers an increase of Nu with ϕ. While in the Maxwell model, a retardation of Nu is seen at high Ra.

6. References

[1] Eiyad Abu-Nada, Ali J. Chamkha, Effect of fluid variable properties on natural convection in closures filled with a Cuo-EG- water nanofluid, International Journal of thermal science 49 (2010) 2339-2352.

[2] Saleh H., Roslan R., Hashim I., Natural convection heat transfer in a nanofluid-filled trapezoidal enclosure, International Journal of heat and mass transfer, 54 (2011) 194-201.
[3] Sheikh Zadeh G.A., Arfmanesh A., Kheirkah M.H., Abdollah R, Natural convection of Cu-water nanofluid in a cavity with partially active side walls, European Journal of Mechanics B/fluids 3 (2011) 166-176.
[4] Massimo Corcione, Empirical correlating equations for predicting the effective thermal conductivity and dynamics viscosity of nanofluids, Energy convection and Management, 52 (2011) 787-793.
[5] Eiyad Abu-Nada, Effect variable viscosity and thermal conductivity of AL_2O_3-water nanofluid on heat transfer enhancement in natural convection, International Journal of heat fluid flow 30 (2009) 679-690.
[6] Farhad Talebi, Amir Houshang Mahmoudi, Mina Shahi, Numerical study of mixed convection flows in a square lid-driven cavity utilizing nanofluid, International communications in Heat and Mass Transfer 37 (2010) 79-90.
[7] Singh A. K, Thermal conductivity of Nanofluids, Defense Science Journal, 58 (2008) 600-607.
[8] Qiang Sun, Ioan Pop, Free convection in a triangle cavity filled with a porous medium saturated with nanofluids with flush mounted heater on the wall, International Journal of thermal Sciences 50 (2011) 2141-2153.
[9] Aminossadati S. M, Ghasemi B., Natural convection cooling of a localized heat source at the bottom of a nanofluid-filled enclosure, European Journal of Mechanics B/fluid 29 (2009) 630-640.
[10] Ali J. Chamkha, Muneer A. Ismael, Conjugate heat transfer in a porous cavity filled with nanofluids and heated by a triangular thick wall, International Journal of Thermal Sciences, 67 (2013) 135-151.
[11] Muneer A. Ismael, Ahmed Abdul Kareem Mahdi, Conjugate heat transfer in a Differentially Heated Porous Cavity Filled with nanofluids, Basrah Journal for Engineering Science (2013) 123-139.
[12] Mostafa Mahmoodi, Seyed Mohammed Hashemi, Numerically Study of Natural convection of a nanofluid in C-shaped enclosures, International journal of thermal Sciences, 55 (2012) 76-89.
[13] Mostafa Mahmoodi, Numerical Simulation of free convection of nanofluid in a square cavity with an inside heater, International journal of thermal Sciences, 50 (2011) 2161-2175.
[14] Hakan F. Oztop, Eiyad Abu-Nada, Numerical Study of natural convection in partially heated rectangular enclosures filled with nanofluids, International Journal of Heat and Fluid Flow, 29 (2008) 1326-1336.
[15] Khanafer K., Vafai K., Lightstone M., Buoyancy-driven heat transfer enhancement in a two-dimensional enclosure utilizing nanofluids, Int. J. Heat Mass Transfer 46 (2003) 3639–3653.
[16] Brinkman H.C., The viscosity of concentrated suspensions and solutions, J. Chem. Phys. 20 (1952) 571-571.

[17] Aminossadati S. M., Ghasemi B., Enhanced natural convection in an isosceles triangular enclosure filled with nanofluid, Computers and Mathematics with Applications 61 (2011) 1739-1753.

[18] Maxwell J. C., A Treatise on Electricity and Magnetism, (Vol. II, Oxford University press, Cambridge).

[19] Khodadadi J.M., Hosseini Zahed S. F., Nanoparticles-enhanced phase change materials (NEPCM) with great potential for improved thermal energy storage, Int. communications in Heat and Mass Transfer 34 (2007) 534-543.

[20] Ean Hin Ooi, Viktor Popov, Numerically Study of influence of nanoparticle shape on the natural convection in CU-water nanofluid, Int. Journal of thermal Science 65 (2013) 178-188.

[21] Raj Kamal Tiwar, Manab Kumar Dos, Heat transfer augmentation in a two-sided lid-driven differentially heated square cavity utilizing nanofluids, Int. communications in Heat and Mass Transfer 50 (2007) 2002-2018).

[22] Yu W. and Choi S.U.S., The role of interfacial layers in the enhanced thermal Conductivity of nanofluids: A renovated Maxwell model, Journal of Nanoparticles Research 5 (2003) 167-171.

[23] Backstrom G., Fields of physics by finite element analysis using Flex PDE, by GB publishing and Gunnar Backstrom Malmo, Sweden.

[24] Baytas A. C., Pop I., Free convection in oblique enclosures filled with porous medium, International journal of Heat and Mass Transfer 42 (1999) 1047-1057.

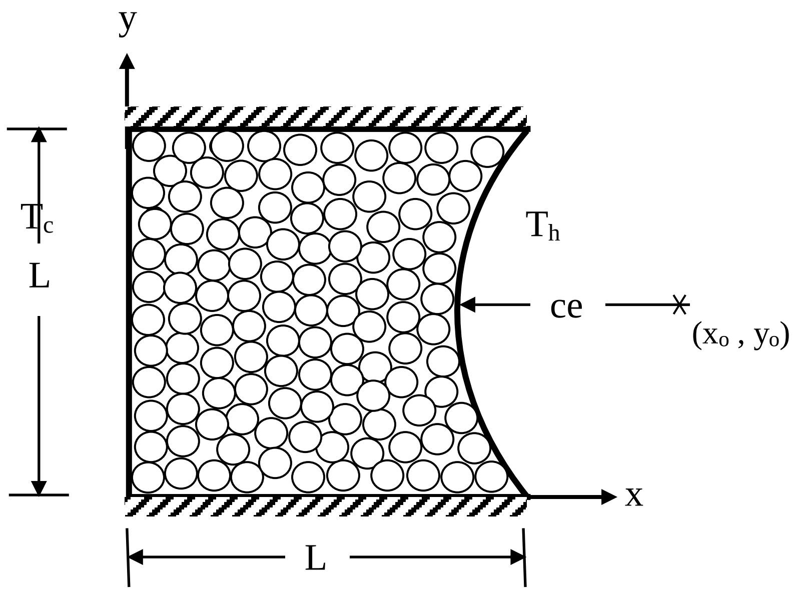


Fig.1 Physical domain.

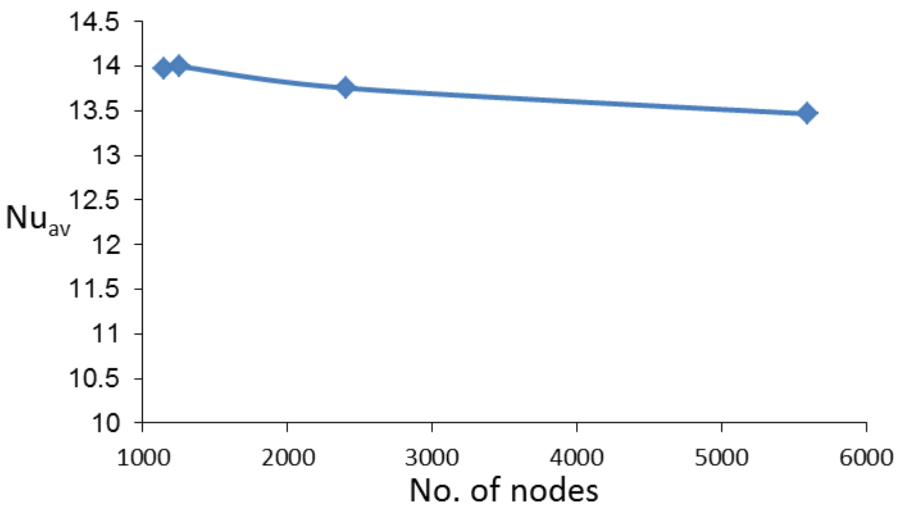
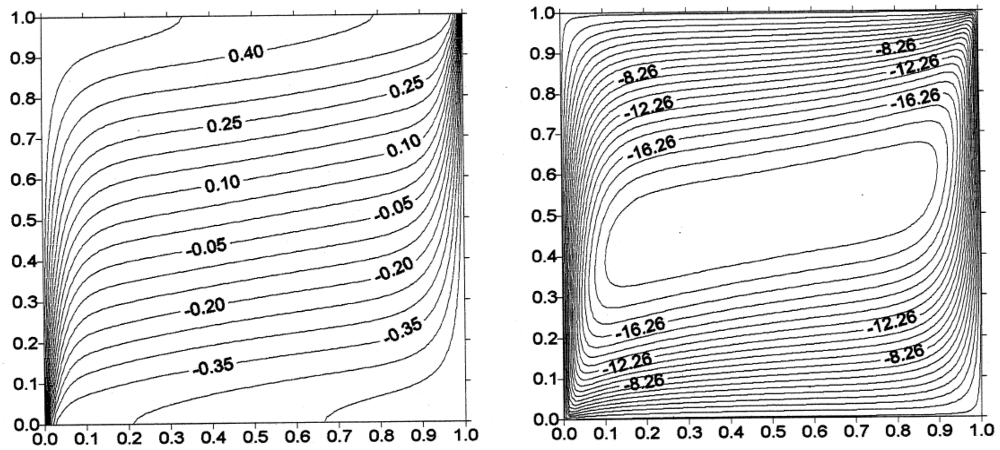


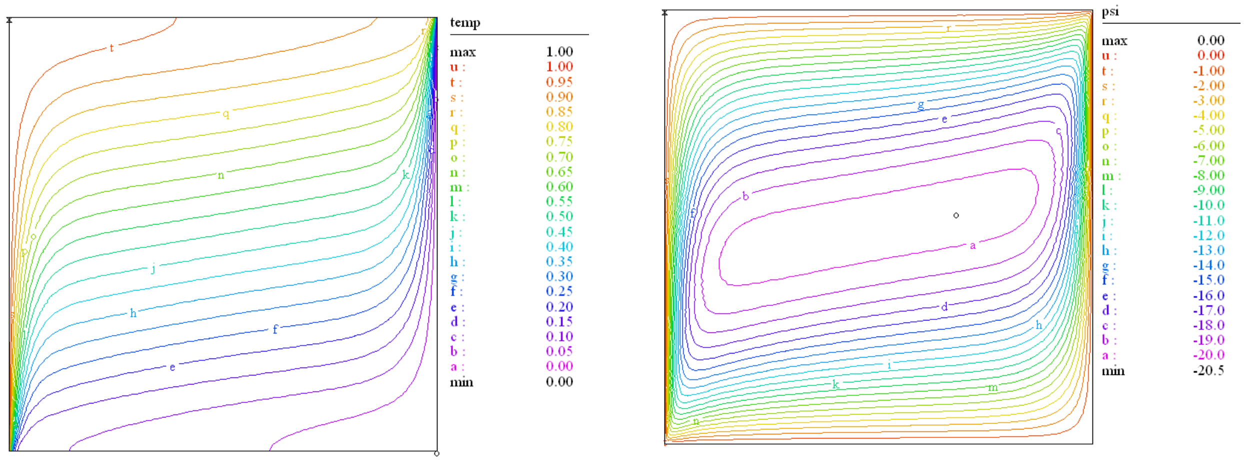
Fig. 2 Grid dependency test.

Table 1: Thermo physical properties of base fluid and Cu nanoparticles [11]

Physical property	Base fluid (water)	Cu
Cp (J/kg.K)	4179	385
ρ (kg/ m^3)	997.1	8933
K (W/m.K)	0.613	401
β (1/K)	21×10^{-5}	1.67×10^{-5}



(b) Baytas and Pop [24]

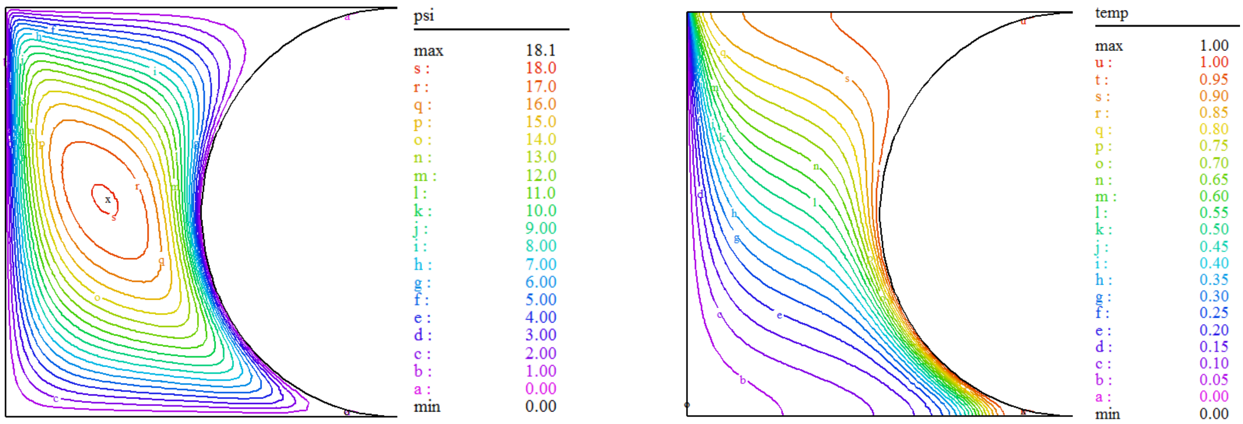


(a) Present

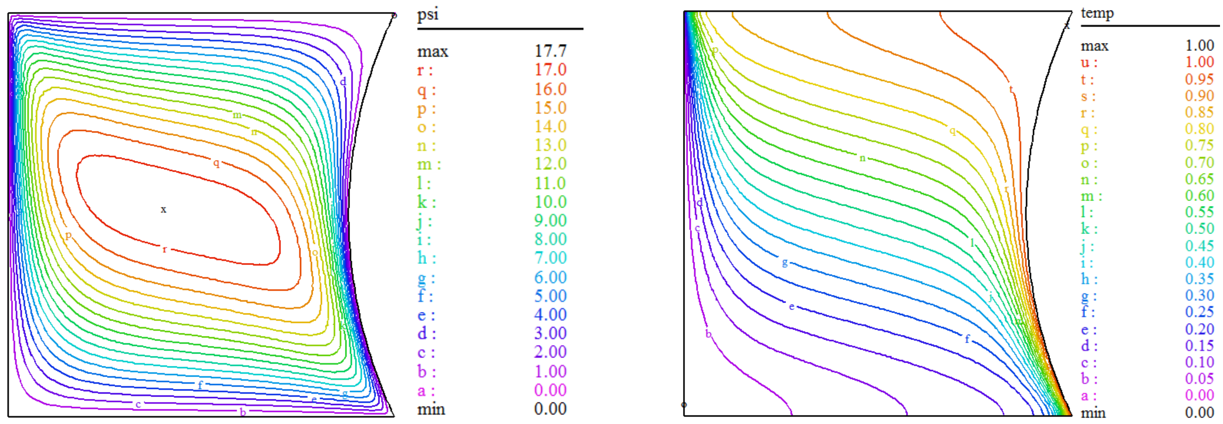
Fig. 3 Comparison of isotherms (left) and streamlines (right) (a) Ref. [24,] (b) Present.

Table 2: Comparison of average Nusselt number with porous square cavity (Baytas and Pop) [24], $\phi = 0$

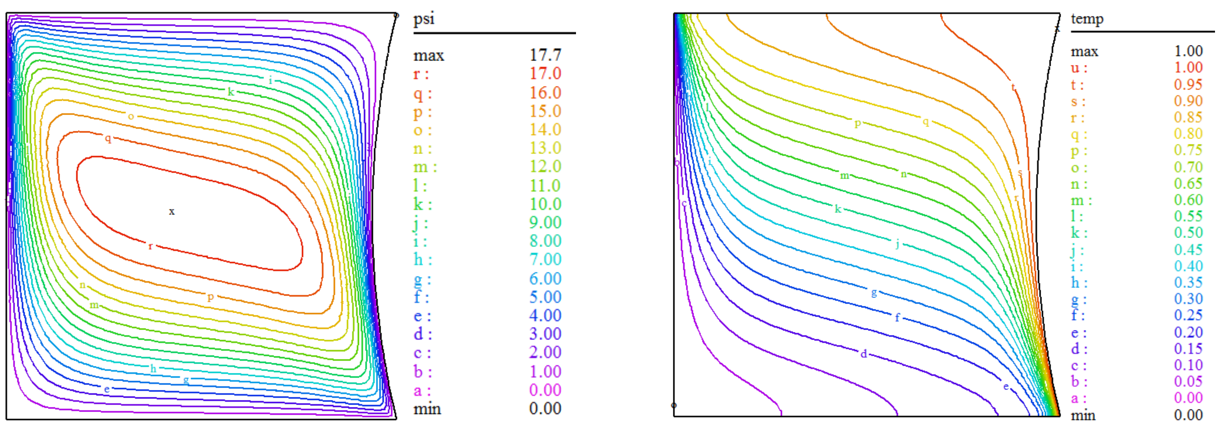
Ra	10	100	1000
Bytas and Pop [24]	1.079	3.16	14.06
Present	1.0813	3.1714	14.24



(a) Ce=1 , Ψ_{max} =18.1

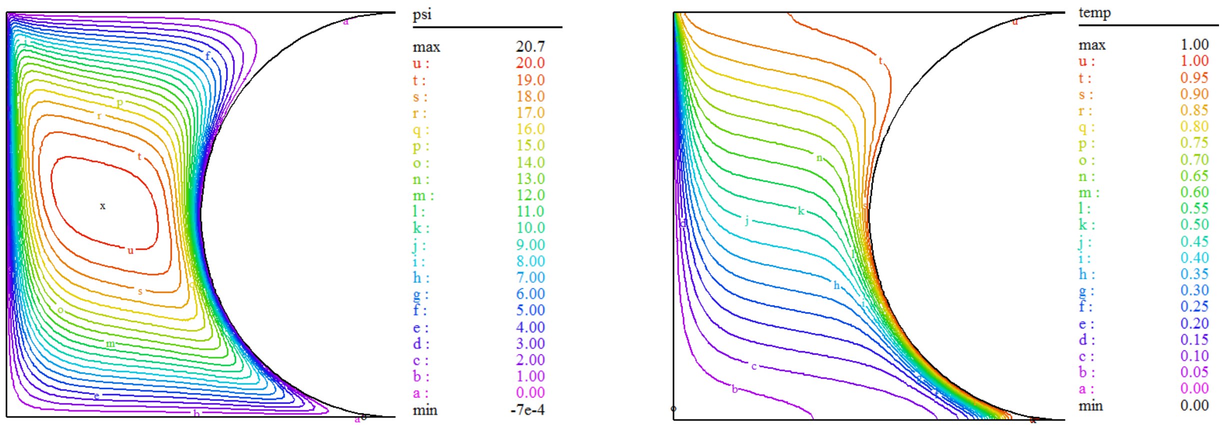


(b) Ce=2, , Ψ_{max} =17.7

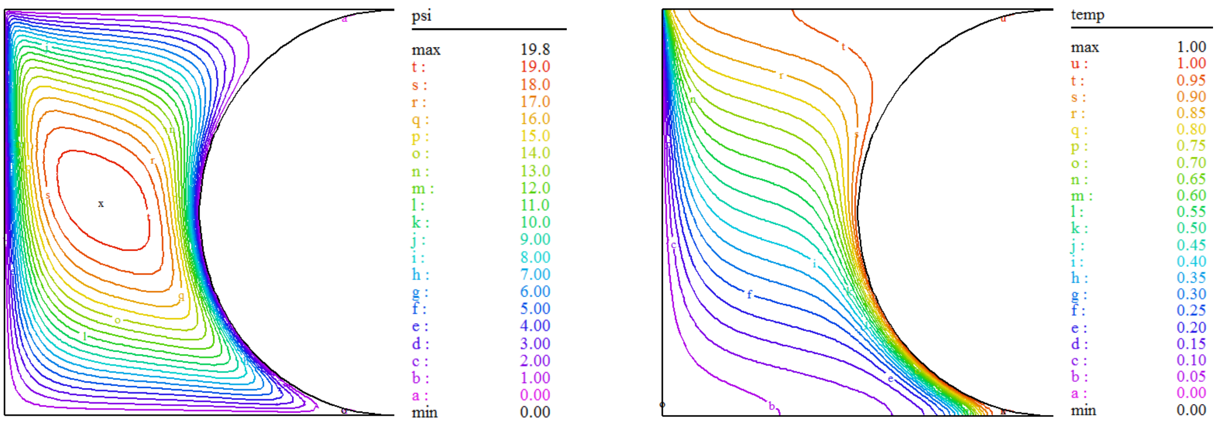


(c) Ce=3 , Ψ_{max} ==17.7

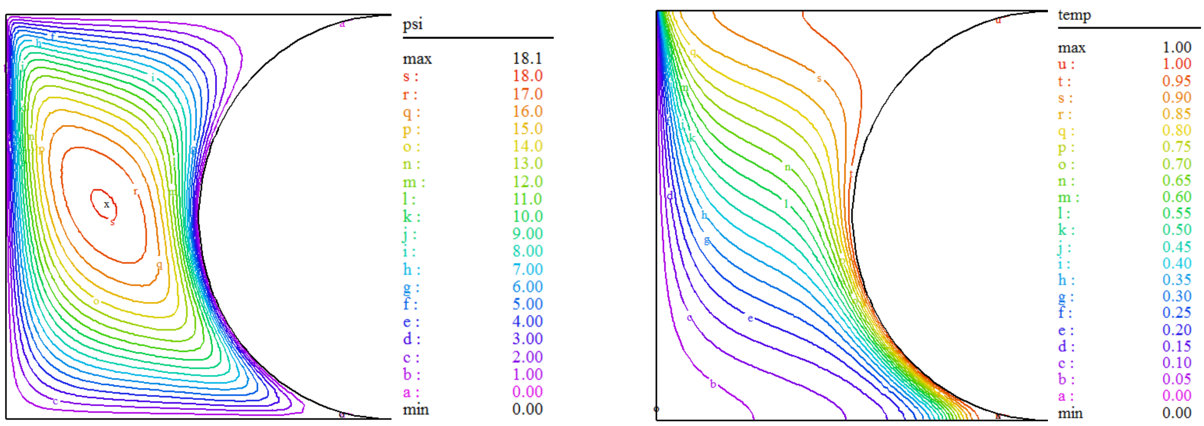
Fig. 4 Streamlines (left) isotherm (right) at Ra=1000 and ϕ = 0.2 for different values of Ce



(a) ϕ =0 , Ψ_{max} =20.7

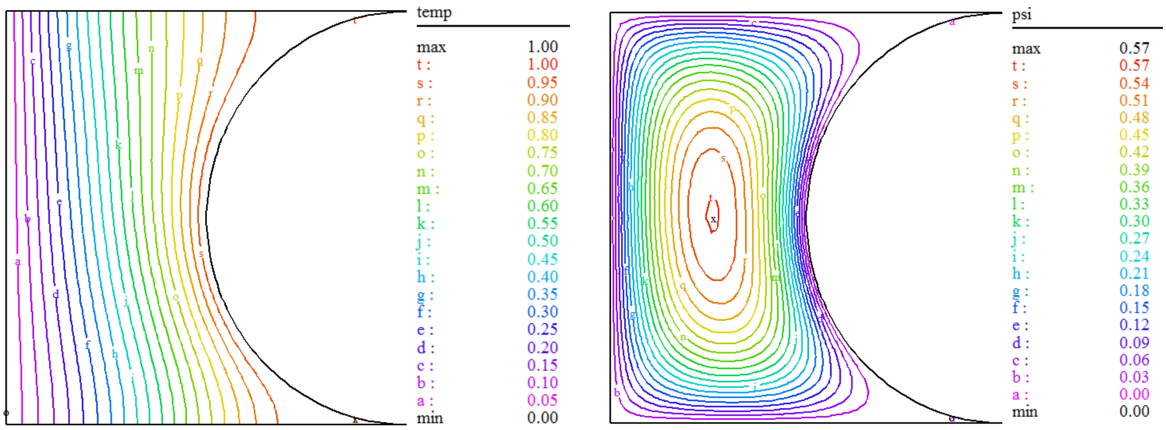


(b) ϕ = 0.1 , Ψ_{max} =19.8

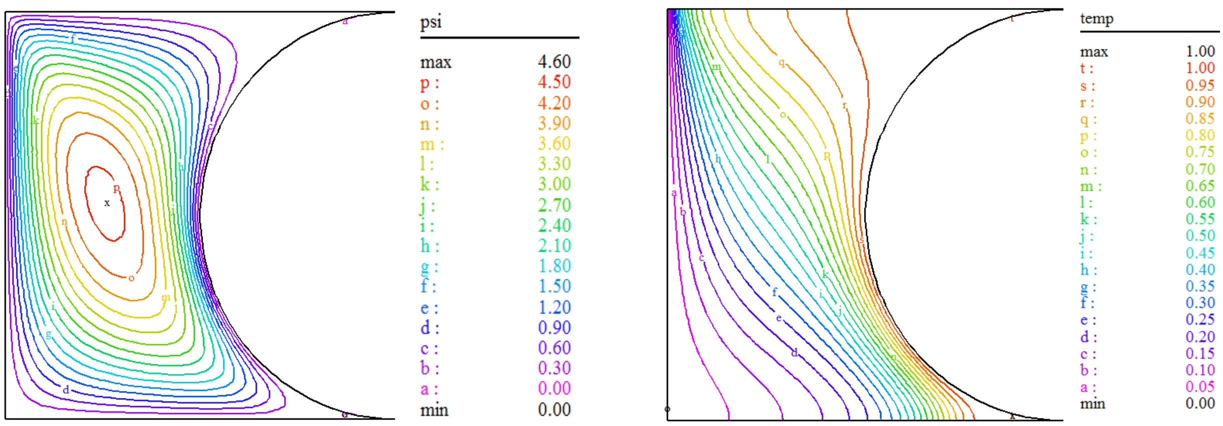


(c) ϕ =0.2 , Ψ_{max} =18.1

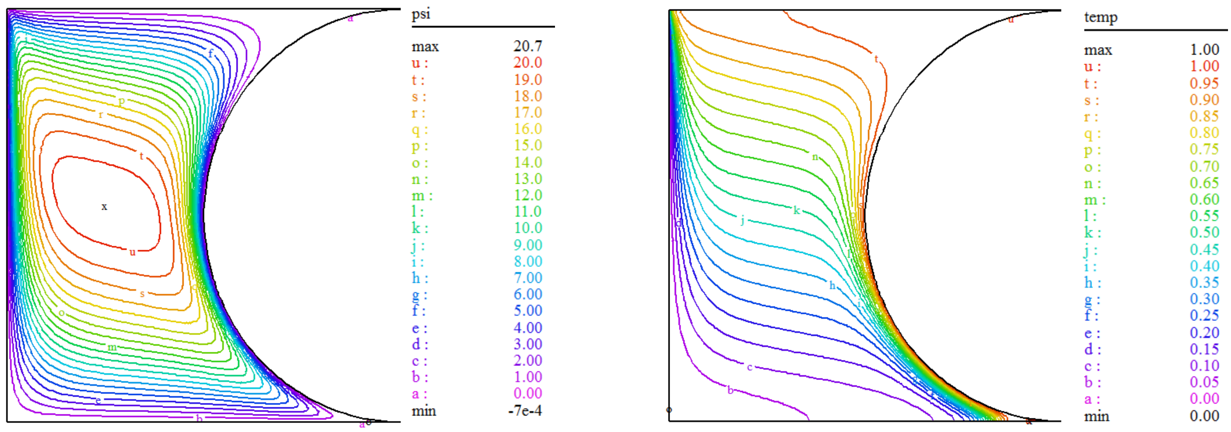
Fig. 5 Streamlines (left) isotherm (right) at Ra=1000 and Ce=1 for different values ϕ .



(a) Ra=10 , Ψ_{max} =0.57

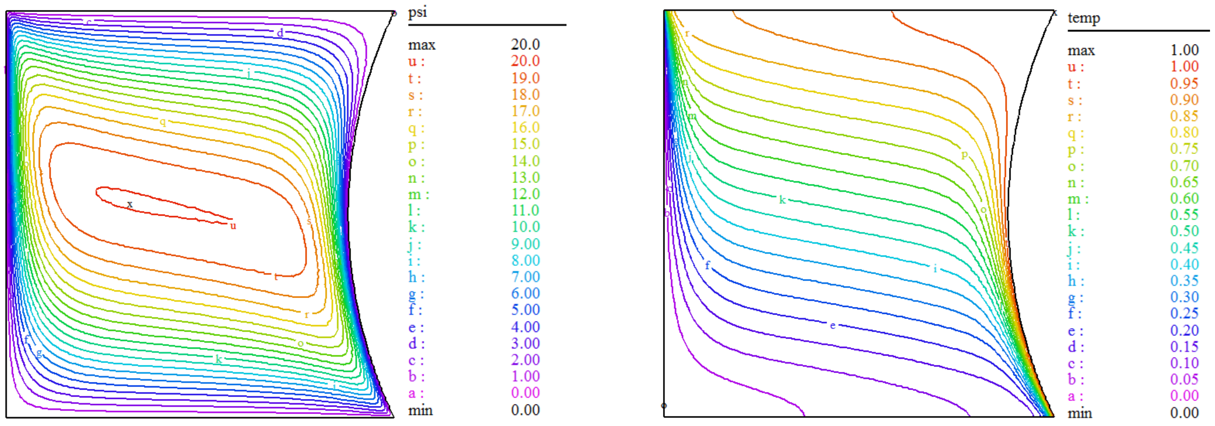


(b) Ra=100 , Ψ_{max} = 4.60

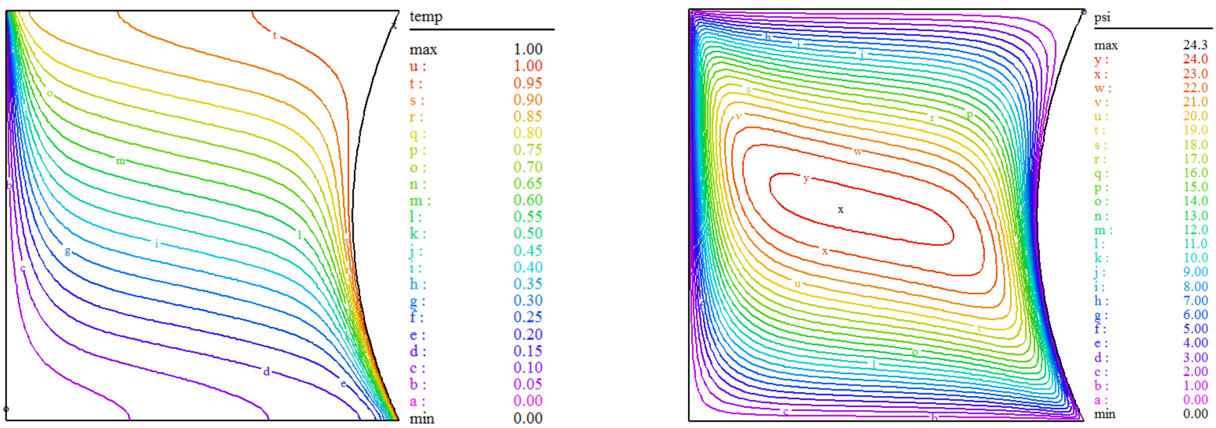


(c) Ra =1000 , Ψ_{max} = 20.7

Fig. 6 Streamline (left) isotherm (right) at Ce=1, ϕ =0 for different values of Ra numbers



(a) Maxwell model at Ra=1000 , ce = 2 and Ø= 0.05, $\Psi_{max} = 20$



(b) Yu and Choi model at Ra=1000 ,Ce=2 and Ø=0.05. $\Psi_{max} = 24.3$

Fig. 7 Streamlines (left) and isotherms (left) for both models.

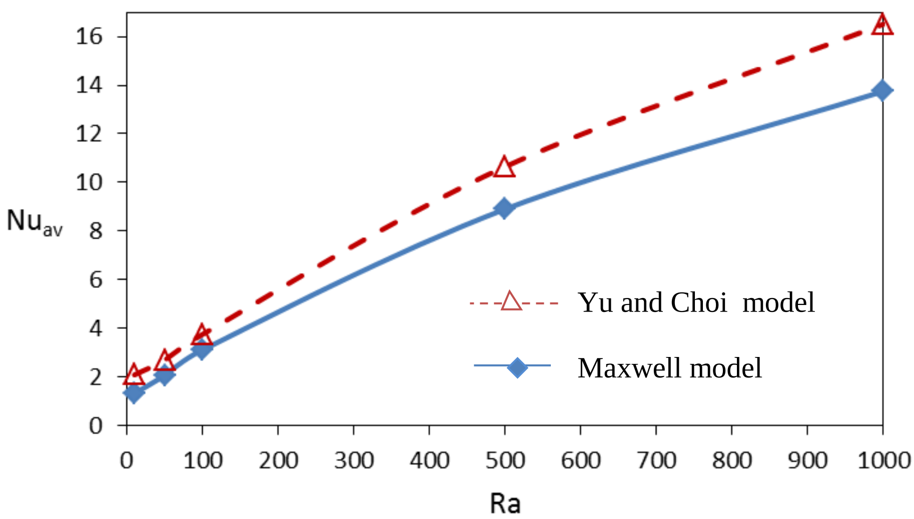


Fig. 8 Variation of Nu_{av} with Ra for Maxwell model and Yu and Choi model.

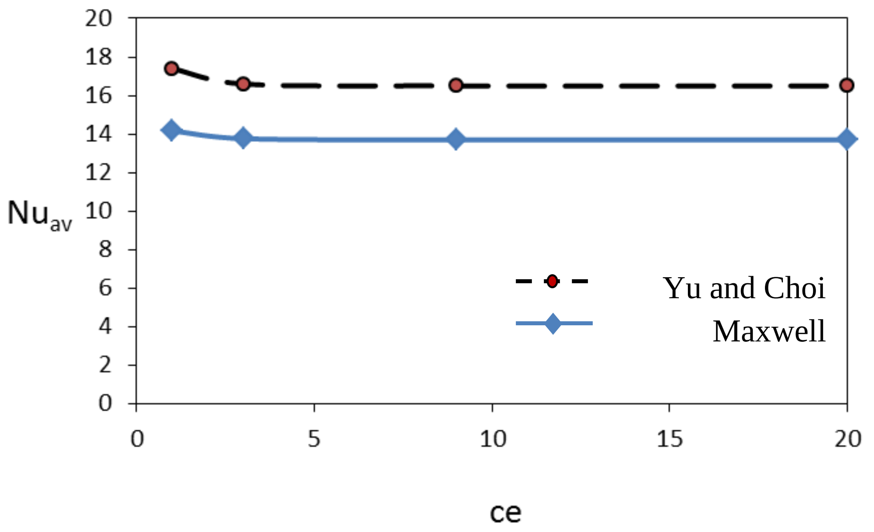


Fig. 9 Variation of the right wall curvature on Nu_{av} for both models.

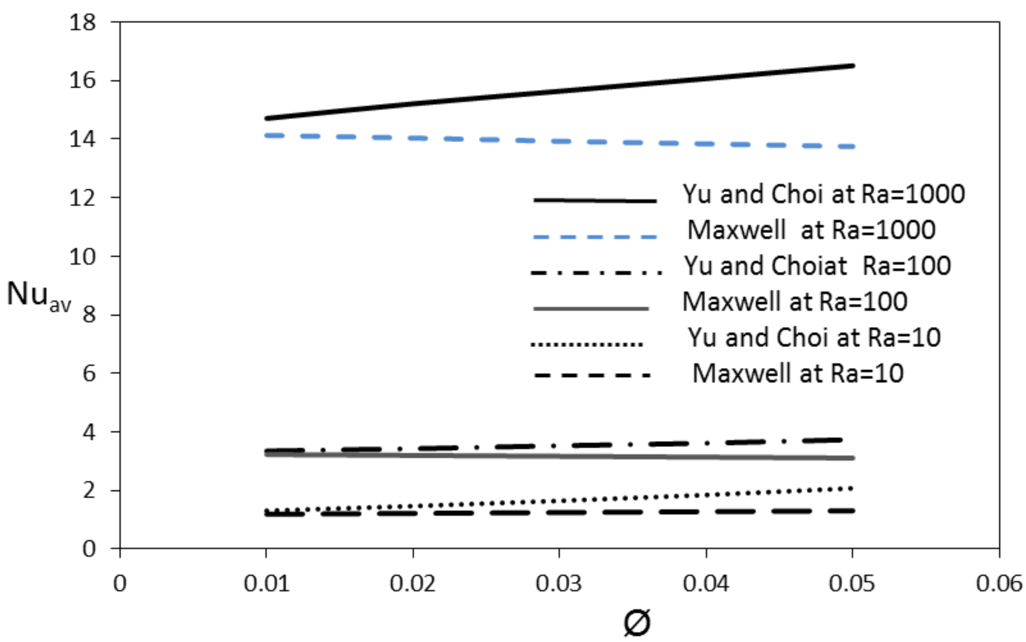


Fig. 10 Variation of nanoparticles volume fraction on Nu_{av} for both models at different Rayleigh number.