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Jordan Homomorphism on ΓM- Modules

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ABSTRACT

The aim object of this paper is present and study the concepts of homomorphism and Jordan homomorphism on ΓM -module and prove that every Jordan homomorphism from Γ-ring M Into prime ΓM -module X is either a homomorphism from M into X or anti-homomorphism from M into X.

Introduction

The definition of Γ-ring presented by Nobusawa [1] and generalization by Barnes[2].Let M and Γ be two additive abelian groups. Suppose that there is a mapping from M×Γ×M→M (the image of (a,α,b) being denoted by aαb, a,b∈M and α∈Γ) satisfying for all a,b,c∈M and α,β∈Γ:

i) (a+b)αc = aαc + bαc
a(α+β)c = aαc + aβc
aα(b+c) = aαb +aαc

ii) (aαb)βc = aα(bβc)

then M is called a Γ-ring. This definition is due to Barnes [2], every ring is Γ-ring. If the following condition holds for a Γ-ring M then M is called a prime Γ-ring [3], aΓMΓb =0 then a=0 or b=0, a,b∈M. M is called 2-torsion free if 2a=0 implies a=0 for all a∈M.

Let M be a Γ-ring and X be an additive abelian group X is a left ΓM- module if there exists a mapping M×Γ×X →X (sending (m,α,x) into mαx where m∈M , α, β ∈Γ and x∈X) satisfying for all $m,m_1,m_2\in M$, α,β∈Γ and $x,x_1,x_2 \in X$:[4]

i) $(m_1+m_2)\,\alpha x= m_1\alpha x+m_2\alpha x$

ii) $m(\alpha+\beta)x=m\alpha x+m\beta x$

iii) $m\alpha(x_1+x_2)=m\alpha x_1+m\alpha x_2$

iv) $(m_1\alpha m_2)\beta x= m_1\alpha(m_2\beta x)$

X is called a right ΓM- module if there exists a mapping X×Γ×M →X, X is called ΓM- module if X is both left and right ΓM- module, X is called a left prime (right prime) if aΓMΓb =0 then a=0 or b=0, a∈M,b∈X (a∈X,b∈M) respectively and X is prime if its both left and right prime. X is called 2-torsion free if 2x=0 imples x=0 for all x∈X. [5]

In this paper the research extend the results of Herstien [6],[7] to ΓM- module by introduce the concepts of homomorphism and Jordan homomorphism on ΓM-module and prove that every Jordan homomorphism from Γ-ring M into prime ΓM-module X is either a homomorphism from M into X or anti-homomorphism from M into X.

1. *Homomorphism on ΓM-module:*

In this section the research introduce the concepts of homomorphism and Jordan homomorphism on ΓM-module. We begin by the following definition:

Definition 1.1:

Let M be Γ-ring and X be ΓM-module θ: M→X be an additive map then θ is called homomorphism on X if

$\theta(a\alpha b)=\theta(a)\alpha\theta(b)$ for all a,b∈M and α ∈Γ.

Now we introduce the definition of Jordan homomorphism on ΓM-module:

Definition 1.2:

Let θ be an additive mapping of Γ-ring M into ΓM-module X then θ is called Jordan homomorphism if

$\theta(a\alpha a)=\theta(a)\alpha\theta(a)$ for all a∈M and α ∈Γ.

Definition 1.3:

Let θ be an additive mapping of Γ-ring M into ΓM-module X then θ is called

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$\theta(a\alpha b\beta a)=\theta(a)\alpha\theta(b)\beta\theta(a)$ Jordan triple homomorphism if
for all a,b $\in$ M and $\alpha,\beta\in\Gamma$.

Definition 1.4:

Let θ be an additive mapping of Γ-ring M into ΓM-module X then θ is called anti- homomorphism if $\theta(a\alpha b)=\theta(b)\alpha\theta(a)$ for all a,b $\in$ M and $\alpha\in\Gamma$.

In the following lemma we present the properties of homomorphism on ΓM-module

Lemma 1:

Let θ be a Jordan homomorphism of Γ-ring M into ΓM-module X , then for all a,b,c $\in$ M and $\alpha,\beta\in\Gamma$.

i) $\theta(a\alpha b+b\alpha a)=\theta(a)\alpha\theta(b)+\theta(b)\alpha\theta(a)$

ii) $\theta(a\alpha b\beta a+a\alpha b\beta a)=\theta(a)\alpha\theta(b)\beta\theta(a)+\theta(a)\beta\theta(b)\alpha\theta(a)$

iii) $\theta(a\alpha b\alpha a)=\theta(a)\alpha\theta(b)\alpha\theta(a)$

iv) $\theta(a\alpha b\alpha c+c\alpha b\alpha a)=\theta(a)\alpha\theta(b)\alpha\theta(c)+\theta(c)\alpha\theta(b)\alpha\theta(a)$

v) $\theta(a\alpha b\beta c+c\alpha b\beta a)=\theta(a)\alpha\theta(b)\beta\theta(c)+\theta(c)\beta\theta(b)\alpha\theta(a)$

proof:

i) $\theta((a+b)\alpha(a+b))=\theta(a+b)\alpha\theta(a+b)$
$\theta(a)\alpha\theta(a)+\theta(a)\alpha\theta(b)+\theta(b)\alpha\theta(a)+\theta(b)\alpha\theta(b)$ …(1)

On the other hand

$\theta((a+b)\alpha(a+b))=\theta(a\alpha a+a\alpha b+b\alpha a+b\alpha b)$
$=\theta(a)\alpha\theta(a)+\theta(b)\alpha\theta(b)+\theta(a\alpha b+b\alpha a)$ …(2)

Compare (1) and (2) we get

$\theta(a\alpha b+b\alpha a)=\theta(a)\alpha\theta(b)+\theta(b)\alpha\theta(a)$

ii) Replace $a\beta b+b\beta a$ for b in (1) we get

$\theta(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$
$=\theta(a)\alpha\theta(a\beta b+b\beta a)+\theta(a\beta b+b\beta a)\alpha\theta(a)$
$=\theta(a)\alpha(\theta(a)\beta\theta(b)+\theta(b)\beta\theta(a))+(\theta(a)\beta\theta(b)+\theta(b)\beta\theta(a))\alpha\theta(a)$

On the other hand

$\theta(a\alpha(a\beta b+b\beta a)+(a\beta b+b\beta a)\alpha a)$
$=\theta(a\alpha a\beta b+a\alpha b\beta a+a\beta b\alpha a+b\beta a\alpha a)$
$=\theta(a)\alpha\theta(a)\beta\theta(b)+\theta(b)\beta\theta(a)\alpha\theta(a)+\theta(a\alpha b\beta a+a\beta b\alpha a)$

Compare (1) and (2) we get

$\theta(a\alpha b\beta a+a\alpha b\beta a)=\theta(a)\alpha\theta(b)\beta\theta(a)+\theta(a)\beta\theta(b)\alpha\theta(a)$

iii) Replace α for β in (ii) and since X is 2-torsion free we get the require result.

iv) Replacing a+c for a in (iii) we get:

$\theta((a+c)\alpha b\alpha(a+c))=\theta(a+c)\alpha\theta(b)\alpha\theta(a+c)$
$=\theta(a)\alpha\theta(b)\alpha\theta(a)+\theta(a)\alpha\theta(b)\alpha\theta(c)+\theta(c)\alpha\theta(b)\alpha\theta(a)+\theta(c)\alpha\theta(b)\alpha\theta(c)$ …(1)

On the other hand

$\theta((a+c)\alpha b\alpha(a+c))$
$=\theta(a\alpha b\alpha a+a\alpha b\alpha c+c\alpha b\alpha a+c\alpha b\alpha c)$
$=\theta(a)\alpha\theta(b)\alpha\theta(a)+\theta(c)\alpha\theta(b)\alpha\theta(c)+\theta(a\alpha b\alpha c+c\alpha b\alpha a)$...(2)

Compare (1) and (2) we get

$\theta(a\alpha b\alpha c+c\alpha b\alpha a)=\theta(a)\alpha\theta(b)\alpha\theta(c)+\theta(c)\alpha\theta(b)\alpha\theta(a)$

v) Replace a+c for a in definition 1.3

$\theta((a+c)\alpha b\beta(a+c))=\theta(a+c)\alpha\theta(b)\beta\theta(a+c)$
$=\theta(a)\alpha\theta(b)\beta\theta(a)+\theta(a)\alpha\theta(b)\beta\theta(c)+\theta(c)\alpha\theta(b)\beta\theta(c)+\theta(c)\alpha\theta(b)\beta\theta(c)$ …(1)

On the other hand

$\theta((a+c)\alpha b\beta(a+c))=\theta(a\alpha b\beta a+a\alpha b\beta c+c\alpha b\beta a+c\alpha b\beta c)$
$=\theta(a)\alpha\theta(b)\beta\theta(a)+\theta(c)\alpha\theta(b)\beta\theta(c)+\theta(a\alpha b\beta c+c\alpha b\beta a)$ …(2)

Compare (1) and (2) we get

$\theta(a\alpha b\beta c+c\alpha b\beta a)=\theta(a)\alpha\theta(b)\beta\theta(c)+\theta(c)\alpha\theta(b)\beta\theta(c)$

Definition 1.5 :

Let θ be Jordan homomorphism of Γ-ring M into ΓM-module X then we define $\psi:M\times\Gamma\times M\rightarrow X$ by :

$\psi(a,b)_\alpha=\theta(a\alpha b)-\theta(a)\alpha\theta(b)$, for all a,b $\in$ M and $\alpha\in\Gamma$.

In the following lemma the research present the properties of $\psi(a,b)_\alpha$

Lemma 2 :

If θ be Jordan homomorphism of Γ-ring M into ΓM-module X then for all a,b,c $\in$ M and $\alpha,\beta\in\Gamma$:

i) $\psi(a+b,c)_\alpha=\psi(a,c)_\alpha+\psi(b,c)_\alpha$

ii) $\psi(a,b+c)_\alpha=\psi(a,b)_\alpha+\psi(a,c)_\alpha$

iii) $\psi(a,b)_{\alpha+\beta}=\psi(a,b)_\alpha+\psi(a,b)_\beta$

Proof:

i) $\psi(a+b,c)_\alpha = \theta((a+b)\alpha c) - \theta(a+b)\alpha\,\theta(c)$

$= \theta(a\alpha c + b\alpha c) - \theta(a)\,\alpha\,\theta(c) - \theta(b)\,\alpha\,\theta(c)$

$= \theta(a\alpha c) - \theta(a)\,\alpha\,\theta(c) + \theta(b\alpha c) - \theta(b)\,\alpha\,\theta(c)$

$= \psi(a,c)_\alpha + \psi(b,c)_\alpha$

ii) $\psi(a,b+c)_\alpha = \theta(a\alpha(b+c)) - \theta(a)\alpha\,\theta(b+c)$

$= \theta(a\alpha b + a\alpha c) - \theta(a)\,\alpha\,\theta(b) - \theta(a)\alpha\,\theta(c)$

$= \theta(a\alpha b) - \theta(a)\,\alpha\,\theta(b) + \theta(a\alpha c) - \theta(a)\alpha\,\theta(c)$

$= \psi(a,b)_\alpha + \psi(a,c)_\alpha$

iii) $\psi(a,b)_{\alpha+\beta} = \theta(a(\alpha+\beta)b) - \theta(a)(\alpha+\beta)\theta(b)$

$= \theta(a\alpha b + a\beta b) - \theta(a)\alpha\,\theta(b) - \theta(a)\beta\,\theta(b)$

$= \theta(a\alpha b) - \theta(a)\alpha\,\theta(b) + \theta(a\beta b) - \theta(a)\beta\,\theta(b)$

$= \psi(a,b)_\alpha + \psi(a,b)_\beta$

Not that θ is homomorphism from Γ-ring M into ΓM-module X if and only if $\psi(a,b)_\alpha = 0$ for all $a,b \in M$ and $\alpha \in \Gamma$.

Now, we present the following lemma:

Lemma 3:

Let θ be a Jordan homomorphism of 2-torsion free Γ-ring M into 2-torsion free ΓM-module X then for all $a,b,m \in M$ and $\alpha,\beta \in \Gamma$.

i) $\psi(a,b)_\alpha\,\beta\,\theta(m)\beta\,\psi(b,a)_\alpha + \psi(b,a)_\alpha\,\beta\,\theta(m)\beta\,\psi(a,b)_\alpha = 0$

ii) $\psi(a,b)_\alpha\,\alpha\,\theta(m)\alpha\,\psi(b,a)_\alpha + \psi(b,a)_\alpha\,\alpha\,\theta(m)\alpha\,\psi(a,b)_\alpha = 0$

iii) $\psi(a,b)_\beta\,\alpha\,\theta(m)\alpha\,\psi(b,a)_\beta + \psi(b,a)_\beta\,\alpha\,\theta(m)\alpha\,\psi(b,a)_\beta = 0$

Proof :

Let $w = a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b$

Since θ is a Jordan homomorphism, then

$\theta(w) = \theta(a\alpha b\beta m\beta b\alpha a + b\alpha a\beta m\beta a\alpha b)$

$= \theta(a)\,\alpha\,\theta(b\beta m\beta b)\alpha\,\theta(a) + \theta(b)\alpha\,\theta(a\beta m\beta a)\alpha\,\theta(b)$

$= \theta(a)\,\alpha\,\theta(b)\beta\,\theta(m)\beta\,\theta(b)\alpha\,\theta(a) + \theta(b)\alpha\,\theta(a)\beta\,\theta(m)\,\beta\,\theta(a)\alpha\,\theta(b)$ …(1)

On the other hand

$\theta(w) = \theta((a\alpha b)\beta m\beta(b\alpha a) + (b\alpha a)\beta m\beta(a\alpha b))$

$= \theta(a\alpha b)\beta\,\theta(m)\beta\,\theta(b\alpha a) + \theta(b\alpha a)\beta\,\theta(m)\beta\,\theta(a\alpha b)$

$= \theta(a\alpha b)\beta\,\theta(m)\beta(\theta(a)\alpha\,\theta(b) + \theta(b)\,\alpha\,\theta(a) - \theta(a\alpha b)) + (-\theta(a\alpha b) + \theta(a)\alpha\,\theta(b) + \theta(b)\alpha\,\theta(a))\beta\,\theta(m)\beta\,\theta(a\alpha b)$

$= -\theta(a\alpha b)\beta\,\theta(m)\beta(\theta(a\alpha b) - \theta(a)\alpha\,\theta(b)) - \theta(a\alpha b)\beta\,\theta(m)\beta(\theta(a\alpha b) - \theta(b)\,\alpha\,\theta(a)) + \theta(a)\alpha\,\theta(b)\beta\,\theta(m)\beta\,\theta(a\alpha b) + \theta(b)\alpha\,\theta(a)\beta\,\theta(m)\beta\,\theta(a\alpha b)$ …(2)

By comparing (1) and (2), we get:

$0 = -\theta(a\alpha b)\beta\,\theta(m)\beta\,\psi(a,b)_\alpha - \theta(a\alpha b)\beta\,\theta(m)\beta\,\psi(b,a)_\alpha + \theta(a)\,\alpha\,\theta(b)\beta\,\theta(m)\beta\,\theta(a\alpha b) + \theta(b)\alpha\,\theta(a)\beta\,\theta(m)\beta\,\theta(a\alpha b) - \theta(a)\,\alpha\,\theta(b)\beta\,\theta(m)\beta\,\theta(b)\alpha\,\theta(a) - \theta(b)\alpha\,\theta(a)\beta\,\theta(m)\beta\,\theta(a)\alpha\,\theta(b)$

$= -\theta(a\alpha b)\beta\,\theta(m)\beta\,\psi(a,b)_\alpha - \theta(a\alpha b)\beta\,\theta(m)\beta\,\psi(b,a)_\alpha + \theta(a)\,\alpha\,\theta(b)\beta\,\theta(m)\beta\,\psi(b,a)_\alpha + \theta(b)\alpha\,\theta(a)\beta\,\theta(m)\beta\,\psi(a,b)_\alpha$

$= -(\psi(a,b)_\alpha - \theta(b)\alpha\,\theta(a))\;\beta\,\theta(m)\;\beta\,\psi(a,b)_\alpha - (\psi(a,b)_\alpha - \theta(a)\alpha\,\theta(b))\theta(a)\;\alpha\,\theta(b)$

Thus we have:

$\psi(a,b)_\alpha\,\beta\,\theta(m)\beta\,\psi(b,a)_\alpha + \psi(b,a)_\alpha\,\beta\,\theta(m)\beta\,\psi(a,b)_\alpha = 0$

ii) Replace β by α in (i) and proceeding in the same way as in the proof of (i) by the similar arguments, we get (ii).

iii) Interchanging α and β in (i), we get (iii).

The research needs the following lemma:

Lemma 4:[5]

Let M be Γ-ring and X a 2-tortion free semiprime ΓM-module and a,b the elements of X. Then the following conditions are equivalent:

i) aΓMΓb=(0)

ii) bΓMΓa=(0)

iii) aΓMΓb+bΓMΓa=(0)

If one of these conditions are fulfilled then aΓb=bΓa=(0).

Lemma 5:

Let θ be a Jordan homomorphism of 2-torsion free Γ-ring M into 2-torsion free ΓM-module X then for all a,b,m ∈ M and $\alpha, \beta \in \Gamma$.

i) $\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = \psi(b,a)_\alpha \beta(\text{m}) \beta \psi(a,b)_\alpha = 0$

ii) $\psi(a,b)_\alpha \alpha \theta(\text{m}) \alpha \psi(b,a)_\alpha = \psi(b,a)_\alpha \alpha \theta(\text{m}) \alpha \psi(a,b)_\alpha = 0$

iii) $\psi(a,b)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta = \psi(b,a)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta = 0$

Proof:

(i) By lemma 3 (i) we get:

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(b,a)_\alpha \beta \theta(\text{m}) \beta \psi(a,b)_\alpha = 0$

By lemma 4 we get:

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = \psi(b,a)_\alpha \beta \theta(\text{m}) \beta \psi(a,b)_\alpha = 0$

ii) By lemma 3 (ii) we get:

$\psi(a,b)_\alpha \alpha \theta(\text{m}) \alpha \psi(b,a)_\alpha + \psi(b,a)_\alpha \alpha \theta(\text{m}) \alpha \psi(a,b)_\alpha = 0$

By lemma 4 we get:

$\psi(a,b)_\alpha \alpha \theta(\text{m}) \alpha \psi(b,a)_\alpha = \psi(b,a)_\alpha \alpha \theta(\text{m}) \alpha \psi(a,b)_\alpha = 0$

iii) By lemma 3 (iii) we get:

$\psi(a,b)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta + \psi(b,a)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta = 0$

By lemma 4 we get:

$\psi(a,b)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta = \psi(b,a)_\beta \alpha \theta(\text{m}) \alpha \psi(b,a)_\beta = 0$

2) The Main Results:

In this section the research introduce the main results and we begin by the following theorem:

Theorem 6:

Let θ be a Jordan homomorphism from Γ-ring M into prime ΓM-module X then for all a,b,c,d,m ∈ M and $\alpha, \beta \in \Gamma$:

i) $\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,c)_\alpha = 0$

ii) $\psi(a,b)_\alpha \alpha \theta(\text{m}) \alpha \psi(d,c)_\alpha = 0$

iii) $\psi(a,b)_\beta \alpha \theta(\text{m}) \alpha \psi(d,c)_\beta = 0$

Proof:

Replace a+c for a in lemma 5(i)

$\psi(a+c,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a+c)_\alpha = 0$

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha + \psi(c,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(c,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha = 0$

By lemma 5

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha + \psi(c,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = 0$

Therefore, we get:

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha \beta \text{m} \beta \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha = 0$

$= - \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha \beta \text{m} \beta \psi(c,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = 0$

Hence, by the primness of X:

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha = 0$ …(1)

Now, replacing b+d for b in lemma 5(i)

$\psi(a,b+d)_\alpha \beta \theta(\text{m}) \beta \psi(b+d,a)_\alpha = 0$

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha + \psi(a,d)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(a,d)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha = 0$

By lemma 5

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha + \psi(a,d)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = 0$

Then the research get

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha \beta \theta(\text{m}) \beta \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha$

$= - \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha \beta \theta(\text{m}) \beta \psi(a,d)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha = 0$

Since X is prime ΓM-module, then

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha = 0$ …(2)

Thus

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b+d,a+c)_\alpha = 0$

$\psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,a)_\alpha + \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(b,c)_\alpha + \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,a)_\alpha + \psi(a,b)_\alpha \beta \theta(\text{m}) \beta \psi(d,c)_\alpha = 0$

By (1) and (2) and lemma 5(i) the research get

$\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\alpha = 0$

ii) Proceeding in the same way as before by similar replacements in lemma 5(ii), the research obtains (ii).

iii) Finally, replacing $\alpha+\beta$ for α in (ii), the research get :

$\psi(a,b)_{\alpha+\beta}\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_{\alpha+\beta} = 0$

$\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\alpha +$

$\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta +$

$\psi(a,b)_\beta\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\alpha +$

$\psi(a,b)_\beta\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta = 0$

By (i) and (ii) the research get

$\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta +$

$\psi(a,b)_\beta\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\alpha = 0$

Therefore

$\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta\ \alpha$

$\theta(\mathrm{m})\ \alpha\ \psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta$

$= -\psi(a,b)_\alpha\ \beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\beta\ \alpha\ \theta(\mathrm{m})\quad \alpha\ \psi(a,b)_\beta$

$\beta\ \theta(\mathrm{m})\ \beta\ \psi(d,c)_\alpha = 0$

Since X is prime ΓM-module, then

$\psi(a,b)_\beta\ \alpha\ \theta(\mathrm{m})\ \alpha\ \psi(d,c)_\beta = 0$

Now, the research able to prove the main result in this paper.

<u>Theorem 7:</u>

Every Jordan homomorphism from Γ-ring M into prime ΓM-module X is either a homomorphism from M into X or anti-homomorphism from M into X.

Proof:

Let θ be Jordan homomorphism of Γ-ring M into prime ΓM-module X. Since X is prime therefore by theorem 6 (i) , $\psi(a,b)_\alpha = 0$ or $\psi(d,c)_\alpha = 0$ for all a,b,c,d$\in$M and $\alpha\in\Gamma$.

If $\psi(d,c)_\alpha \neq 0$ for all c,d$\in$M and $\alpha\in\Gamma$ then $\psi(a,b)_\alpha = 0$ for all a,b$\in$M and $\alpha\in\Gamma$ and hence θ is homomorphism of Γ-ring M into prime ΓM-module X.

But if $\psi(d,c)_\alpha = 0$ for all c,d$\in$M and $\alpha\in\Gamma$ then θ is anti-homomorphism from M into X.

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تشاكل جوردان للمقاسات من النمط-ΓM

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الخلاصة

الهدف الاساسي من العمل تقديم ودراست المفاهيم التالية التشاكل وتشاكل جوردان على المقاسات X من النمط- ΓM وقد أثبتنا أن كل تشاكل جوردان من الحلقة M من النمط- Γ الى المقاس X من النمط- ΓM هو تشاكل من M الى X .