On iS*- Separation Axioms In Topological Space

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Abstract:

This paper discussed anew types of sets and how to deal with them .All these concepts depend on the concept of is*-open set. The most important of these concepts that presented in this paper are new types of separation axioms in topological space that called is*- separation axioms .Additionally we investigated the relationship between these types of separation axioms which presented in this paper.

Keywords: is*-open set , is*-separation axioms , iS* Ƭo - space , iS* Ƭ1 - space , iS* Ƭ2 - space.

في الفضاء التبولوجيis* بديهيات الفصل من النمط

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الخلاصة:

يناقش البحث الحالي أنواع جديدة من المجموعات وكيفية التعامل معها . وكل هذه المفاهيم تعتمد على مفهوم المجموعة المفتوحة من النمط is* واهم هذه المفاهيم التي قدمت في هذا البحث هي أنواع جديدة من بديهيات الفصل في الفضاء التبولوجي سميت بديهيات الفصل من النمط is* بالاضافة الى ذلك تم دراسة العلاقة بين هذه الأنواع من بديهيات الفصل المطروحة في هذا البحث .

1. Introduction

"AL - Meklafi [4] generalized the concept of closed set to the S*- closed , the complement of S* - closed set is called S*- open set , Askander[2] introduce the concept of i-open set . In this paper we introduce a new concept of open set namely iS*- open set , and study their properties , also we introduce a new types of separation axioms namely iS* separation axioms , and study the relations between (standard separation axioms , i-separation axioms , S* - separation axioms) with iS*- separation axioms . we obtain the definition from standard separation properties by replacing open set by iS* - open set in their definitions , moreover , we study the relation between this type of separation axioms."

2. Preliminaries

" Throughout this paper (X,Ʈ), (X,Ʈx) and (Y, Ʈ y) mean topological spaces. For a subset A of X, the interior and closure of A are denoted by int (A) and cl (A) respectively. Now we recall the following definitions" .

Definition 2 . 1 [3] : A subset A of a topological space (X, Ʈ x) is called a semi-open set (S - open for short) if A ⊆ cl (int A). The complement of a semi-open set is defined to be semi-closed set (S - closed for short).

Definition 2 . 2 [4]: A subset A of a topological space (X , Ʈ x) is called S*- open set If F ⊆ int A whenever F ⊆ A and F is semi - closed in (X, Ʈx). The complement of a S* - open set is defined to be S*- closed set.

The class of all S*- open set in (X, Ʈx) is denoted by S* O(X, Ʈx) .

Definition 2.3 [2] A subset A of a topological space (X, Ʈx) is called i- open set if there exists open set (O ≠ X,Ø) such that A ⊆ Cl (A ∩ O). The complement of an i - open set is called i - closed set.

The class of all i - open set in (X, Ʈ x) is denoted to be io(X, Ʈ x).

3. iS*- open set.

In this section we introduce a new concept namely iS*- open set.

Definition 3.1 : A subset A of a topological space (X , Ʈ x) is called is*- open set if there exists S*- open set (O≠ X,Ø) such that A ⊆ cl (A ∩ O).

The complement of an is*- open set is denoted by is*- closed set.

The class of all is*- open set in (X , Ʈx) is denoted to be is*o(X , Ʈx)

Definition 3.2 : Let A be a subset of a topological). space (X, Ʈx) then :

1. The intersection of all is*- closed sets containing A is called is*- closure of A, denoted by cl is* (A).

2.The union of all is*- open sets of X containing in A is called the is*- interior denoted by int is* (A).

Remarks 3 . 3

(i) Every open (closed) set is an S*- open (S*- closed) set respectively [5].

(ii) Every S*- open (S* - closed) set is an is*_open (is*_closed) set respectively.

(iii)Every S - open (S - closed) set is an i - open (i - closed) set respectively.[1]

(iv)Every i - open (i - closed) set is an is*- open (is*- closed) set respectively.

(v)S{ - open set and S*- open set are independent [5].

(vi)The union of two is*- open sets are is*- open set.

(vii)The intersection of two is*- open set does not necessary is*- open set.

Proof : (ii) let A ≠ X,Ø is, since A ⊆ cl A , then A ⊆ cl (A ∩ A),then A is an is*- open set .

Proof: (iv)) let A ≠ X,Ø is an i- open set , then there exist B is open set such that

A ⊆ cl (A ∩ B) since every open set is an S*- open set [5], then B is an S*- open set , then A is an is*- open set .

The converse of (i), (ii), (iii) and (iv) are not true in general consider the following examples.

Example 3.4: Let X = { a, b, c}, Ʈx = {Ø, X, {a, b}}, Then SO (X) ={Ø, X, {a, b}}

iO (X) = { Ø, X, {a}, {b},{a, b},{a, c}, {b, c}} , S* O(X) = { Ø, X, {a}, {b}, {a, b}}

iS* O (X) = { Ø, X, {a}, {b},{a, b}, {a, c}, {b, c}}

Example 3 . 5: Let X = {a, b, c, d} , Ʈ x = { Ø, X, {a, b}, {c, d}}

SO (X) = {Ø, X, {a, b} , {c, d}}

iO (X) = {Ø, X, {a},{ b}, {c}, {d}, {a, b}, {c, d}}

S* O (X) = P (X), iS* O (X) =P (X)

Then we have

From example 3.4{a}, {b} are S*- open sets but Not open sets.

From example 3.4 {a, c}, {b, c} are iS*- open sets but Not S*- open sets.

From example 3.4 {a}, {b} is an i - open sets but Not S - open sets.

From example 3.5 {a, b, c} {a, b, d}, {a, c, d}, {b, c, d} are iS*- open sets but Not i-open sets.

From example 3.4 {a} U {b}={a,b} is an is*_open set.

From example 3.4 {a,c} ∩ {b,c}={c} is not is* _open set.

Note: Let (X,Ʈ) be a topological space. Then the family of all iS* O(X) are supra topology on X but Not topology on X.

The example 3 . 4 show that.

4. iS* Separation Axiom

Definition 4 . 1 : A topological space (X, Ʈ) is called an iS* Ʈo - space and denoted by (iS* Ʈo) if for any two distinct points x,y in X there is an iS*- open set in X containing one of them but Not the other.

Example 4 . 2: Let X = {1, 2, 3} , Ʈx = { Ø, X, {1,2}}

Then iS*O (X) ={ Ø, X , {1}, {2}, {1,2} , {1,3}, {2,3}}

It clearly that (X ,Ʈx) is iS* Ʈo - space.

Since every open (closed) set is an iS*- open (iS*- closed) set, then we have the following theorem.

Theorem 4 .3 : Every Ʈo - space is an iS* Ʈo - space.

Proof : It is obvious

{Considering the example (3 . 2) show that (X , Ʈx) is an iS* Ʈo - space but Not Ʈo - space.

Since every S*- open (closed) set is an iS*- open (iS*- closed) set then we have the following theorem.

Theorem 4 . 4 : Every S* Ʈo - space is an iS* Ʈo - space.

Proof: It is obvious

But the converse above theorem may not true in general consider the following example.

Example 4 . 5 : Let X = {a, b, c}, Ʈx = {Ø, X, {a, c}} then S*O(X) = { Ø,X, {a,}, {c}, {a, c}}

iS* O(X)={Ø,x,{a},{c},{a,c},{a,b},{b,c}}

Then (X, Ʈx) is an iS*Ʈo - space but Not S*Ʈo - space.

Since every i - open (i - closed) set is an iS*- open (closed) set then we have the following theorem.

Theorem 4 . 6 : Every iƮo - space is an iS*Ʈo - space

Proof: It is obvious

The following example show that the converse of theorem 4.6 are not true in general.

Example 4 . 7 : Let X = {a, b, c,} , Ʈx = { Ø, X } , iO(X) = { Ø, X } , iS* O(X) = P (X)

Then (X, Ʈx) is an iS*Ʈo- space but Not iƬo -space.

Theorem 4 . 8 : A topological space (X, Ƭ) is iS*Ƭo - space if and only if for each pair of distance points x,y of X . CliS*{ x} ≠CliS* {y}.

Proof : For each x, y ∈ X, x ≠ y Cl_{iS}* {X} ≠cl_{iS}* {y}, let G ∈ X Such that G ∈ Cl_{iS}* {X} but

G ∈ Cl_{iS}* {y}. Assume that x ∈ Cl_{iS}* {y}, if x ∈ Cl_{iS}* {y} then Cl_{iS}* {x} ⊆ Cl_{iS}* {y} this contradict that fact that G ∉ Cl {y} consequently x ∈ Cl_{iS}* $\{y\}^c$ to which y does not belong. Necessity let (X,Ƭ) be iS*Ƭo-space and x, y ∈ X, x ≠ y, ∈ iS*- open set U.

Such that x ∈ u or y ∈ U then $U^c$is an iS*- closed set such that x ∈ U and y ∈U^c, since Cl_{iS}* {y} is the Smallest iS*- closed set containing y Cl_{iS}* {y} ⊆ U^c and there for x ∉ Cl_{iS}* {y}. Hence CliS* {y} ≠cl_{iS}* {x}.

"Theorem 4 .9 : Let (X, Ƭx) be any is* Ƭo _ space then every relative topological space is is* Ƭo

"Proof: let (y, Ƭy) be relative topological space. To show that (Y,Ƭy) is is* Ƭo _ space let y_1,y_2 ∈ y and y_1 ≠ y_2 then y_1,y_2 ∈ x . since (X, Ƭx) is an is* _ Ƭo then there exists is* _ open set U ⊆ X such that U containing one of y_1, y_2 but not both.

Now , if y_1 ∈ U then y_1 ∈ y ∩ U = U* if y2 ЄU then y_2 Є y ∩ U = U* therefore (Y, Ƭy) is an is* Ƭo _ space."

Definition 4.10: A topological space (X, Ƭ) is an is* $Ƭ_1$ _ space and denoted by (is*$Ƭ_1$) if

for any two distinct points x,y in X there exists two is* _ open sets U,V in X such that x Є U , y ∉ U and y ∉ V , x ∉ V.

Example 4 . 11 : Let X = {a, b, c, d} , ∉ x ={Ø, X, {a, b}, {c, d}}

Then iS*O (X) = P(X)

It is clearly that (X, Ƭx) is an iS* $Ƭ_1$ - space.

Since every open (closed) set is an iS*- open (closed) set, than we have the following theorem.

Theorem 4 . 12 : Every Ƭo - space is an iS* Ƭo - space.

Proof: It is obvious.

But the converse of above theorem may not be true in general.

Consider the example 4.13 show that (X, Ƭx) is an iS*Ƭ1 - space but Not $Ƭ_1$ - space.

Example 4 . 13 : Let X = {a, b, c, d} ,

Ƭx = { Ø, X, , {a, d} , {b, c} }

, iS*O (X) = P (X)

Then (X, Ƭx) is an iS*$Ƭ_1$ - space but Not $Ƭ_1$ - space.

Since every S*- open (closed) set is an iS*- open (closed) set then we have the following theorem.

Theorem 4 . 14 : Every S*$Ƭ_1$ - space is an iS*$Ƭ_1$ - space.

Proof: It is obvious

But the converse above theorem may not be true in general.

Consider example 4.15 which shows that (X, Ƭx) is an iS*$Ƭ_1$ - space but not S*$Ƭ_l$ - space.

Example 4 . 15 : Let X = {a, b, c, } , Ƭx = { Ø, X, {a, b} }

S*O (X) = { Ø, X , {a}, {b}, {a, b} }

iS*O (X) ={Ø, X, {a} , {b}, {a, b}, {a, c} , {b, c} }

Then (X, Ƭx) is an iS*$Ƭ_1$ - space but Not S*$Ƭ_1$ - space.

Since every i - open (closed) set is an iS* - open (closed) set then we have the following theorem

Theorem 4.16 : Every i$Ƭ_1$ - space is an iS*$Ƭ_1$ - space

Proof: It is obvious.

The following example show that the converse of theorem 4.16 are not true in general.

Example 4 . 17 : Let X = { a, b, c, d, e, f}, Ƭx = { Ø, X} , iO(X) = {Ø, X}

iS*O (X) = P (X)

Then (X, Ƭx) is an iS*$Ƭ_1$ - space but Not i$Ƭ_1$ - space.

Theorem 4 . 18 : A topological space (X, Ƭ) is an iS*$Ƭ_1$ if every singleton is an iS*- closedX, Ƭ)

is an iS*$Ƭ_1$ - space. Let x and y be any two distinct points of (

Proof: suppose that every singleton is an iS*- closed in (X, Ƭ), to prove (X, Ƭ). Assume that U = X / {x} and V = X / {y} since {x} and {y} are iS*- closed set in (X, Ƭ) then U and V are iS*- open sets in (X, Ƭ), since y ∈ U , x ∉ U and x ∈ V , y ∉ V then (X, Ƭ) is an iS*$Ƭ_1$ - space.

Theorem 4 . 19 : Let (X, Ƭ) be any iS*$Ƭ_1$ - space. Then every relative topological space (Y, Ƭy) of (X, Ƭ) is an iS*$Ƭ_1$

Proof: Since a topological space (X, Ƭ) is an iS*$Ƭ_1$. Let y_1 , y_2 ∈ Y, such that $y_1 \neq y_2$, then y_1 , y_2 ∈ X, hence there exists two iS*- open sets U, V such that y_1 ∈ U and y_1 ∈ Y then y_1 ∈ Y ∩ U = U* and y_2 ∈ V and y_2 ∈ Y then y_2 ∈ Y ∩ V =V* then U*, V* are iS*- open sets from definition (2.4) then (Y, Ƭy) is an iS*$Ƭ_1$ - space.

Theorem 4 . 20 : Every iS* $Ƭ_1$ -space is an iS*Ƭo - space.

Proof: Let (X, Ƭ) be an iS*$Ƭ_1$ - space and let x , y ϵ X, x ≠ y.

Then there exists two iS*- open sets U, V such that

X ∈ U and y ∉ U ,Y ∈ V and x ∉ V

This mean there exists iS*- open set contain one point and not contain another , then (X, Ƭ) is an iS*Ƭo - space.

The following example show that the converse of theorem (4.20) are not true in general .

Example 4.21 : let X = {a , b} , Ƭx = { Ø, X , {a } }

iS*O(X) = { Ø, X , {a } } , then (X, Ƭx) is an iS*Ƭo - space but not iS*$Ƭ_1$ - space .

Definition 4.22: A topological space (X, Ƭ) is called an iS*$Ƭ_2$ - space if for any distinct points x and y in X there exists two disjoint iS*- open sets U , V in X such that x ϵ U and y ϵ V.

Example 4 . 23 : Let X = {a, b, c, d} , Ƭx = { Ø, X, {a, b} , {c, d}}

iS*O (X) = P (X)

then (X, Ƭx) is an iS*$Ƭ_2$ - space.

Since every open (closed) set is an iS*- open (closed) set, then we have the following theorem.

Theorem 4.24: Every $Ƭ_2$ - space is an iS*$Ƭ_2$ - space.

Proof: It is obvious.

The converse of theorem 4.24 may not be true in general.

The following example shows that

Example 4 . 25 : Let X = {a, b, c, d} , Ƭx = { Ø, X, {a, b} , {c, d} }

iS*O (X) = P (X) ,Then (X, Ƭx) is an iS*$Ƭ_2$ - space but Not $Ƭ_2$ - space.

Since every S* - open (closed) set is an iS*- open (closed) set then we have the following theorem.

Theorem 4 .26 : Every S*t2 - space is an iS*t2 - space

Proof : It is obvious

The following example show that the converse of theorem (4.26) are not true in general .

Example 4 . 27 : Let X = {a, b, c} , Ƭx ={Ø, X, {a, b} }

S*O(X) = { Ø, X, {a}, {b}, {a, b}}

iS*O(X) = { Ø, X, {a}, {b}, {a, b}, {a, c}, {b, c} }

Then (X, Ƭx) is an iS*$Ƭ_2$ - space but Not S*$Ƭ_2$ - space.

Since every i - open (closed) set is an iS*- open (closed) set then we have the following theorem.

Theorem 4.28 : Every i$Ƭ_2$ - space is an

$iS^*\mho_2$ - space.

Proof: It is obvious.

The following example show that the converse of theorem 4.28 are not true in general.

Example 4.29 : Let X = {a, b, c, d, e, f} , Ʈx = { Ø, X } , iO (X) = {Ø, X } , iS*O (X) = P (X)

then (X, Ʈx) is an $iS^*\mho_2$ - space but Not $i\mho_2$ - space.

Theorem 4.30 Let (X, Ʈx) be any iS* $\mho_2$ - space then every relative topological space (Y, Ʈy) of (X, Ʈ) is an iS* $\mho_2$.

Proof: Let(Y, Ʈy) be relative topological space of (X, Ʈx) and let $y_1, y_2 \in y$ such that $y_1 \neq y_2$. since $Y \subseteq X$ so $y_1, y_2 \subseteq X$.

But (X, Ʈx) is an is* $\mho_2$ _space , then there exists tow disjoint is* _ open set U,V in X, such that $y_1 \in U$ and $y_2 \in V$ and $U \cap V = \emptyset$ then $y_1 \in Y \cap U = U^*$ and $y_2 \in Y \cap V = V^*$ and $U^* \cap V^* = \emptyset$ then (Y, Ʈy) is an iS*- space.

Theorem 4.31 Every $iS^*\mho_2$ - space is an iS* $\mho_1$- space.

Proof: Let (X, Ʈ) be an $iS^*\mho_2$ - space and let x, y be two distinct points in X since (X, Ʈ) is an $iS^*\mho_2$ - space then there exists two iS* - open sets U, V in X such that $x \in U$, $y \in V$ and $U \cap V = \emptyset$. Since $y \notin U$ and $x \notin V$ then (X, Ʈ) is an $iS^*\mho_1$ - space.

The following example show that the converse of theorem 4.31 are not true in general.

Example 4. 32Let X ={a,b,c,d}, Ʈx ={ Ø,X,{a,b,} }

Then iS* O(X)={ Ø,X,{a},{b},{a.b},{a,c} ,{a,d},{b,c},{b,d},{a,b,c},{a,b,d},{a,c,d},{b,c ,d}}

Then (X, Ʈx) is an iS* $\mho_1$- space but not $iS^*\mho_2$ - space.

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