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ON GENERALIZED τ*- HOMEOMORPHISMS IN TOPOLOGICAL SPACES

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ABSTRACT

In this paper, we introduce a new class of closed sets called generalized-τ*-closed set, also introduce a new class of homeomorphisms called g-τ*-homeomorphisms and a class of maps which is included in the class of g-τ*-homeomorphisms, and study the relation between it and another mapping of homeomorphism ,and prove that the set of all g-τ*-homeomorphisms forms a group under the operation composition of maps.

1. Introduction

In 1970, Levine [1] introduced the concept of generalized closed sets as a generalization of closed sets in topological spaces. Using generalized closed sets, Dunham [2] introduced the concept of the closure operator cl* and a new topology τ* and studied some of their properties. Balachandran, Sundaram and Maki [3] introduced and studied g-continuous maps. Pushpalatha, Eswaran and Rajarubi [4] introduced and investigated τ*-generalized closed sets. Eswaran and Pushpalatha [5] introduced and studied τ*-generalized continuous maps in a topological spaces.

The notion homeomorphism plays a very important role in topology. By definition a homeomorphism between two topological spaces X and Y is a bijective map f : X →Y when both f and f^{-1} are continuous, so in this paper, we introduce a new class of sets called generalized-τ*-closed set, and introduce a new class of maps called generalized-τ*-closed, generalized-τ*-continuous and generalized-τ*-irresolute ,and study some properties about them. Also we introduce a new class of homeomorphisms called generalized-τ*-homeomorphisms (g-τ*-homeomorphisms) and a class of maps which is included in the class of g-τ*-homeomorphisms and study the relation among it and homeomorphism,τ*-homeomorphism, g-homeomorphism and τ*-g-homeomorphism ,and prove that the set of all g-τ*-homeomorphism forms a group under the operation composition of maps.

Throughout this paper (X, τ) and (Y, σ) are topological spaces on which no separation axioms are assumed unless otherwise explicitly stated. For a subset A of a topological space X, cl(A), cl*(A) and A^c denote the closure, closure* and complement of A respectively.

2. Preliminaries

We recall the following definitions:

Definition 2.1. [1].

A subset A of a topological space (X, τ) is called generalized closed (briefly g-closed) in X if cl (A) $\subseteq$G whenever A$\subseteq$G and G is open in X.

A subset A is called generalized open (briefly g-open) in X if its complement Ac is g-closed.

Definition 2.2.[2].

For a subset A of a topological space (X, τ),

(i) The generalized closure operator cl*(A) is defined by the intersection of all g- closed sets containing A.

(ii) The topology τ* is defined by τ* ={ G : cl*(Gc) = Gc }

Theorem 2.3.[2] For any subsets A and B of a topological space (X, τ)

(i) If A$\subseteq$B then cl*(A)$\subseteq$ cl*(B).
(ii) If A is g-closed then cl*(A)=A .
(iii) cl*(A$\cup$B) = cl*(A)$\cup$cl*(B) .
(iv) cl*(A$\cap$B) $\subseteq$ cl*(A)$\cap$cl*(B) .

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Definition 2.4.[4].

A subset A of a topological space (X, τ) is called τ*-generalized closed set (briefly τ*-g-closed) if cl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is τ*-open. The complement of τ*-generalized closed set is called the τ*-generalized open set (briefly τ*-g-open).

Now we introduce a new class of sets called generalized τ*-closed set (briefly g-τ*-closed) ,and study the relation between this type of g-τ*-closed set and each of closed, τ*-closed, g- closed set and τ*-g-closed set.

Definition 2.5.

A subset A of a topological space (X,τ) is called generalized-τ*-closed set(briefly g-τ*-closed) if cl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is open in X. The complement of g-τ*-closed set is called the generalized-τ*-open set (briefly g-τ*-open).

Proposition 2.6.

(i) Every closed (τ*-closed) set is g-τ*-closed .

(ii) Every g-closed set is g-τ*-closed.

(iii) Every g-τ*-closed set is τ*-g-closed .

Proof : Let (X, τ) be a topological space.

(i) Let A be a closed set in X, such that A $\subseteq$ G, G is an open set in X. Since A is closed, cl(A) = A $\subseteq$ G. But cl*(A) $\subseteq$ cl(A). Thus, we have cl*(A) $\subseteq$ G ,G is open. Therefore A is g-τ*-closed.

(ii) Let A be a g-closed set in X. Assume that A $\subseteq$ G, G is an open in X. Then cl(A) $\subseteq$ G, since A is g-closed. But cl*(A) $\subseteq$ cl(A). Therefore cl*(A) $\subseteq$ G. Hence A is g-τ*-closed.

(iii) Let A be a g-τ*-closed in X, then there exist an open set G in X such that cl*(A) $\subseteq$ G whenever A $\subseteq$ G .Since every open set is τ*-open (see[4]) , then cl*(A) $\subseteq$ G whenever A $\subseteq$ G and G is τ*-open in X. Therefore A is τ*-g-closed.

The converse of proposition(2.6) need not be true in general as seen from the following example.

Example 2.7. Consider the space X = {a, b, c} with topology τ = {X, ϕ, {a}}. Then the set {a} is not closed and not g-closed but {a} is g- τ*-closed.

Remark 2.8. From all the above statements ,[1] and [4] we have the following diagram:

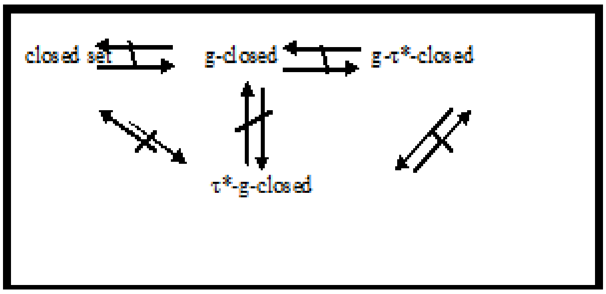

Diagram (1)

3. The generalized τ*- Homeomorphism

In this section we introduce a new class of maps namely generalized-τ*-closed , generalized-τ*-continuous ,generalized-τ*-irresolute, and generalized-τ*-homeomorphisms. Also we study some of their properties and relations among them and other maps .

Definition 3.1.[6]

A map f : (X, τ)→(Y, σ) is called a generalized closed map (respectively generalized open map) if for each closed set (open set) V of X , f(V) is g-closed (g-open) in Y .

Definition 3.2.

A map f : (X, τ)→(Y, σ) is called a generalized τ*-closed map (respectively generalized τ*-open map) if for each closed set (open set) V of X , ,f(V) is g- τ*-closed (g- τ*-open) in Y .

A generalized τ*-closed map is written shortly as g- τ*-closed map.

Example 3.3. Let the spaces X=Y= {a, b, c} with topologies τ ={X, ϕ ,{a}} and σ ={Y, ϕ ,{b}}.Define a map f : (X, τ)→(Y, σ) by : f(a) = b, f(b) = a, f(c) = c. Then f is g-τ*-closed since the set {b, c} is closed in X, and f ({b, c}) = {a, c} is g-τ*-closed in Y.

Proposition 3.4.

(i) Every closed (τ*-closed) map is g-τ*-closed map .

(ii) Every g-closed map is g-τ*-closed map .

(iii) Every g-τ*-closed map is τ*-g-closed map .

Proof : Clear from Proposition(2.6) .

Definition 3.5.[3].

A map f : (X, τ)→(Y, σ) is called generalized continuous (g-continuous) if the inverse image of every closed set in Y is g-closed in X.

Definition 3.6.[5].

A map f : (X, τ)→(Y, σ) is called τ*-generalized continuous (τ*-g-continuous) if the inverse image of every g-closed set in Y is τ*-g-closed in X.

Definition 3.7.

A map f : (X, τ)→(Y, σ) is called a generalized-τ*-continuous map (written shortly as g-τ*-continuous map) if the inverse image of every closed in Y is g-τ*-closed in X .

Proposition 3.8.

(i) Every continuous (τ*-continuous) map is g-τ*-continuous map .

(ii) Every g-continuous map is g-τ*-continuous map .

(iii) Every g-τ*-continuous map is τ*-g-continuous map .

Proof : Clear from Proposition(2.6) .

Theorem 3.9. Let (X, τ) and (Y, σ) be two topological spaces. Then f : X→Y is g- τ*-continuous if and only if the inverse image of every open set in Y is g- τ*-open in X.

Proof: Let U be any open set in Y. Then Uc is closed in Y. Since f is g- τ*-continuous, f -1(Uc) is g- τ*-closed in X. But f -1(Uc)= (f -1(U))c. This implies that f -1(U) is g- τ*-open in X.

Conversely, let V be any closed set in Y. Then Vc is open in Y. By assumption, f -1(Vc) is g- τ*-open in X. But f -1(Vc) = (f -1(V))c ,this implies that f -1(V) is g- τ*-closed in X. Hence f is g- τ*-continuous.

Proposition 3.10. For any bijective f : (X, τ)→(Y, σ) the following statements are equivalent:

(i) f -1 is g-τ*-continuous, f -1 : (Y, σ) → (X, τ)

(ii) f is a g-τ*-open map and

(iii) f is a g-τ*-closed map.

Proof : (i)→ (ii): Clear from Theorem (3.9) ..

(ii) → (iii): Let V be a closed set in X. Then Vc is open in X. By assumption, f (Vc) is g- τ*-open in Y. But f (Vc) = (f (V))c ,this implies that f (V) is g- τ*-closed in Y. Hence f is a g-τ*-closed .

(iii) → (i): Let V be a closed set in X. By assumption, f (V) is g- τ*-closed in Y.

But f (V) = f -1(f -1(V)) ,and therefore f -1 is g-τ*-continuous on Y.

Definition 3.11.[7].

A map f : (X, τ)→(Y, σ) is called a generalized irresolute (g-irresolute) if f -1 (V) is g-closed set in X for every g-closed set V in Y .

Definition 3.12.

A map f : (X, τ)→(Y, σ) is called a generalized τ*-irresolute (g-τ*-irresolute) if f -1 (V) is g-τ*-closed set in X for every g-τ*-closed set V in Y .

Theorem 3.13. A map f : (X, τ)→(Y, σ) is g-τ*-irresolute if and only if f is g-τ*-continuous .

Proof : Assume that f is g-τ*-irresolute. Let V be any closed set in Y. By Prop.(2.6)(i), V is g-τ*-closed in Y. Since f is g-τ*-irresolute, f -1(V) is τ*-g-closed in X. Therefore f is

g-τ*-continuous.

Conversely, assume that f is g-τ*-continuous. Let F be any closed set in Y. By Prop.(2.6)(i), F is g-τ*-closed in Y. Since f is τ*-g-continuous, f -1(F) is g-τ*-closed in X. Therefore f is

g-τ*-irresolute.

Definition 3.14.

A bijection f : (X,τ) → (Y,σ) is called a generalized τ*-homeomorphism (abbreviated g-τ*-homeomorphism) if f is both g-τ*-continuous and g-τ*-open .

Proposition 3.15.

(i) Every homeomorphism (τ*-homeomorphism) is g-τ*-homeomorphism.

(ii) Every g-homeomorphism is g-τ*-homeomorphism.

(iii) Every g-τ*-homeomorphism is τ*-g-homeomorphism.

Proof : Clear by Proposition (3.4) and Proposition (3.8) .

The converse of the above Proposition need not be true in general, as seen from the following example.

Example 3.16. Let X =Y = {a, b, c} with topologies

τ1 = {X, ϕ, {a},{a, b}, {a, c}} , σ1= {Y, ϕ, {b},{a, b}},. τ2 = {X, ϕ, {a}} and

σ2= {Y, ϕ, {a},{b, c}}. The mapping f1 : (X,τ1) → (Y,σ1) which is defined by: f1(a) = b, f1(b) = a, f1(c) = c is g-τ*-homeomorphism but not g-homeomorphism since the set {a, c} is open in X while f1 ({a, c}) = {b, c} is not g-open in Y .And the identity map f2 : (X,τ2) → (Y,σ2) is τ*-g-homeomorphism but not g-τ*-homeomorphism since the set {b} is not closed in Y while f2 -1 ({b}) = {b} is g-τ*-closed in X .

Theorem 3.17. Let f : (X, τ) → (Y, σ) and h : (Y, σ) →(Z, η) be two maps ,then

(i) hof is g-τ*-continuous if f is g-τ*-irresolute and h is g-τ*-continuous.

(ii) hof is g-τ*-continuous if f is g-τ*-continuous and h is continuous.

(iii) hof is g-τ*-irresolute if f and h are both g-τ*-irresolute .

Proof : (i) Let V be a closed set in Z. Since h is g-τ*-continuous, then h-1(V) is g-τ*-closed in Y. Since f is g-τ*-irresolute, then (hof)-1 (V) = f -1(h-1(V)) is g-τ*-closed in X .Therefore hof is g-τ*-irresolute.

(ii) Let V be a g-τ*-closed set in Z. Since h is continuous, then h-1(V) is closed in Y. Since f is g-τ*-continuous, then (hof)-1 (V) = f -1(h-1(V)) is g-τ*-closed in X .Therefore hof is g-τ*-continuous.

(iii) Let V be a closed set in Z. Since h is g-τ*-irresolute, then h-1(V) is g-τ*-closed in Y. Since f is g-τ*-irresolute, then (hof)-1 (V) = f -1(h-1(V)) is g-τ*-closed in X .Therefore hof is g-τ*-irresolute.

Remark 3.18. The composition of two g-τ*-homeomorphism is not alway g-τ*-homeomorphism as seen from the following example.

Example 3.19. Let X =Y = Z ={a, b, c} with topologies τ = {X, ϕ, {b},{a, b}} , σ = {Y, ϕ, {a}} and η = {Z , ϕ, {b}, {a, b}}.

Let f : (X, τ) → (Y, σ) be a map defined by f(a) =b ,f(b) =a ,f(c) =c ,and a map h : (Y, σ) →(Z, η) defined by h(a) =a , h(b) =c ,h(c) =b . Then f and h are g-τ*-homeomorphism but hof : X → Z is not g-τ*-homeomorphism since the set {a, b} is open in X while f ({a, b}) = {a, c} is not g-τ*-open in Z .

4. The class of g-τ*-Homeomorphism

Biswas in [8] define the class of generalized homeomorphism (gc-homeomorphism) when f and f -1 are g-irresolute .In this section we introduce a class of maps which is included in the class of g-τ*-homeomorphism and denoted it by (g- τ*c-homeomorphism) ,and study the relation between it and another homeomorphism .

Definition 4.1.

A bijection f : (X, τ) → (Y, σ) is said to be g-τ*c-homeomorphism if both f and f -1 are g-τ*-irresolute.

We say that X and Y are g-τ*c-homeomorphic if there exists a g-τ*c-homeomorphism from X onto Y .

Proposition 4.2.

(i) Every homeomorphism (τ*-homeomorphism) is g-τ*c-homeomorphism.

(ii) Every g-homeomorphism is g-τ*c-homeomorphism.

(iii) Every g-τ*c-homeomorphism is g-τ*-homeomorphism.

Proof : (i) Let f : (X, τ) → (Y, σ) be a homeomorphism .

Let V be a closed set in Y. Since f is homeomorphism, then f -1(V) closed in X. By Prop.(2.6.(i)) V is g-τ*-closed in Y, and f -1(V) is g-τ*-closed in X. Therefore f is g-τ*-irresolute.

And ,let U be a closed set in X. Since f is homeomorphism, then f (U) closed in Y. By Prop.(2.6.(i)) U is g-τ*-closed in X, and f (U) is g-τ*-closed in Y. Therefore f -1 is g-τ*-irresolute. Hence f is g-τ*c-homeomorphism .

(ii) Let f : (X, τ) → (Y, σ) be a g-homeomorphism .

Let U be a closed set in X. Since f is g-homeomorphism, then f (U) is g-closed in Y. By Prop.(2.6.(ii)) U is g-τ*-closed in X, and f (U) is g-τ*-closed in Y. Therefore f -1 is g-τ*-irresolute

And ,let V be a closed set in Y. Since f is g-homeomorphism, then f -1(V) is g-closed in X. By Prop.(2.6.(ii)) V is g-τ*-closed in Y, and f -1(V) is g-τ*-closed in X. Therefore f is g-τ*-irresolute. Hence f is g-τ*c-homeomorphism.

(iii) Let f : (X, τ) → (Y, σ) be a g-τ*c-homeomorphism .

Since f is g-τ*c-homeomorphism, then f is g-τ*-irresolute and by Theorem (3.13), f is g-τ*-continuous .

Let U be an open set in X, then Uc is closed in X.By Prop.(2.6.(i)) Uc is g-τ*-closed in X. Since f is g-τ*c-homeomorphism, then f -1 is g-τ*-irresolute, then f (Uc) is g-τ*-closed in Y.But f (Uc) = (f (U))c that mean f(U) is g-τ*-open in Y. Therefore f is g-τ*-open .Hence f is g-τ*-homeomorphism.

Remark 4.3. The following example show that the converse of Prop. (4.2) is not true in general.

Example 4.4. Let X =Y = {a, b, c} with topologies

τ 1= {X, ϕ, {a, b}},σ 1= {Y, ϕ, {b},{a, b}}, τ 2= {X, ϕ, {a}}, σ 2= {Y, ϕ, {b, c},{a}},

τ 3= {X, ϕ, {b},{a, b}}and σ 3= {Y, ϕ, {a}}.

The identity mapping f 1: (X, τ1) → (Y, σ1) is g-τ*c-homeomorphism but it is not homeomorphism since f1 -1({b}) = {b} is not open set in X, while {b} is open in Y.

The identity mapping f 2: (X, τ2) → (Y, σ2) is g-τ*c-homeomorphism but it is not g-homeomorphism since f2 -1({a}) = {a} is not g-closed set in X, while {a} is closed set in Y.

And the mapping f 3: (X, τ3) → (Y, σ3) which is defined by : f3(a) = b, f3(b) = a, f3(c) = c, is

g-τ*-homeomorphism but it is not g- τ*c-homeomorphism since f3 -1({c}) = {c} is not g-τ*-open sets in X, while {c} is g-τ*-open sets in Y.

Proposition 4.5. The composition of two g-τ*c-homeomorphism is g-τ*c-homeomorphism.

Proof : Let f : (X, τ) → (Y, σ) and h : (Y, σ) →(Z, η) be two g-τ*c-homeomorphism .To prove h o f : (X, τ) →(Z, η) is also g-τ*c-homeomorphism .

Since f and h are g-τ*c-homeomorphism, then f, f -1, h and h -1 are g-τ*-irresolute, then by Theorem (3.17(iii)), (h o f) is g-τ*-irresolute and (f -1o h-1) = (h o f)-1 is g-τ*-irresolute .Hence h o f is g-τ*c-homeomorphism.

Remark 4.6. From Proposition(3.15) ,Example(3.16) ,Proposition(4.2) and Examples(4.4) ,we have the following diagram .

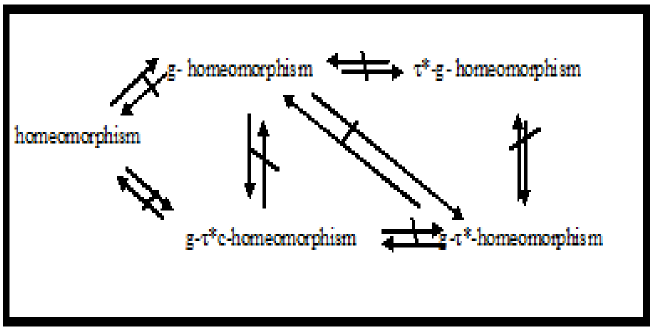

Diagram (2)

Now we will denote to the family of all g-τ*c-homeomorphism of (X, τ) onto itself by g-τ*c-h(X, τ) ,where g-τ*c-h(X, τ) = {f | f : X→X is a g-τ*c-homeomorphism }.

Theorem 4.7. The set g-τ*c-h(X, τ) is a group under the composition of maps.

Proof: Define a binary operation *: g-τ*c-h(X, τ) ×g-τ*c-h(X, τ) → g-τ*c-h(X, τ) by

f * g= h o f , for all f,h ϵ g-τ*c-h(X, τ) ,

and o is the usual operation of composition of maps.Then by Proposition (4.6), hof is

g-τ*c-homeomorphism .Therefor hof ϵ g-τ*c-h(X, τ) .Let I: (X, τ) → (X, τ) be the identity map belonging to g-τ*c-h(X, τ) servers as the identity element ,and we know that the composition of

maps is associative . If f ϵ g-τ*c-h(X, τ) ,then f-1 ϵ g-τ*c-h(X, τ) such that f o f -1= f -1of =I and so inverse exists for each element belonging to g-τ*c-h(X, τ). Therefore g-τ*c-h(X, τ) is a group under the operation of composition of maps.

Theorem 4.8. Let f : (X, τ) → (Y, σ) be a g-τ*c-homeomorphism. Then f induces an

isomorphism from the group g-τ*c-h(X, τ) onto the group g-τ*c-h(Y, σ).

Proof : Let we define a map θ: g-τ*c-h(X, τ) →g-τ*c-h(Y, σ) by

θ (h) = f o h o f -1 , for every h ϵ g-τ*c-h(X, τ). Then θ is a bijection. Further, for all

h1, h2 ϵ g-τ*c-h(X, τ) , θ(h1o h2)= f o (h1oh2) of -1 =(f o h1 o f -1) o (f o h2 o f -1) = θ(h1) o θ(h2) .Therefore, θ is a homeomorphism and so it is an isomorphism induced by f.

Remark 4.9. By using theorem (4.8.) the space (X, τ) where X = {a, b, c} and τ = {X, ϕ , {b},

{a ,b}}is not g-τ*c-homeomorphic to the space (Y, σ) where Y= {a, b, c}and σ = {Y, ϕ , {a}},since f: X → Y which defined by f(a) =b ,f(b) =a ,f(c) =c is not g-τ*c-homeomorphism.

Remark 4.10. The following example show that the converse of theorem (4.8.) is not always true, that is there exist a map not g-τ*c-homeomorphism induces as isomorphism.

Example 4.11. Let X =Y = {a, b, c} with topologies τ = {X, ϕ , {a, b}} and

σ = {Y, ϕ , {b},{c},{b, c }}. Let f : X → Y be an identity map. The g-τ*-closed sets in X are {{c},{a,c},{b,c}} ,and the g-τ*-closed sets in Y are {{a,c},{a},{a,b}}.Then f is not

g-τ*c-homeomorphism.

Let hx : X → X defined by hx(a)=b ,hx(b)=a ,hx(c)=c ,and hy : Y → Y defined by hy(a)=a ,hy(b)=c ,hy(c)=b ,then g-τ*c-h(X,τ) = {1x ,hx} and g-τ*c-h(Y, σ) ={1y,hy} .The induced homeomorphism f* : g-τ*c-h(X, τ) →g-τ*c-h(Y, σ) is an isomorphism since f*(hx) =fo (hx) of -1 = hy ,and

f*(1x) =1y is the identity on Y.

Theorem 4.12. The g-τ*c-homeomorphism is an equivalence relation in the collection of all

topological spaces.

Proof: Reflexivity and symmetry are immediate and transitivity follows from Proposition(4.5).

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حول التشاكلات المعممة –t* في الفضاءات التبولوجيـــة

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الخلاصة:

في هذا البحث قدمنا نوعا جديدا من المجموعات المغلقة اسميناه بالمجموعة المعممة–t* المغلقة ، وقدمنا ايضا نوعا جديدا من التشاكلات اسميناه التشاكل المعمم–t* ، وصف من التطبيقات الذي يكون متضمن في صف التشاكل المعمم–t* ، درسنا العلاقة بينه وبين تشاكلات اخرى. كما برهنا ان مجموعة كل التشاكلات المعممة–t* تشكل زمرة تحت عملية تركيب الدوال.