Estimating Parameters of Non-Gaussian Mixed Model ARMA(1,1) by using a Non-Linear Newton Raphson Procedure

- Simulation Study-

Dr. Abdul-Majeed H. Alnassir
Professor of Statistics
Admin. & Econ. College
University of Baghdad

Mr.Mohammed Q. Al-Khudhairy
Assistant Professor
Dept of Ban. & Comm.
Al-Mansour University College

ABSTRACT

To estimate non-linear parameters of mixed model ARMA(1,1) in moving averages, the researchers have applied a non-linear Newton-Raphson procedure to estimate these parameters of this model, and to test its efficiency, to compare it with non-linear least Square method by simulation.

As a main conclusion, the researchers have found that this applied procedure is good, efficient and reliable as a method for estimating parameters of the non-linear mixed model ARMA (1,1).

Introduction :

From 1968 to 1970, Box-Watts and Box-Jenkins proposed models of seasonal Series, and then made mathematical formulae to determine, estimate, test, and forecast these models. From that time, non-linear least Square method has remained the most common and usable to estimate the non-linear parameters of the mixed model ARMA(1,1).

This research aims at applying a non-linear Newton-Raphson procedure to estimate the non-linear parameters of the mixed model from first order and comparing it with non-linear least square method by simulation. Models random errors have Non-Gaussian distribution.

2- Mixed Model ARMA (1,1)

By using backshift operator B in the following formula:

$$\phi(B)x_t = \theta(B)a_t \quad \ldots(2.1)$$

$$(1-\phi_1 B)x_t = (1-\theta_1 B)a_t$$

So, the general formula of the mixed (Autoregressive-moving Average) Model ARMA(1,1) can be written as follows:

$$X_t = \phi_1 X_{t-1} + a_t - \theta_1 a_{t-1} \quad \ldots(2.2)$$

where:

$X_{t-i}, i = 0,1$ $,t = 1,2,\ldots,n$	values of time Series observations
ϕ_1	Autoregressive parameter
θ_1	Moving Average parameter
$a_{t-i}, i = 0,1$	random error

2-1: The Theoretical Aspects of the ARMA(1,1):

a. stationary:

To achieve stationary roots of the equation $\phi(B) = 1 - \phi_1 B = o$ Must be outside the Unit circle, where:

$$|B| > 1$$

This leads to make the parameter:

$$|\phi_1| = \frac{1}{|B|}$$

$$\therefore |\phi_1| \langle 1$$

To make The model stationary, it must be:

$$-1 \langle \phi_1 \langle 1 \ldots. \quad (2.1.1)$$

b- Inevitability:

To achieve inevitability, root of the equation $\theta(B) = 1 - \theta_1 B = o$ must be outside the unit circle, where: $|B| \rangle\ |$

This leads to make the parameter:

$$|\theta_1| = \frac{1}{|B|}$$

$$\therefore |\theta_1| < 1$$

To make the model inevitable it must be:

$$-1 < \theta_1 < 1 \qquad \ldots.(2.1.2)$$

c. Auto Covariance:

$$(1-\theta_1 B)X_t = (1-\theta_1 ?)a_t$$

$$X_t = \phi_1 X_{t-1} + a_t - \theta_1 a_{t-1}$$

$$EX_t X_{t-1} = \phi_1 EX_{t-1xt-1} + Ea_t X_{t-1} - \theta_1 Ea_{t-1} X_{t-1}$$

The general formula of The auto covariance of the mixed model ARMA(1,1) can be written as follows:

$$\wp_k = \begin{cases} \dfrac{1-2\phi_1\theta_1+\theta_1^2}{1-\phi_1^2} & ,\ k=0 \\ \dfrac{(\phi_1-\theta_1)(1-\phi_1\theta_1)}{1-\phi_1^2} & ,k=1 \\ \phi_1 \wp_{k-1} & ,k\geq 2 \end{cases} \qquad \ldots\ldots(2.1.3)$$

d. Auto Correlation:

The general formula of The autocorrelation of The mixed model ARMA(1.1) can be written as follows:

$$\rho_k = \begin{cases} 1 & ,\ k=0 \\ \dfrac{(\phi_1-\theta_1)(1-\phi_1\theta_1)}{(1-2\phi_1\theta_1+\theta_1^2} & ,k=1 \\ \phi_1 \rho_{k-1} & ,k\geq 2 \end{cases} \qquad \ldots\ldots(2.1.3)$$

2-2: Estimation of The Parameters:

The model:

$$a_t = \theta^{-1}(B)\phi(B)X_t$$

We notice that the above model is non-linear in its parameters.
This is because of the moving averages $\theta(B)$.

a. non-linear Least Square Method (NL):

The mixed model ARMA(1,1):

$$a_t = (1-\theta_1 B)^{-1}(1-\phi_1 B)X_t$$

can be written as follows:

$$a_t = \frac{1-\phi_1 B}{1-\theta_1 B}X_t \qquad \text{....(2.2.1)}$$

To implement Taylor's series, we account The first derivation of $a_t to \phi_1, \theta_1$ and find Their initial values $\phi_{1,0}, \theta_{1,0}$ as follows:

$$X_{1,t} = -\frac{\partial a_t}{\partial \phi_1}\Big|_{\phi_1=\theta_{1,0}} = \frac{B}{(1-\theta_{1,0}B)}X_t \qquad \text{....(2.2.2)}$$

$$X_{2,t} = -\frac{\partial a_t}{\partial \theta_1}\Big|_{\theta_1=\theta_{1,0}} = \frac{B-\phi_{1,0}B^2}{(1-\theta_{1,0}B)^2}X_t \qquad \text{....(2.2.3)}$$

by simplifying the two equations (2.2.2) and (2.2.3), we have:

$$X_{1,t} = \theta_{1,0}X_{1,t-1} + X_{t-1} \qquad \text{....(2.2.4)}$$

$$X_{2,1} = 2\theta_{1,0}X_{2,t-1} - \theta_{1,0}^2 X_{2,1-2} - X_{t-1} + \theta_{1,0}X_{t-2} \qquad \text{....(2.2.5)}$$

we can get the value of $a_{t,0}$ as follows:

$$a_{t,o} = (\hat{\phi}_1 - \phi_{1,0})X_{1,t} + (\hat{\theta} - \theta_{1,0})X_{2,t} + a_t \qquad \text{....(2.2.6)}$$

It is a linear regression equation. So we can implement ordinary least square to estimate ϕ_1, ϕ_1 as follows:

$$\phi_1^{\wedge} = \phi_{1,0} + \frac{\sum_{t=0}^{n} a_{t,0} X_{1,t}}{\sum_{t=0}^{n} X_{1,t}^2} \qquad \text{....(2.2.7)}$$

$$\theta_1 = \theta_{1,0} + \frac{\sum_{t=0}^{n} a_{t,0} X_{2,t}}{\sum_{t=0}^{n} X_{2,t}^2} \qquad \text{....(2.2.8)}$$

These obtaining values can be used as estimating initial values. This operation can be repeated till values of $\phi_1^{\wedge}, \theta_1^{\wedge}$ be fixed

b. Newton-Raphson Procedure (NR):

The researchers have applied a non-linear Newton-Raphson procedure to estimate the parameters of the mixed ARMA(1,1). This is because the equation $a_t = (1-\theta_1 B)^{-1}(1-\phi_1 B)X_t$ is non-linear in moving average. This method depends on a non-linear procedure, namely Newton Raphson, as follows:

$$\because X_t = (1-\phi_1 B)^{-1}(1-\theta_1 B)a_t$$

$$\therefore a_t = (1-\theta_1 B)^{-1}(1-\phi_1 B)X_t \qquad \text{....(2.2.9)}$$

by squaring and taking summation of the two sides:

$$\sum_{t=1}^{n} a_t^2 = \sum_{t=1}^{n} \left[(1-\theta_1 B)^{-1}(1-\phi_1 B)X_t\right]^2$$

$$\frac{\partial \Sigma_{t=1}^{n} a_t^2}{\partial \phi_{1,0}} = 2\sum_{t=1}^{n} \left[(1-\theta_{1,o} B)^{-1}(1-\phi_{1,0} B)X_t\right](-B)(1-\theta_{1,0} B)^{-1} X_t$$

make the derivation equals zero, and divide on (-2), we have:

$$F(x)=\sum_{t=1}^{n}\left[\frac{X_tX_{t-1}-\phi_{1,0}X_{t-1}^2}{(1-\theta_{1,0}B)^2}\right]=o \qquad ....(2.2.10)$$

$$\frac{\partial F(X)}{\partial\phi_{1,0}}=-\sum_{t=1}^{n}\left[\frac{X_{t-1}^2}{(1-\theta_{1,0}B)^2}\right]$$

$$F'(X)=\sum_{t=1}^{n}\left[\frac{X_{t-1}^2}{(1-\theta_{1,0}B)^2}\right]=0 \qquad ....(2.2.11)$$

by an implementing Newton Raphson as in the following formula:

$$X_{n+1}=Xn-\frac{F(X)}{F'(X)}$$

$$\hat{\phi}_1=\phi_{1,0}-\left[\frac{\sum_{t=1}^{n}X_tX_{t-1}-\phi_{1,0}\sum_{t=1}^{n}X_{t-1}^2}{(1-\theta_{1,0}B)^2}\Big/\frac{\sum_{t=1}^{n}X_{t-1}^2}{(1-\theta_{1,0}B)^2}\right]$$

$$\hat{\phi}_1=2\phi_{1,0}-\frac{\sum_{t=1}^{n}X_tX_{t-1}}{\sum_{t=1}^{n}X_{t-1}^2} \qquad ....(2.2.12)$$

$$\frac{\partial\Sigma_{t=1}^{n}a_t^2}{\partial\theta_{1,0}}=2\sum_{t=1}^{n}\left[(1-\theta_{1,0}B)^{-1}(1-\phi_{1,0}B)X_t\right](-1)(1-\theta_{1,0}B)^{-2}(-B)(1-\phi_{1,0}B)X_t$$

make the derivation equals zero, and divide on (-2), we have:

$$\therefore F(X) = \sum_{t=1}^{n} \left[\frac{(1-\phi_{1,0}B)^2 X_t X_{t-1}}{(1-\theta_{1,0}B)^2} \right] = 0 \qquad ....(2.2.13)$$

$$\frac{\partial F(X)}{\partial \theta_{1,0}} = \sum_{t=1}^{n} \left[\frac{(1-\theta_{1,0}B)^3 (0) - (1-\phi_{1,0}B)^2 X_t X_{t-1} (3)(1-\theta_{1,0}B)^2 (-B)}{(1-\theta_{1,0}B)^6} \right]$$

$$F'(X) = \sum_{t=1}^{n} \left[\frac{(1-\phi_{1,0}B)^2 X_{t-1}^2}{(1-\theta_{1,0}B)^4} \right] = 0 \qquad ....(2.2.14)$$

by an implementing Newton Raphson as in the following formula:

$$X_{n+1} = X_n - \frac{F(X)}{F'(X)}$$

$$\hat{\theta}_1 = \theta_{1,0} - \left[\frac{(1-\phi_{1,0}B)^2}{(1-\theta_{1,0}B)^3} \sum_{t=1}^{n} X_t X_{t-1} \Big/ \frac{(1-\phi_{1,0}B)^2}{(1-\theta_{1,0}B)^4} \sum_{t=1}^{n} X_{t-1}^2 \right]$$

$$\hat{\theta}_1 = 2\theta_{1,0} - \frac{\sum_{t=1}^{n} X_t \, Xt-1}{\sum_{t=1}^{n} X_{t-1}^2} \qquad ....(2.2.15)$$

where:

$\phi_{1,o}, \theta_{1,0}$ are initial values

Table (1): MAPE (ϕ) and MAPE (θ) Values

N	ϕ/θ		Discrete						Continuous	
			Poi (0.5)		Geo (0.5)		Gam (1,2)		Cau (1,2)	
			NR	NL	NR	NL	NR	NL	NR	NL
25	ϕ	0.9	46.976	50.305	54.898	55.899	124.257	101.360	108.706	62.344
		0.1	16.921	18.769	25.368	28.536	70.783	76.422	14.402	16.239
		-0.1	25.481	26.382	30.998	40.380	68.445	77.014	21.517	22.656
		-0.9	47.661	27.418	41.088	35.556	99.027	101.765	110.148	35.257
	θ	0.8	46.215	55.659	65.722	48.830	117.476	88.767	75.614	45.446
		0.2	16.740	19.194	25.416	29.460	62.881	72.160	13.256	15.316
		-0.2	23.707	26.597	32.755	40.994	63.017	72.543	19.153	21.523
		-0.8	46.156	28.779	40.838	31.493	118.249	90.142	57.224	27.502
50	ϕ	0.9	45.950	40.894	57.677	51.091	123.916	101.314	93.831	35.252
		0.1	16.750	18.878	24.487	27.106	62.256	76.855	18.461	16.235
		-0.1	22.951	25.823	28.080	36.840	64.750	76.462	18.942	22.061
		-0.9	41.496	39.553	42.281	43.313	121.141	100.319	82.732	32.110
	θ	0.8	45.426	73.536	60.403	51.262	107.462	88.716	91.977	55.482
		0.2	16.581	18.639	24.026	26.018	61.429	65.482	14.076	16.089
		-0.2	22.573	25.424	27.869	32.549	61.347	63.786	19.066	21.816
		-0.8	28106	25.289	43.944	44.098	107.645	86.342	51.111	31.042
100	ϕ	0.9	35.649	33.834	76.954	54.333	113.144	87.325	74.995	40.291
		0.1	15.853	16.734	22.092	22.339	62.201	68.145	13.251	14.278
		-0.1	17.934	22.805	21.466	24.094	62.125	70.36	14.953	19.355
		-0.9	26.104	27.746	55.469	28.798	112.528	88.360	55.609	33.306
	θ	0.8	35.085	32.765	67.354	50.822	102.514	82.138	73.993	39.114
		0.2	15.429	16.852	19.995	24.163	56.009	59.647	12.890	14.342
		-0.2	17.670	24.663	19.152	27.674	55.006	60.130	14.739	20.841
		-0.8	24.837	27.700	42.698	28.385	102.182	28.263	53..077	33.284

3- Simulation:

Four Simulation experiments are designed to estimate the parameter of The mixed model ARMA(1,1) by supposing initial Values to the parameters $(\phi = \pm o.9, \pm 0.1), (\theta = \pm 0.8, \pm 0.2)$ to sample size (n=25,50,100). Each experiment is repeated 1000 times.

The random error a_t of the mixed model ARMA(1,1) has discrete distribution (Poisson, Geometric) on the one hand, and continuous distribution (Gamma, Cauchy) on the other. After the parameter of the mixed model are estimated by non-linear least square method and the applied proccedure, a comparison of the results will be bone by using mean absolute percentage Error (MAPE) to the estimated parameters.

Simulation results is given in the following table:

4- Conclusions:

a. Results show that the applied procedure is good and reliable as a method to estimate the parameters of the mixed model-from first order ARMA(1,1).

b. MAPE values of the parameters ϕ,θ decrease when the sample size is in creased in most cases.

c. MAPE values of the parameters ϕ and θ in crease when the initial values of the parameters approach ± 1. MAPE Values of the parameters decrease when the initial values approach zero.

d. Non-linear least square method gives less MAPE values of the parameters ϕ and θ then the proposed method when the parameters of the initial values approach ± 1.

e. The applied procedure gives less MAPE values of the parameters ϕ and θthen the non-linear square method when the parameters of the initial values approach zero.

References:

1. Box, G.E.P. & Jenkins, G.M. (1976) Time Series Analysis: Forecasting and control, 2nd, Holden-Day, San-Francisco.
2. Brockwell, P.J. & Davis, R.A.(1991) Time Series Theory and Methods, 2nd ed., Sprinnger Verlag New York Inc., New York.
3. Cragg, J.G.(1990) "Monte Carlo Methods", in Eatwell, J.et. al. (eds.) Time Series and Statistics, Mac mill Press Limited, London, PP. (118-125)
4. Dent, W.& min, A.S. (1978) "Monte Carlo Study of Autoregressive Intcgrated Moving Average Process", J. of Econometrics, Vol. 7, pp. (23-53)
5. Draper, N.R.& Smith, H. (1981) Applied Regression Analysis (2nd ed.), John Wiley & Sons, New York.
6. Groybeal, W.J. & Pooch, V.W. (1980) Simulation: Principles and Methods, Eir throp publishers, Inc., U.S.A
7. Hamilton, J.D. (1994) Time Series Analysis, Princeton University Press, Princeton, New Jersey.
8. Moran, B.J.T. (1984) Elements of Simulation, Chapman and Hall, London.
9. Newbold, p. (1975) "The principles of the Box – Jenkins Approach", Operational Research Quarterly, Vol. 26, pp. (320-337).
10. Priestley, M.B. (1988) Non-Linear and Non-Stationary Time Series Analysis, Academic Press.
11. Tong, H. Non-linear Time Series, A Dynamical System Approach, Oxford Science Publication, Clarendon press, Oxford,1990.
12. Wei, W.W.S.(1990) Time Series Analysis: Univariate and Multivariate methods, Addison-Wesley publishing comp. Inc., New York.
13. White, H.(1990) "least square" in Eatwell, J.et al. (eds.) Time Series and Statistics, Macmillan Press Limited, London, pp. (193-203).

تقدير معالم النموذج المختلط غير الطبيعي (١،١) ARMA باستخدام أحد اساليب نيوتن - رافسن غير الخطية -

أ. م . محمد قدوري الخضيري
قسم العلوم التجارية والمصرفية
كلية المنصور الجامعة

الاستاذ الدكتور عبدالمجيد حمزة الناصر
عميد كلية الادارة والاقتصاد
جامعة بغداد

المستخلص

لتقدير معالم النموذج المختلط (١،١)ARMA غير الخطية في الأوساط المتحركة استخدم الباحثان أحد الأساليب غير الخطية وهو أسلوب نيوتن - رافسن لتقدير هذه المعالم ، وكذلك اختبار كفاءة هذا الاسلوب لمقارنته بطريقة المربعات الصغرى غير الخطية وباستخدام اسلوب لمحاكاة .

ومن أهم الاستنتاجات التي توصل اليها الباحثان هي أن اسلوب نيوتن غير الخطية للنموذج المختلط ARMA (1,1) غير الطبيعي .