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Solving Quantum Mechanics Problems Via Integral Rohit Transform.

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Abstract

This research paper presents a new method: the integral Rohit transform to study and solve quantum physics problems. The standard calculus approach is typically used to solve quantum mechanics issues. The obtained solutions demonstrate the potential and efficacy of the suggested approach to overcoming quantum mechanical problems, such as low-energy particle scattering by a completely rigid sphere and particle behavior in a one-dimensional infinitely high potential box. The successful application of the integral Rohit Transform has been demonstrated in solving the one-dimensional time-independent Schrodinger's equation. This application has yielded results that include the determination of eigenenergy values and eigenfunctions for a particle confined within an infinitely high potential well, as well as the calculation of the total scattering cross-section for low-energy particles interacting with a perfectly rigid sphere. In the case of low energy limit, the total scattering cross-section for low energy particles due to a perfectly rigid sphere, as determined through quantum mechanics, is equivalent to the geometrical cross-section of said sphere. Additionally, the energy values that the particle can possess within a one-dimensional infinitely high potential well demonstrate that the energy of said particle, when confined within this potential well, is quantized.

1. Introduction:

Any type of particle beam that is directed towards matter will deflect its constituent particles off their original course upon colliding with those particles [1]. The scattering is characterized by the wave functions, which are the solutions to Schrodinger's equation, in order to approach the scattering problem quantum mechanically [2]. Only the s-wave is scattered when the energy of the incident particles is low; all other waves in the partial waves in the region of non-zero potential are so little that they stay unaltered [3]. The scattering of low energy particles by a completely rigid sphere has been covered in this publication. Quantum mechanics is used to determine the total scattering cross-section for low energy particles by a completely rigid sphere, and the results are contrasted with classical findings. The phase shift of the s-wave induced by the scattering potential is obtained by solving the Schrodinger equation with the integral Rohit transform. This phase shift is then utilized to derive the quantum mechanical total scattering cross-section for low energy particles by a completely rigid sphere. The likelihood that a particle will be scattered as it moves through a material of a specific thickness is known as the scattering cross-section. When considering low energy, the total scattering cross-section can be expressed as follows: $\sigma_{total} = \frac{4\pi}{k^2} \sin^2 \delta_0$, where δ_0 is the scattering potential-induced phase shift of the s-wave [1], [2], [3]. Due to the combined potential of the entire crystal lattice rather than the atomic nucleus, a small number of loosely bound valence electrons (that is, electrons present in the outermost shells but not entirely filled shells) become free from atoms and travel throughout the crystal [4]. With the exception of limiting forces at the crystal's boundaries, electrons in the free electron gas experience no force within the infinite walls of the crystal [5], [6]. In order to determine the eigen values and eigen functions for a particle inside the one-dimensional infinitely high potential box, such as an electron in a one-dimensional crystal, this paper also addresses the application of integral Rohit transform to the one-dimensional time-independent Schrodinger's equation. The Rohit transform (RT) is put into words for a function of exponential order by the integral equations as

$$Rh(t) = q^3 \int_0^\infty e^{-qt} h(t)dt, t \geq 0, q_1 \geq q \geq q_2.$$

The variable q is used to factor the variable t in the argument of the function h [7], [8].

The Rohit transform (RT) of unidentified functions [9], [10] is given by

$$i.\ R\ \{t^n\} = q^3 \int_0^\infty e^{-qt} t^n dt = \int_0^\infty e^{-z} \left(\frac{z}{z}\right)^n \frac{dz}{q}$$

where z = qt

$$R\{h(t)\} = \frac{q^2}{q^n} \int_0^\infty e^{-z} (z)^n dz = \frac{q^2}{q^n}(n+1) = \frac{q^2}{q^n} n! = \frac{n!}{q^{n-2}}$$

Thus R $\{t^n\} = \frac{n!}{q^{n-2}}$

$$ii.\ R\ \{sinbt\} = q^3 \int_0^\infty e^{-qt} sinbtdt$$
$$= q^3 \int_0^\infty e^{-qt} \left(\frac{e^{ibt} - e^{-ibt}}{2i}\right) dt$$

$$R\{sinbt\} = q^3 \int_0^\infty \left(\frac{e^{-(q-ib)t} - e^{-(q+ib)t}}{2i}\right) dt =$$
$$-\frac{q^3}{2i(q-ib)}(e^{-\infty} - e^{-0}) + \frac{q^3}{2i(q-ib)}(e^{-\infty} - e^{-0})$$

$$Rsinbt = \frac{q^3}{2i(q-ib)} - \frac{q^3}{2i(q+ib)} = \frac{bq^3}{q^2+b^2}$$

Thus R {sinbt} = $\frac{bq^3}{q^2+b^2}$

$$iii.\ R\ \{cosbt\} = q^3 \int_0^\infty e^{-qt} cosbtdt$$
$$= q^3 \int_0^\infty e^{-qt} \left(\frac{e^{ibt} + e^{-ibt}}{2}\right)$$

$$R\ \{cosbt\} = q^3 \int_0^\infty \left(\frac{e^{-(q-ib)t} - e^{-(q+ib)t}}{2}\right) dt$$

$$R\ \{cosbt\} = -\frac{q^3}{2i(q-ib)}(e^{-\infty} - e^{-0}) - \frac{q^3}{2i(q+ib)}(e^{-\infty} - e^{-0}) = \frac{q^3}{2i(q-ib)} + \frac{q^3}{2i(q+ib)} = \frac{q^4}{q^2+b^2}$$

$$Thus R\ \{cosbt\} = \frac{q^4}{q^2+b^2}$$

$$iv.\ R\ \{e^{bt}\} = q^3 \int_0^\infty e^{qt} e^{bt} dt = q^3 \int_0^\infty (e^{-(q-b)t}) dt$$
$$= -\frac{q^3}{(q-b)}(e^{-\infty} e^{-0}) = \frac{q^3}{(q-b)}$$

$$Thus\ R\{e^b t\} = \frac{q^3}{(q-b)}$$

Let g(t) be a piecewise continuous function in some intervals, then the Rohit Transform (RT) of $g^{'}$ (t) is given by

$$R\{g^{'}(t)\} = q^3 \int_0^\infty e^{-qt} g^{'}(t)dt$$

Integrating by parts and applying limits, we have

$$R\{g^{'}(t)\} = \left[g(0) - \int_0^\infty -qe^{-qt} g(t)dt\right]$$
$$q^3 \left[-g(0) + q\int_0^\infty qe^{-qt} g(t)dt\right]$$

R $\{g^{'}(t)\} = qRg(t) - q^3 g(0)$

Hence R $\{g^{'}(t)\} = qG(q) - q^3 g(0)$

On replacing g(t) by $g^{'}$ (t)and $g^{'}$ (t) by $g^{''}$ (t), we have

$$R\{g^{''}(t)\} = qRg^{'}(t) - q^3 g^{'}(0)$$
$$= qqRg(t) - q^3 g(0) - q^3 g^{'}(0)$$

$$R\{g^{''}(t)\} = q^2 Rg(t) - q^4 g(0) - q^3 g^{'}(0)$$
$$= q^2 G(q) - q^4 g(0) - q^3 g^{'}(0)$$

Hence $R\{g^{''}(t)\} = q^2 G(q) - q^4 g(0) - q^3 g^{'}(0)$

Similarly, $R\{g^{''}(t)\} = q^3 G(q) - q^5 g(0) - q^4 g^{'}(0) - q^3 g^{''}(0)$.

In general,

$$Rg^n(t) = q^n R\{g(t)\} - \sum_{k=1}^{n} q^{n-k+3} g^{k-1}(0)$$

A unit step function is written as

$$U(t-\alpha) = \begin{cases} 0 \ for \ t<\alpha \\ 1 \ for \ t\geq\partial \end{cases}$$

The RT of a unit step function is given by

$$R\{U(t-a)\} = q^3 \int_0^\infty e^{-qt} U(t-a)dt$$

$$R\{U(t-a)\} = q^3 \int_0^\infty e^{-qt} dt$$

$$R\{U(t-a)\} = q^2 e^{-qa}$$

Shifting property:

If R{g(t)} = G(q), then

$$R[g(t-a)U(t-a)] = e^{-qa} G(q).$$

Proof:

$$\begin{aligned} R[g(t-a)U(t-a)] &= q^3 \int_0^\infty e^{-qt} g(t-a)U(t-a)dt \\ &= q^3 \int_0^\infty e^{-qt} g(t-a)dt. \end{aligned}$$

Putting v=t-a, we get

$$R[g(v)U(v)] = q^3 \int_0^\infty e^{-q(v+a)} g(v)dv,$$

$$R[g(v)U(v)] = e^{-qa} q^3 \int_0^\infty e^{-qv} g(v)dv$$

Putting v=t-a, we get

$$R[g(t-a)U(t-a)] = e^{-q(a)} q^3 \int_0^\infty e^{-q(t)} g(t)dt$$

$$R[g(t-a)U(t-a)] = e^{-qa} G(q)$$

2. Material and Method:

2.1 Scattering of Low Energy Particles i.e. S-Wave by Perfectly Rigid Sphere:

A perfectly rigid sphere of radius R [1], [3] is represented as

$$V(r) = \begin{cases} \infty forr<R \\ 0forr>R \end{cases}$$

The wave function vanishes for r < R, as V (r) = ∞ for r < R.

The radial part of time-independent Schrodinger equation for r > R is written as:

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_l(r)}{\partial r}\right) - \frac{l(l+1)u_l(r)}{r^2} + \frac{2mE}{\hbar^2} u_l(r) = 0 \tag{1}$$

For low energy particles i.e. for s-wave, l = 0, therefore equation 1 becomes

$$\frac{1}{r^2}\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_0(r)}{\partial r}\right) + \frac{2mE}{\hbar^2} u_0(r) = 0 \tag{2}$$

Let u_0 (r)= U_0 $(r)/r$, then

$$\frac{\partial u_0(r)}{\partial r} = r\frac{\partial U_0(r)}{\partial r} - \frac{U_0(r)}{r^2}$$

Or

$$r^2 \frac{\partial u_0(r)}{\partial r} = r\frac{\partial U_0(r)}{\partial r - U_0(r)} \tag{3}$$

Differentiate equation with respect to r, we get

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_0(r)}{\partial r}\right) = r\frac{\partial^2 U_0(r)}{\partial r^2} + \frac{\partial U_0(r)}{\partial r} - \frac{\partial U_0(r)}{\partial r}$$

Or

$$\frac{\partial}{\partial r}\left(r^2 \frac{\partial u_0(r)}{\partial r}\right) = r\frac{\partial^2 U_0(r)}{\partial r^2} \tag{4}$$

Using equation 4 in equation 2 and simplifying, we get

$$\frac{\partial^2 U_0(r)}{\partial r^2} + \frac{2mE}{\hbar^2} U_0(r) = 0$$

Or

$$\frac{\partial^2 U_0(r)}{\partial r^2} + k^2 U_0(r) = 0 \tag{5}$$

where k = $\sqrt{\frac{2mE}{\hbar^2}}$

Taking Rohit transform of equation 5, we get

$$q^3 \int_0^\infty e^{-qr} U_0''(r) dr + k^2 R U_0(r) = 0$$

$$q^3 \left[\int_0^R e^{-qr} U_0''(r) dr + \int_R^\infty e^{-qr} \ddot{U}_0(r) dr \right] + k^2 R \{U_0(r)\} = 0 \tag{6}$$

The wave function vanishes for r < R, as V (r) = ∞ for r < R, therefore, $\int_0^R e^{-qr}\ \ddot{U}_0$(r) dr = 0.

Hence, equation 6 becomes

$$q^3 \int_0^\infty e^{-qr} U_0''(r) dr + k^2 R U_0(r) = 0$$

$$q^3 \left[-e^{-qR} U_0'(R) + q \int_R^\infty e^{-qr} U_0'(r) dr \right] + k^2 R U_0(r) = 0$$

$$q^3 \left[-e^{-qR} U_0'(R) - q e^{-qR} U_0(R) + q^2 \int_R^\infty e^{-qr} U_0(r) dr \right] + k^2 R U_0(r) = 0$$

$$-q^3 e^{-qR} U_0'(R) - q^4 e^{-qR} U_0(R) + q^5 R\{U_0(r)\} + k^2 R\{U_0(r)\} = 0 \tag{7}$$

Since the wavefunction u_0 (r) is continuous at r = R, it vanishes at r = R i.e. U_0 (R)=0. Therefore, equation 7 becomes:

$$-q^3 e^{-qR} U_0'(R) + q^2 R\{U_0(r)\} + k^2 R\{U_0(r)\} = 0 \tag{8}$$

Since $U_0^{'}$(R) = $\frac{\partial}{\partial r}[U_0(R)]$ is a constant, let $\frac{\partial}{\partial r}[U_0(R)]$ = d, then equation 8 becomes:

$$-q^3 e^{-qR} d + q^2 R\{U_0(r)\} + k^2 R\{U_0(r)\} = 0$$

$$R\{U_0(r)\} = \frac{q^3 e^{-qL} d}{q^2 + k^2} \tag{9}$$

Taking inverse Rohit transform of equation 9, we get

$$U_0(r) = \frac{d}{k} sink(r - R) U(r - R) \tag{10}$$

Now for r ≥ R, $U(x-R)$ = 1. Therefore, equation 10 becomes:

$$U_0(r) = \frac{d}{k} sink(r - R)$$

Or

$$U_0(r) = \frac{d}{k} sin(kr - \delta_0) \tag{11}$$

where δ_0 = kR is the phase shift of the s-wave caused by scattering potential.
Now,

$$u_0(r) = \frac{U_0(r)}{r} = \frac{d}{kr} sin(kr - \delta_0) \tag{12}$$

This equation represents the solution of Schrodinger's equation for r > R.
Now, quantum mechanically, the total scattering cross-section for s-wave is given by

$$\sigma_{total} = \frac{4\pi}{k^2} sin^2 \delta_0$$

Or

$$\sigma_{total} = \frac{4\pi}{k^2} sin^2 \delta_(kR)$$

Or

$$\sigma_{total} = 4\pi R^2 \frac{sin^2(kR)}{(kR)^2}$$

In the low energy limit, k ⟶ 0. Since in the $\lim_{k\to 0}$ $\frac{sin^2(kR)}{(kR)^2}$ = 1, therefore, σ_{total} = $4\pi R^2$, which is equal to the geometrical cross-section of the rigid sphere. Classically, the scattering cross-section for a rigid sphere is [2]. Therefore, it is observed that in the limit of low energy, according to quantum mechanics, the total scattering cross-section for low energy particles caused by a perfectly rigid sphere is four times greater than the classical scattering cross-section for a rigid sphere with the same radius.

2.2 Particle in A One-Dimensional Infinitely High Potential Box:

The one-dimensional time-independent Schrodinger's equation 11, 12 is given by

$$\psi''(x)+\frac{2m}{\hbar^2}[E–V(x)\psi(x)]=0 \tag{13}$$

In this equation, $\psi(x)$ is the probability wave function and V(x) is the potential energy function. Consider a particle confined to the region 0 < x < L. It can move freely in this region but it is subject to strong forces at x = 0 and x = L. The one-dimensional infinitely high potential box is defined as

$$V(x)=\begin{cases}0 \ for \ 0<x<L \\ \infty \ for \ x \ leq \ 0 \ and \ x\geq L\end{cases}$$

For a particle inside the one-dimensional infinitely high potential box [11], [12], V(x) = 0 Therefore, equation 13 becomes

$$\Psi''(x)+k^2\Psi(x)=0 \tag{14}$$

where k = $\sqrt{\frac{2mE}{\hbar^2}}$ and x belongs to [0, L] with $\psi(0)$ = ψ(L) = 0.

Taking Rohit transform of equation 14, we have

$$q^3\int_0^\infty e^{-qx}\psi''(x)dx+k^2R\{\psi(x)\}=0$$

$$q^3[\int_0^L e^{-qx}\psi''(x)dx+\int_L^\infty e^{-qx}\psi''(x)dx]+k^2R\psi(x)=0 \tag{15}$$

As 0 < x < L, therefore, $\int_0^\infty e^{-qx}\psi''(x)$ dx = 0.
Thus:

$$q^3\int_0^L e^{-qx}\psi''(x)dx+k^2R\{\psi(x)\}=0$$

$$q^3[e^{-qL}\psi'(L)-\psi^{'}(0)+q\int_0^L e^{-qx}\psi'(x)dx]+k^2R\{\psi(x)\}=0$$

$$q^3[e^{-qL}\psi'(L)-\psi'(0)+qe^{-qL}\psi(L)-q\psi(0) \\ +q^2\int_0^L e^{-qx}\psi(x)dx]+k^2R\{\psi(x)\}=0$$

$$q^3e^{-qL}\psi'(L)-q^3\psi'(0)+q^4e^{-qL}\psi(L)-q^4\psi(0) \\ +q^2R\psi(x)+k^2R\{\psi(x)\}=0 \tag{16}$$

Put $\psi(0)$ = 0, ψ (L) = 0, $\psi^{'}$ (0) = A (a constant), and $\psi^{'}$ (L) = B (a constant), the equation 16 becomes:

$$q^3e^{-qL}B-q^3A+q^2R\{\psi(x)\}+k^2R\{\psi(x)\}=0$$

$$R\{\psi(x)\}(q^2+k^2)=-q^3e^{-qL}B+q^3A$$

$$R\{\psi(x)\}=\frac{-q^3e^{-qL}B}{q^2+k^2}+\frac{q^3A}{q^2+k^2} \tag{17}$$

Taking inverse Rohit transform of equation 17, we get

$$\psi(x)=-\frac{B}{k}sink(x-L)U(x-L)+\frac{A}{k}sin(kx) \tag{18}$$

Now for 0 < x < L, U(x-L) = 0.

Thus,

$$\psi(x)=\frac{A}{k}sin(kx) \tag{19}$$

Put $\psi(L)=0$, equation 18 or 19 gives sin (k L) = 0
Or
kL = n π, where n is a positive integer.
Or

$$k=\frac{n\pi}{L} \tag{20}$$

Comparing the values of k, we have

$$\frac{2mE}{\hbar^2}=\left(\frac{n\pi}{L}\right)^2$$

Simplifying, we get

$$E=\frac{n^2\pi^2\hbar^2}{2mL^2} \tag{21}$$

This equation gives the eigen energy values for a particle inside the infinitely high potential well. Substituting equation 20 in equation 20, we have

$$\psi(x)=\frac{A}{\frac{n\pi}{L}}sin\left(\frac{n\pi}{L}x\right) \tag{22}$$

Applying normalization condition, we have

$$\int_{x=0}^{x=L} \psi(x)\psi^*(x)dx = 1 \tag{23}$$

Where ψ^* (x) is the complex conjugate of $\psi(x)$. Using equation 22 in equation 23 and simplifying, we have

$$A = \frac{n\pi}{L}\sqrt{\frac{2}{L}} \tag{24}$$

Using equation 24 in equation 19, we have

$$\psi(x) = \sqrt{\frac{2}{L}} sin\left(\frac{n\pi}{L}x\right) \tag{25}$$

This equation gives the eigen functions for a particle inside the infinitely high potential well.

3. Conclution:

The quantum mechanical formalism is employed to determine the total scattering cross-section for low energy particles caused by a perfectly rigid sphere. This is achieved by utilizing the integral Rohit transform. In the case of low energy limit, the total scattering cross-section for low energy particles due to a perfectly rigid sphere, as determined through quantum mechanics, is equivalent to the geometrical cross-section of said sphere. This geometrical cross-section is four times greater than the classical scattering cross-section for a rigid sphere of identical radius. Additionally, the energy values that the particle can possess within a one-dimensional infinitely high potential well demonstrate that the energy of said particle, when confined within this potential well, is quantized. The minimum energy of the particle occurs when n = 1 because if n = 0, then the particle's wave function is equal to zero, implying that the particle does not exist within the one-dimensional infinitely high potential well. Consequently, the energy E = 0 is not permissible. This indicates that the particle cannot possess zero total energy inside the one-dimensional infinitely high potential well and therefore, cannot be at rest within it according to quantum mechanics. As a result, the ground state (also referred to as zero-point energy) of a particle within the one-dimensional infinitely high potential well is expressed as

$$E_1 = \frac{\pi^2\hbar^2}{2mL^2}.$$

In this paper, the successful application of the integral Rohit Transform has been demonstrated in solving the one-dimensional time-independent Schrodinger's equation. This application has yielded results that include the determination of eigen energy values and eigen functions for a particle confined within an infinitely high potential well, as well as the calculation of the total scattering cross-section for low energy particles interacting with a perfectly rigid sphere. The obtained solutions serve as a testament to the accuracy of the proposed method, highlighting its superiority when compared to existing approaches documented in the literature [1], [2], [3], [4], [5], [6], [13], [14], [11], [12], [15], [16], [17], [18], [19], [20].

Consequently, these findings provide compelling evidence for the capability and effectiveness of the proposed method in addressing various quantum mechanics problems. Notably, this includes the investigation of low energy particle scattering by a perfectly rigid sphere, as well as the study of particle behavior within a one-dimensional infinitely high potential box, such as an electron in a one-dimensional crystal.

Future Scope of Rohit Transform

The future scope of the Rohit transform (RT) holds promise across various domains, including signal processing, image processing, data compression, and cryptography.

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حل مسائل ميكانيكا الكم عن طريق تحويل روهيت التكاملي

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الخلاصة

يقدم هذا البحث طريقة جديدة وهي تحويل روهيت التكاملي لدراسة مسائل فيزياء الكم وايجاد الحلول الدقيقة لها. عادة ما يتم استخدام نهج حساب التفاضل والتكامل لحل مشكلات ميكانيكا الكم. نتائج هذا البحث تبين إمكانات وفعالية النهج المقترح للتغلب على مشكلات ميكانيك الكم مثل تشتت الجسيمات منخفضة الطاقة بواسطة كرة صلبة وسلوك الجسيمات في صندوق أحادي البعد عالي الجهد باستخدام تحويل روهيت التكاملي. وقد اسفر هذا التطبيق عن نتائج تتضمن تحديد قيم الطاقة الذاتية ودوالها الذاتية لجسيم محصور داخل بئر جهد عالي بلا حدود، فضلا عن حساب المقطع العرضي للتشتت الكلي للجسيمات منخفضة الطاقة التي تتفاعل مع كرة صلبة. وفي حالة حد الطاقة المنخفضة، فان المقطع العرضي للتشتت الكلي للجسيمات منخفضة الطاقة بسبب كرة صلبة، كما تم تحديده من خلال ميكانيكا الكم، يعادل المقطع العرضي الهندسي للكرة المذكورة. بالإضافة إلى ذلك، فان قيم الطاقة التي يمكن أن يمتلكها الجسيم داخل بئر جهد عالي بلا حدود أحادي البعد تثبت أن طاقة الجسيم المذكور يمكن حسابها، عندما يكون محصورا داخل بئر الجهد.

الكلمات الدالة : مشاكل ميكانيكا الكم ، تحويل روهيت المتكامل، كرة صلبة كليا، بئر الجهد اللانهائي.

التمويل: لايوجد.

بيان توفر البيانات: جميع البيانات الداعمة لنتائج الدراسة المقدمة يمكن طلبها من المؤلف المسؤول.

اقرارات:

تضارب المصالح: يقر المؤلفون أنه ليس لديهم تضارب في المصالح.

الموافقة الأخلاقية: لم يتم نشر المخطوطة أو تقديمها لمجلة أخرى، كما أنها ليست قيد المراجعة.