

Analysis of CFRP Strengthened Reinforced Concrete Beams under Monotonic and Cyclic Loads

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Abstract

A layered technique formulation through the depth of reinforced concrete beam section is devoted to develop an incremental-iterative algorithm suitable for the analysis of beams strengthened by CFRP laminates under monotonic and cyclic loading conditions. Concrete nonlinear behavior in compression through loading, unloading, and reloading stages is considered with a tension-stiffening model to represent concrete in tension. A bilinear behavior with strain hardening model of steel reinforcement bars through loading, unloading, and reloading stages is used. A computer program Matlab code is developed and verified through comparisons with given experimental case studies available in literature, which show good agreement. Extending the present algorithm to include different sections, hybrid beams, and long term effects are recommended as future work.

Keywords: Layered technique, CFRP strengthening, concrete beams, cyclic loads.



تحليل العتبات الخرسانية المسلحة المقواة بالياف الكربون تحت تأثير احمال

جاسئة ودورية

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الملخص

تم استخدام تقنية الطبقات خلال مقطع العتبات الخرسانية المسلحة لأنشاء خوارزمية تزايدية – تكرارية لتحليل العتبات المقواة بصفائح الياف الكريون والمعرضة الى نمطي احمال جاسئة ودورية. تم اعتبار التصرف اللاخطي للخرسانة في حالة الانضغاط لمراحل التحميل، التفريغ، واعادة التحميل بالإضافة الى اعتبار انموذج التصلب – التوتر للخرسانة في حالة الشد. تم استخدام تصرف ثنائي مع انموذج تصلب الاجهاد لقضبان التسليح لمراحل التحميل، التفريغ، واعادة التحميل. تم انشاء برنامج حاسوب باستخدام الماتلاب وتم تحقيقه من خلال مقارنات مع نتائج عملية متوفرة في دراسات سابقة، والتي بينت توافقا جيدا. كدراسات مستقبلية، تم التوصية بتطوير الخوارزمية الحالية كي تشمل مقاطع مختلفة، عتبات هجينة، والتأثير طويل الامد.

الكلمات المفتاحية: تقنية الطبقات، تقوية بألياف الكربون، عتبات خرسانية، احمال دورية.

1. Introduction

The moment-curvature relationship for a strengthened reinforced concrete section subjected to flexural load can be derived assuming that plane sections before bending remain plane after bending and that the stress-strain curves for concrete and steel are known. The curvatures and corresponding bending moments may be determined using these assumptions and from the requirements of strain compatibility and equilibrium of forces [1, 2]. Trisegmental relationships predict the actual behavior of reinforced concrete beams better than bilinear relationships, which tend to overestimate deformations [3]. In some comparisons between moment-curvature theory and experimental data, the overestimation was between 10 to 100% [4]. El-Tawil et al. [5] developed an analytical model to simulate the static and incremented fatigue behavior of reinforced concrete beams strengthened with CFRP (Carbon Fiber Reinforced Polymer) laminates. Fiber section model, including the nonlinear timedependent behavior of concrete in addition to yielding of steel and rupture of CFRP laminates, is used to carry out the calculations. Then the results gained from analysis were compared with experimental data done by Shahawy and Beitelman [6, 7] for two groups of CFRP strengthened beams. They concluded that fatigue loads caused a time-dependent stresses redistribution, which led to a mild increase in steel and CFRP laminate stresses as fatigue life was exhausted. CFRP materials are an excellent choice for externally strengthening reinforced concrete beams [8] because they behave linearly elastic up to failure with high ultimate strengths that are much greater than the yield strength of reinforcing steel and do not show degradation when exposed to moisture and typical outdoor temperatures [9, 10].

2. Material Constitutive Relationships

2.1. Basic Assumptions

The followings are the main basic assumptions used in the present algorithm: plane sections normal to the beam longitudinal axis remain plane after bending, the CFRP laminate and the longitudinal steel bars reinforcement are assumed to be perfectly bonded to the concrete, and deformations resulting from shear forces are neglected throughout the present analytical method.



2.2. Concrete Behavior in Compression

An empirical monotonic model suggested by Kent and Park [11] is adopted in this analysis, as shown in Fig. 1. This model describes the stress–strain relationship of concrete in compression as a function of the concrete compressive strength and its corresponding strain. The model consists of a curvilinear ascending and a bilinear descending branches. This model is then extended by Otter and Naaman [12] and has been selected to represent the cyclic behavior of concrete in direct compression. Some of the key points used to describe the response under cyclic loading are shown in Fig. 2.



Fig. 1: Monotonic stress-strain relationship for concrete in compression [11].





2.3. Concrete Behavior in Tension

Concrete is assumed to crack when it reaches its tensile strength calculated according to the ACI-318 Code 2014 [13]. After concrete cracks, gradual release of tensile stress takes place in concrete members reinforced with steel bars or CFRP laminates. As the crack widens, this tension-stiffening effect accounts for the mechanism of load transfer which exist between reinforcing bars (or CFRP laminate) and linear degradation of the surrounding concrete is used to describe its tensile strength after cracking stage. It is appropriate to consider that CFRP will generate a higher concrete tension stiffening effect as compared with steel reinforcement because it is directly attached to a bigger surface area of concrete. The residual stress due to the presence of steel bar reinforcement alone is assumed to decrease linearly from 70% from its cracking stress to zero at strain value equals to five times cracking strain of concrete [6]. Tension-stiffening effect due to the presence of both steel and CFRP laminate is assumed to be decreased to zero at 20 times cracking strain of concrete. These mentioned models are shown in Fig. 3. For the cyclic behavior; unloading and reloading of a cracked layer are supposed to follow a secant path [14], the secant modulus can be calculated using the previously kept maximum strain developed in the cracked layer.



Fig. 3: Tension-stiffening models for concrete in tension, monotonic, and cyclic behavior [14]

2.4. Monotonic and Cyclic Uniaxial Stress-Strain Relationship of Steel

A bilinear elasto-plastic model with a strain hardening part is used for the steel bars [15]. The response in tension and compression is assumed to be identical. In case of unloading when the steel is stressed beyond the yield stress, a path with the same elastic modulus could be followed.

2.5. Modeling the Behavior of CFRP Laminates

The CFRP laminate material is assumed to be a unidirectional brittle material [16], with high yield tensile strength f_{yf} , and with E_f modulus of elasticity. CFRP laminates have no compressive strength; therefore, its response under tensile stresses is a matter of concern. An elastic behavior followed by sudden failure is assumed to occur during loading, unloading, and reloading stages.

3. Moment-Curvature Relationship

The determination of theoretical moment-curvature for cyclically loaded reinforced concrete rectangular beams externally strengthened with CFRP laminates involves a large amount of computational effort [17]. Therefore a Matlab code is developed and verified to perform the required computations. The code implements a discrete (layered) element technique to simulate the complex stress distribution through the section that occurs due to cyclic loading. In this technique, the rectangular cross section of the beam is divided into a number n of horizontal elements (layers). Each element has a width equal to the section width, Fig. 4. If there are n elements in the section, each element will have a depth d_e , described by;

$$d_e = \frac{h}{n}$$

where, h is the overall depth of the section.



Fig. 4: Discrete element technique for rectangular strengthened beam section

Assuming that the strain at top fiber of the concrete section is ε_c and the neutral axis depth measured from this fiber is d_n , the average strain of an element *i* is given by;



$$\varepsilon_i = \varepsilon_c \frac{d_n}{d_n} \tag{1}$$

where; d_i = is the depth measured from top concrete fiber to the center of the concrete element *i*, given by;

$$d_i = (i - 0.5)\frac{h}{n} \tag{2}$$

Consequently, the strain at top or bottom steel bars is;

$$\varepsilon_s = \varepsilon_c \, \frac{d_n \pm d' (or \, d)}{d_n} \tag{3}$$

where; d, d' are the depth of bottom and top steel bars measured from top concrete fiber, respectively. The concrete tensile strength may be calculated from ACI-318 Code 2014 [13], as:

$$f_t = 0.7\sqrt{f_c'} \tag{4}$$

and the corresponding strain is calculated using Hook's Law and concrete modulus of elasticity (E_o) [13] as;

$$\mathcal{E}_t = \frac{f_t}{E_o} \tag{5}$$

Also, strains at top and bottom CFRP laminates are:

$$\varepsilon'_{f} = \varepsilon_{c} \frac{d_{n} + d_{f}}{d_{n}}$$
 for top laminate (6)

$$\varepsilon_f = \varepsilon_c \frac{d_n - d_f}{d_n}$$
 for bottom laminate (7)

where; d_f , and d_f are depth of top and bottom CFRP laminates measured from top concrete fiber, respectively. Now, for an element *i* within the section, having ε_i , the corresponding force is calculated from;

$$F_i = \frac{f_i b h}{n} \tag{8}$$

where; f_i = the calculated corresponding stress for the element *i*,

h = total depth concrete section, and,



b = width of rectangular beam, or element.

Applying the equilibrium equation for all horizontal forces within the section;

$$\sum C - \sum T = 0 \tag{9}$$

leads to the following expression;

$$F_{cc} + F_{sc} - F_{ct} - F_{st} - F_{f} = 0 ag{10}$$

where; $\sum C$, and $\sum T$ total compressive and tensile forces, respectively.

F_{cc} = sum of forces of all concrete layers in the compression zone, = $\sum_{i=1}^{n_c}$	F_i (11)
F_{cc} = sum of forces of all concrete layers in the compression zone, = $\sum_{i=1}^{n_c}$	F_i (11)

$$F_{ct} = \text{sum of forces of all concrete layers in the tension zone,} = \sum_{i=n_c+1}^{n} F_i$$
 (12)

$$F_{sc}$$
 = compressive force in compression steel bars, = $A_{sc}f_{sc}$ (13)

$$F_{st}$$
 = tensile force in tension steel bars, = $A_{st} f_{st}$ (14)

$$F_f$$
 = tensile force in CFRP laminate, = $A_f f_f$ (15)

 A_{sc} , A_{st} = areas of compression and tension steel bars, respectively,

 f_{sc} , f_{st} = compression and tension steel bars stresses, respectively,

 $A_{f}, f_f = CFRP$ laminate cross sectional area and stress, respectively, and,

 n_c = number of concrete layers (elements) in compression zone.

Now, if condition of equilibrium for all horizontal forces is satisfied, the moment M_i , of a concrete element *i*, about the neutral axis is calculated using;

$$M_i = F_i \left(d_n - d_i \right) \tag{16}$$

then total internal resisting moment for the whole section can be obtained;

$$M = \sum_{i=1}^{n} M_{i} + F_{sc} (d_{n} - d' or d) + F_{st} (d_{n} - d or d') + F_{f} (d_{n} \pm d'_{f} or d_{f})$$
(17)

and the corresponding curvature is;

$$\phi = \arctan \frac{\varepsilon_c}{d_n} \tag{18}$$

Once moment and curvature values at a specific strain increment are calculated, load and deflection values can be calculated using the following basic relationship:

$$\phi = \frac{M}{EI} = -\frac{d^2 y}{dx^2} \tag{19}$$

After integrating this formula twice, the following final equation for a simple beam with two points loading is obtained:



$$\delta = \phi \left(\frac{3L^2 - 4a^2}{24} \right) \tag{20}$$

where; EI = flexural rigidity

and,
$$P = \frac{4M}{(L - MZL)}$$
 (21)

L = beam length, support to support distance,

a = support to load distance,

 δ = mid-span deflection,

P = total load on the beam, and,

MZL = pure moment zone length, or distance between two-point loading.

4. Numerical Method of Analysis

An incremental-iterative technique is used to compute the theoretical moment–curvature for strengthened rectangular reinforced concrete beams, and then load-deflection relationships, using the computer program developed in this research work. The computer program is coded using Matlab language. The main input data required for the analysis includes:

- The cross sectional properties of the reinforced concrete beam.
- The yield stress f_y , the modulus of elasticity E_s , the area of top and bottom steel bars, and location of steel bars, (d' and d).
- Cross-sectional area A_v , spacing *s*, and core dimensions of confined stirrups.
- Cross-sectional area A_f , Young modulus E_f , and yield strength f_{yf} of CFRP laminate.
- Number of discrete elements *n* within the concrete section.

An outline of the procedure for determining the moment–curvature relationship and then the force–displacement relationship is given below:

1. The initial value of the strain ε_c at the top fiber of the cross-section is prescribed.

- **2.** For the given value of ε_c , the neutral axis depth d_n is estimated, and stresses in each discrete concrete element is computed according to the mentioned strain profiles.
- **3.** The horizontal forces in each concrete element, in the steel bars, and in the CFRP laminate in tension zone are calculated.
- **4.** The equilibrium of all these horizontal forces is checked according to $\sum C \sum T = 0$.

- **5.** If the equilibrium conditioned mentioned in the previous step as given in equation (9) is not satisfied, the estimated depth to the neutral axis is adjusted accordingly until the equilibrium condition of forces is achieved.
- 6. If the equilibrium of horizontal forces is satisfied, the bending moment M, and the corresponding curvature ϕ , are calculated for that particular value of ε_c using equations (17) and (18), then the load and deflection values are calculated.
- 7. The strain ε_c at the top fiber is incremented, and the procedure through steps 2 to 6 is then repeated for the new prescribed value of ε_c .

5. Results and Discussion

To verify of the developed Matlab code, two groups of data gained from two different previous researches are used. The first group consists of two reinforced concrete beams tested monotonically up to failure by Spadea et al. [18]; the first rectangular beam A1 is a conventional reinforced concrete beam while the second beam A1.1 is a CFRP bottom face strengthened reinforced concrete beam. The strengthening is done using 80 mm x 1.2 mm x 4700 mm CFRP laminate. Beams important data are shown in Table 1. The present developed algorithm gives good agreement of load versus mid-span deflection, Fig. 6, when compared with the experimental data in spite of the fact that the computed ultimate capacities for these beams are less than the experimentally recorded values.

Beam	Cross-	Internal Reinforcement			CFRP	Cube Comp.	Load (kN)		
Deam	(mm^2)	Тор	Bott.	Shear	Usage	Strength (MPa)	Crack	Yield	Ultimate
A1	140+200	2-Ф16 2-Ф16	516 2 016	∆ 6@150	-	30.1	10.2	45.2	54.0
A1.1	140x300		2-910	$\Phi 0 @ 150$	Bottom	34.3	10.0	55.3	86.8

Table 1: Experimental data of the two tested beams [18]



Fig. 6: Experimental and analytical load-deflection curves for beams A1 and A1.1

The second group for verification of the proposed algorithm is consists of the data given in Table 2. for ten reinforced concrete beams, which are cast and tested by Al-Shaarbaf and Hasan [19] under monotonic and different cyclic loading histories and by different amounts of CFRP laminates, as shown in Fig. 7. The main input data used for the computer program executions of these tested beams are shown in Table 3.



Fig. 7: Reinforcement details and dimensions of a typical tested beam [19]

Beam	CFRP Width (mm)	Loading Type	Number of Cycles	Applied Cyclic Load (kN)	% Cyclic Load (from ultimate)	Ultimate Capacity (kN)
BR1	-	Monotonic	-	-	-	54

_

-

_

50

60

70

64

70

70

_

-

_

2

2

5

5

5

2

75

69

76

44

74

72

67

73

62

_

-

_

93

80

93

93

93

93

Bean

BF1

BF2

BF3

BR1R1

BF1R1

BF1R2

BF2R1

BF3R1

BF1C1

50

30

70

_

50

50

30

70

50 top, &

50 bottom

Monotonic

Monotonic

Monotonic

Repeated

Repeated

Repeated

Repeated

Repeated

Cyclic

Table 2: Bean	ns strengthening	and loading sche	eme [19]
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Conc	rete						
E_c^*		Young's Modulus (GPa)	28				
$f_{c'}^{**}$		Compressive Strength (MPa)	30				
f_t^*		Tensile Strength (MPa)	3.4				
E _{cu} *		Uniaxial Crushing Strain	0.004				
v **		Poisson's Ratio	0.2				
<i>n</i> **		Total Layers through the Depth	30				
\mathfrak{a}_2^{**}			0.3				
α ₁ ^{**} Conventional Strengthened		Tension-Stiffening Parameters	5				
			20				
Longitudinal Reinforcement							
E_{s}^{**}		Young's Modulus (GPa)	200				
f_{v}^{**}		Yield Stress (MPa)	518				
		Area of Top Steel Bars (mm ²)	157				
A_{s}^{**}		Area of Bottom Steel Bars (mm ²)	157				
H_{s}^{**}		Hardening Parameter	0.1				
CFR	CFRP Laminates						
E_f^{**}		Young's Modulus (GPa)	230				
f_{yf}^{**}		Yield Stress (MPa)	3500				

Table 3: Main input data used in the developed algorithm

* Calculated

** Input Data

The main experimental and layered technique analysis results for these tested beams are shown in Table 4, and Table 5. Comparison between experimental and layered formulation load versus mid-span deflection curves are shown in Fig. 8, and Fig. 9. The layered formulation (L. F.) curve for the monotonically tested conventional reinforced concrete beam BR1, Fig. 8a, shows stiffer response compared with the experimentally recorded curve for the three stages of behavior mentioned by Al-Shaarbaf and Hasan [19]. The computed ductility ratio, which is defined as the ratio of ultimate to yield deflections, for layered technique analysis method is 3.3, Table 5, and is lower than the recorded experimental ductility ratio which is 3.7. The present analytical method shows a decrease of 11% in ductility ratio. Also, an increase of 2% is noticed in the computed over the experimentally recorded ultimate monotonic load capacity for the conventional reinforced concrete beam BR1. Fig. 8b, shows a slightly stiffer behavior in the pre-cracking and post-cracking stages, but it shows also a relatively

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early failure. This early failure reduced the computed ductility ratio, as shown in Table 5. The percentage reduction in the computed ductility ratio over the experimentally recorded ductility ratio is 22%, because the layered formulation produced a ductility ratio of 2.1, which is less than the experimentally recorded ductility ratio of 2.7. But, a reduction of 2% is noticed in the computed ultimate monotonic load capacity. A slight stiffer response, is also noticed in the pre-crack and post-crack regions in the layered technique analysis of the strengthened beam BF2, shown in Fig. 8c. The computed ductility ratio is 2.3, which is less than the experimentally recorded ductility ratio of 2.7 (i.e., by a reduction of 15% in ductility ratio). While the computed ultimate monotonic load capacity increased by 17% compared with the experimentally recorded ultimate monotonic load capacity. The computed ultimate load capacity for beam BF3, shown in Fig. 8d [19], increased by 3% compared with the experimental ultimate monotonic capacity, while, the computed ductility ratio is 2.0, which is less than the experimentally recorded ductility ratio of 2.6 (i.e., by a reduction of 23% in ductility ratio). The comparison shows an indicated response slightly stiffer than the experimental one, especially at stages close to the ultimate monotonic load. This stiffer response is due to the fact that the models considered in layered technique are more conservative than the real behavior.

		Deflecti	on (mm)	(A)		(P)			
Beam Experimer		Experimental Experimental Formulation		d ion $\frac{(\Delta_u)_{L.F.}}{(\Delta_u)_{Exp.}}$		imental	Laye Formu	ered lation	$\frac{(\mathbf{r}_u)_{L.F.}}{(\mathbf{P}_u)_{Exp.}}$	
	\varDelta_y	Δ_u	\varDelta_y	Δ_u	(70)	P_y	P_u	P_y	P_u	(70)
BR1	7.1	26.3	6.7	22.1	84	50	54	48.1	55.2	102
BF1	7.4	19.9	7.8	16.4	82	66	75	64.5	73.9	98
BF2	7.2	19.5	7.6	17.3	89	58	69	58.4	68.1	117
BF3	7.7	20.1	8.3	16.2	80	70	76	68.9	78.3	103
BR1R1	7.0	26.9	6.2	21.1	78	49	44	48.1	46.1	105
BF1R1	9.3	19.4	8.2	15.9	82	67	74	63.7	72.9	98
BF1R2	7.6	18.9	7.3	16.7	88	66	72	64.5	70.8	98
BF2R1	7.2	18.1	7.9	16.4	90	58	67	58.4	66.2	98
BF3R1	9.7	15.9	11.2	15.7	99	68	73	68.1	72.4	99
BF1C1	7.9	19.6	8.2	18.5	94	65	62	64.5	68	110

Table 4: Comparison of experimental and layered formulation results



Table 5. Experimental and layered formulation ductility ratios

	Duct	tility Ratio	Calculated		
Beam	Experimental Layered Formulation		Reduction in Ductility Ratio (%)		
BR1	3.7	3.3	11		
BF1	2.7	2.1	22		
BF2	2.7	2.3	15		
BF3	2.6	2.0	23		
BR1R1	3.8	3.4	11		
BF1R1	2.1	1.9	10		
BF1R2	2.5	2.3	8		
BF2R1	2.5	2.1	16		
BF3R1	1.3	1.1	15		
BF1C1	2.5	2.3	8		



Fig. 8: Experimental and analytical load-deflection curves for beams tested under monotonic loads



Experimental and computed load versus mid-span deflection comparison is made in Fig. 9a for the conventional reinforced concrete beam BR1R1 subjected to two constant repeated load cycles. The computed ultimate load capacity is 5% higher than the recorded experimentally ultimate load capacity, while the computed ductility ratio is reduced by 11%. The experimental ductility ratio is 3.8, while the predicted ductility ratio is 3.4. Fig. 9b presents a comparison between the experimental and computed load versus mid-span deflection curves for beam BF1R1, which is subjected to two constant repeated load cycles followed by a monotonic loading up to failure. The comparison indicates a reduction of 2% in the computed ultimate load, and by a reduction of 10% in the computed ductility ratio. A comparison for beam BF1R2, which is subjected to five constant repeated load cycles followed by monotonic loading up to failure, is shown in Fig. 9c. The computed ultimate load increased by 3% compared with the experimentally recorded ultimate load. The computed ductility ratio is 2.3, which is less than the experimentally recorded ductility ratio of 2.5 (i.e., by a reduction of 8% in ductility ratio). A comparison is made in Fig. 9d for the beam BF2R1 which is subjected to five constant repeated load cycles followed by monotonic loading up to failure. The computed ultimate load is 2% less than the experimental ultimate load, while the computed ductility ratio is reduced by 16%. The experimental ductility ratio is 2.5, while the calculated ductility ratio is 2.1. Fig. 9e presents a comparison between the experimental and computed load versus mid-span deflection curves for beam BF3R1, which is subjected to five constant repeated load cycles followed by monotonic loading up to failure.

The comparison indicates a reduction by 1% in the computed ultimate load, by a reduction of 15% in the computed ductility ratio. The experimental recorded ductility ratio is 1.3, while the computed ductility ratio is 1.1. The comparison shows that the effect of repeated load cycles on the beam stiffness is less in layered technique analysis. This may be due to the use of idealized concrete and steel bars behavior through unloading and reloading stages. A comparison for beam BF1C1 is shown in Fig. 9f [19]. This beam is strengthened by 50 mm width top and 50 mm width bottom CFRP laminates and subjected to constant reversed cyclic loads. The computed analytical ultimate load increased by 10% compared with the experimentally recorded load for the conventionally reinforced beam BR1R1 subjected to two repeated load cycles. The computed ductility ratio is 2.3, which is less than the experimentally recorded ductility ratio of 2.5 (i.e., by a reduction by 8% in ductility ratio). The comparison shows that the analytical response is slightly stiffer than the experimental one



during loading, unloading, and reloading schemes, especially near the ultimate load region. This stiffer response may be attributed to the relatively stiff models used in the layered formulation to represent the concrete in compression and tension.



Fig. 9: Experimental and analytical load-deflection curves for beams tested under cyclic loads

6. Conclusions and Recommendations

The monotonic and cyclic behavior of the strengthened beams is investigated well by using the adopted layered technique method of analysis through the developed computer program. The load versus mid-span deflections, and the yield and ultimate loads predicted are close to those measured during the experimental tests available in literature. The maximum difference between experimental and computed ultimate load capacities for the conventional reinforced concrete beams tested by Al-Shaarbaf and Hasan [19] is 5%, while the maximum difference in ultimate load capacities for the strengthened beams is 17%. The maximum difference in the computed ductility ratio is 11% for the conventional beams, and is 23% for the strengthened beams. Extending the present algorithm to analyze different cross sections and hybrid beams may be recommended as future work. Long term effects and the inclusion of steel fibers in normal and high strength concrete with the appropriate constitutive cyclic model may be also studied.

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