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Recovering Jackknife Ridge Regression Estimates from OLS Results

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ABSTRACT

The aim of this paper is addressing or recalculate the estimation methods in multiple linear regression model when there is a problem of Multicollinearity in this model like the ridge regression for Hoerl and Kannard, Baldwin estimator (HKB) and Jackknifed ridge regression estimator (JRR) using least-squares estimators which the last are the best unbiased estimators, consistent and linear. In this paper we proposed a formula to calculate the above estimators easily depending on the least-squares estimator, this treatment as a mathematical formula faster than the HKB estimator that depend on reducing the variance and JRR estimator that depend on reducing the bias. We used numerical examples of the pricing method in comprehensive quality and environmental quality as air pollution in places as pricing environment. After the comparison JRR and HKB estimates are superior to the OLS estimates under the mean squared error (MSE) criterion. 2000 Mathematics Subject Classification: Primary 62J07.

1. Introduction

Under Multicollinearity, there are alternative estimators, such as the ridge estimators, that can provide more precision in parameter estimation. The ridge estimators are derived based on different criteria from those for the method of ordinary least squares (OLS) in that additional conditions of parameter estimation are imposed to ensure better precision. Huang [7, 8] proposes an alternative criterion to the goodness of fit of the overall model, which will select regression estimators that yield more reliable and stable marginal effect estimates of the variable of interest. Such estimator can be a good choice for deriving a more precise marginal benefit estimate of air quality. The purpose of this paper is to provide alternative, more precise benefit estimates of air quality. Three estimators, the traditional ridge estimator proposed by Hoerl, Kennard, and Baldwin [6], the jackknife ridge estimator proposed by Singh, Chaubey and Dwivedi [11]and the OLS estimator are compared. The properties of these estimators are discussed. Simple formulae to calculate the two estimates and their MSEs for a particular coefficient based on ordinary least squares (OLS) results are presented. The proposed ridge estimates associated with the marginal effects of air pollution on property values are calculated based on results from Twelve published property value/air pollution studies and are compared with the original OLS estimates. It is found that the proposed ridge estimators provide more precise good estimates than the OLS estimator in all Twelve studies.

2. The Model

A typical multiple linear regression model has more than one explanatory variable. Suppose that the policy makers are particularly interested in the effect of one variable, X_1. Consider the following linear regression model:

$$Y = X_1B_1 + X_2B_2 + \varepsilon = XB + \varepsilon, \qquad (2.1)$$

where $X = (X_1,\ X_2)$, $\beta = \begin{pmatrix} B_1 \\ B_2 \end{pmatrix}$, Y is an (n×1) vector of the dependent variable; X_1 is the explanatory variable of interest; X_2 is an (n×(q-1)) matrix that includes all other independent variables; B_1 and B_2 are (1×1) and ((q-1)×1) parameter vectors, respectively; ε is an (n×1) vector of independent and identically distributed random errors that follow a joint normal distribution $N(0, \sigma^2 I)$. I is the (n×n) identity matrix. Assume that $X_1^T X_1$ and $X_2^T X_2$ are non-singular. The OLS estimator of B_1 can be written as follows [9].

$$\hat{B}_{1LS} = \left(X_1^T M_2 X_1\right)^{-1} X_1^T M_2 Y \qquad (2.2)$$

where $M_2 = I - X_2\left(X_2^T X_2\right)^{-1} X_2^T$ and $X_1^T M_2 X_1 > 0$. Under the assumption that the model in (2.1) is correctly specified, $\hat{B}_{LS}$ is unbiased. Its variance is the mean squared error (MSE) and is equal to $\sigma^2\left(X_1^T M_2 X_1\right)^{-1}$. In order to control inflation of standard errors and general instability of the least squares estimates when the multicollinearity problem is present, Hoerl and Kennard [5], Gruber [4], Huang [7, 8], and Batah [1] are proposed ridge estimators tend to shrink the least square estimator toward zero.

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2.1 Huang and Jackknife Ridge Estimators.

Huang [7] proposes an estimator for the parameter of interest (i.e., the coefficient of X1) that is derived from an MSE criterion:

$$\hat{B}_1(K) = \left(X_1{}^T M_2 X_1 + K\right)^{-1} X_1{}^T M_2 Y \qquad (2.3)$$

The Huang ridge estimator is a potential reduction of variance. As shown in Huang [7] that the Huang ridge estimator $\hat{B}_1(K)$ has a smaller MSE than the OLS estimator $\hat{B}_{1LS}$ for any positive K value if $\sigma^2 - B_1^2\left(X_1{}^T M_2 X_1\right)$ is non-negative, or, $B_1^2\left(X_1{}^T M_2 X_1\right)/\sigma^2 \le 1$. The researchers tended to ignore its biasedness. Batah [1] suggested a new estimator by multiplying Huang [7] estimator by the scalar $\left(1 + k\left(X_1{}^T M_2 X_1 + k\right)^{-1}\right)$ followed in obtaining the jackknife ridge regression estimator as follows (see Batah [1]):

$$\hat{B}_1^J(K) = \left(1 + K\left(X_1{}^T M_2 X_1 + K\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + K\right)^{-1} X_1{}^T M_2 Y . \qquad (2.4)$$

The Mean square error of can be expressed as

$$MSE\left(\hat{B}_1^J(K)\right) = \frac{\sigma^2\left[\left(X_1{}^T M_2 X_1\right)^3 + 4K\left(X_1{}^T M_2 X_1\right)^2 + 4(K)^2\left(X_1{}^T M_2 X_1\right)\right] + (K)^4 B_{1LS}^2}{\left(X_1{}^T M_2 X_1 + K\right)^4} . \qquad (2.5)$$

The proposed jackknifed ridge estimator $\hat{B}_1^J(K)$ would be expected to be smaller MSE than the OLS estimator $\hat{B}_{1LS}$ for positive K value if $\sigma^2 - B_1^2\left(X_1{}^T M_2 X_1\right)$ is non-negative, or, $B_1^2\left(X_1{}^T M_2 X_1\right)/\sigma^2 \le 1$. The optimal choice of K can be determined by minimizing the MSE of $\hat{B}_1^J(K)$ as $K^* = \sigma^2 / B_{1LS}^2$ to substitute OLS estimator into K, denoted $\hat{K}^* = \hat{\sigma}^2 / \hat{B}_{1LS}^2$ and derived by substituting $\hat{K}^*$ into $\hat{B}_1^J(K)$ as:

$$\hat{B}_1^J(K^*) = \left(1 + \frac{\hat{\sigma}^2}{\hat{B}_{1LS}^2}\left(X_1{}^T M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{B}_{1LS}^2}\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{B}_{1LS}^2}\right)^{-1} X_1{}^T M_2 Y . \qquad (2.6)$$

The adaptive Jackknife $\hat{B}_1^J(\hat{K}^*)$ ridge estimator is not linear estimators. Hoerl, et al. [6] applied the biasing parameter $K_h^* = \frac{q\sigma^2}{B^T B}$ to deriving the $\hat{B}_1^J(K^*)$ as,

$$\hat{B}_1^J(K_h^*) = \left(1 + K_h^*\left(X_1{}^T M_2 X_1 + K_h^*\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + K_h^*\right)^{-1} X_1{}^T M_2 Y , \qquad (2.7)$$

the mean square error of $\hat{B}_1^J(K_h^*)$ can be shown as follows.

$$MSE\left(\hat{B}_1^J(K_h^*)\right) = \frac{\sigma^2\left[\left(X_1{}^T M_2 X_1\right)^3 + 4K_h^*\left(X_1{}^T M_2 X_1\right)^2 + 4(K_h^*)^2\left(X_1{}^T M_2 X_1\right)\right] + (K_h^*)^4 B_1^2}{\left(X_1{}^T M_2 X_1 + K_h^*\right)^4} . \qquad (2.8)$$

Substituting the estimator of the biasing parameter $\hat{K}_h^* = \frac{q\hat{\sigma}^2}{\hat{B}^T\hat{B}}$ into $\hat{B}_1^J(K_h^*)$, the adaptive Jackknife ridge estimator.

$$\begin{aligned}\hat{B}_1^J(\hat{K}_h^*) &= \left(1 + \hat{K}_h^*\left(X_1{}^T M_2 X_1 + \hat{K}_h^*\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + \hat{K}_h^*\right)^{-1} X_1{}^T M_2 Y \\ &= \left(1 + \frac{q\hat{\sigma}^2}{\hat{B}_{1LS}^2}\left(X_1{}^T M_2 X_1 + \frac{q\hat{\sigma}^2}{\hat{B}_{1LS}^2}\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + \frac{q\hat{\sigma}^2}{\hat{B}_{1LS}^2}\right)^{-1} X_1{}^T M_2 Y\end{aligned} . \qquad (2.9)$$

We are used adaptive jackknife ridge regression estimators especially when the problem of collinearity is severe. For its well known properties, the HKB estimator $\hat{B}_1^J(\hat{K}_h^*)$ is adopted for comparison with the proposed estimator $\hat{B}_1^J(\hat{K}^*)$. Both estimates can be recovered from the OLS results and are used to recalculate the regression coefficient of air quality variable in the property value equations estimated by OLS in Twelve published studies.

3. Improving Jackknife Ridge Regression Estimates from OLS Results

In this section, formulae to compute $\hat{B}_1^J(\hat{K}^*)$, $\hat{B}_1^J(\hat{K}_h^*)$, and the estimated MSEs based on reported OLS results are presented. To recover these estimates from OLS results, two segments, $X_1{}^T M_2 X_1$ and $X_1{}^T M_2 Y$ must be expressed in terms of the OLS results as follows.

$$X_1{}^T M_2 X_1 = \frac{\sigma^2}{S^2_{\hat{B}_{1LS}}}, \quad X_1{}^T M_2 Y = \hat{B}_{1LS} \frac{\sigma^2}{S^2_{\hat{B}_{1LS}}}. \qquad (3.1)$$

Substituting (3.1) into the expressions of $\hat{B}_1^J(\hat{K}^*)$ and $\hat{B}_1^J(\hat{K}_h^*)$, the two estimators can be calculated from the reported OLS results.

$$\hat{B}_1^J(\hat{K}^*) = \left(\frac{\hat{B}^3_{1LS} + 2S^2_{\hat{B}_{1LS}}}{\hat{B}^3_{1LS} + S^2_{\hat{B}_{1LS}}}\right)\left(\frac{\hat{B}^3_{1LS}}{\hat{B}^3_{1LS} + S^2_{\hat{B}_{1LS}}}\right) = \left(\frac{t^2+2}{t^2+1}\right)\left(\frac{t^2}{t^2+1}\right)\hat{B}_{1LS} = \left(\frac{t^2(t^2+2)}{(t^2+1)^2}\right)\hat{B}_{1LS}, \qquad (3.2)$$

and

$$\hat{B}_1^J(\hat{K}_h^*) = \left(\frac{\sum_{i=0}^{q} \hat{B}^2_{iLS} + 2qS^2_{\hat{B}_{1LS}}}{\sum_{i=0}^{q} \hat{B}^2_{iLS} + qS^2_{\hat{B}_{1LS}}}\right)\hat{B}_{1LS}, \qquad (3.3)$$

Where t is the t statistics $\frac{\hat{B}_{1LS}}{S_{\hat{B}_{1LS}}}$ for testing B_1 to be zero. $S_{\hat{B}_{1LS}}$ is the square root of $\hat{\sigma}^2\left(X_1{}^T M_2 X_1\right)^{-1}$. Hence, t follows a noncentral student's t distribution with noncentarlity parameter $\frac{\hat{B}_{1LS}}{\sqrt{\hat{\sigma}^2(X_1{}^T M_2 X_1)^{-1}}}$ and q is the number of parameters estimated in the original model. The corresponding estimated MSEs are calculated based on the formulae for $MSE\left(\hat{B}_1^J(\hat{K}^*)\right)$ and $MSE\left(\hat{B}_1^J(\hat{K}_h^*)\right)$ in (2.6) and (2.9). They can be written in terms of the OLS as.

$$MSE\left(\hat{B}_1^J(\hat{K}^*)\right) = \frac{\hat{\sigma}^2\left[\left(\frac{\hat{\sigma}^2}{S^2_{\hat{B}_{1LS}}}\right)^3 + 4\frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\left(\frac{\hat{\sigma}^2}{S^2_{\hat{B}_{1LS}}}\right)^2 + 4\left(\frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\right)^2\left(\frac{\hat{\sigma}^2}{S^2_{\hat{B}_{1LS}}}\right)\right] + \left(\frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\right)^4 B^2_{1LS}}{\left(\frac{\hat{\sigma}^2}{S^2_{\hat{B}_{1LS}}} + \frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\right)^4},$$

$$MSE\left(\hat{B}_1^J(\hat{K}^*)\right) = \frac{\hat{B}^2_{1LS}\left(t^6 + 4t^4 + 4t^2 + 1\right)}{\left(t^2+1\right)^4}, \qquad (3.4)$$

$$MSE\left(\hat{B}_1^J(\hat{K}_h^*)\right) = \frac{S^2_{\hat{B}_{1LS}}\left(\sum_{i=1}^{q} B^2_{iLS}\right)^2\left[\left(\sum_{i=1}^{q} B^2_{iLS}\right)^2 + 4qS^2_{\hat{B}_{1LS}}\left(\sum_{i=1}^{q} B^2_{iLS}\right) + 4\left(qS^2_{\hat{B}_{1LS}}\right)^2\right] + \left(qS^2_{\hat{B}_{1LS}}\right)^4 \hat{B}^2_{1LS}}{\left(\sum_{i=1}^{q} B^2_{iLS} + qS^2_{\hat{B}_{1LS}}\right)^4}. \qquad (3.5)$$

The small and large sample properties of $\hat{B}_1^J(K^*)$

$$\hat{B}_1^J(K^*) = \left(1 + \frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\left(X_1{}^T M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\right)^{-1}\right)\left(X_1{}^T M_2 X_1 + \frac{\hat{\sigma}^2}{\hat{B}^2_{1LS}}\right)^{-1} X_1{}^T M_2 Y$$

Assume that

$$\lim_{n\to\infty} \frac{X_1^T M_2 X_1}{n} = \Psi, \ \operatorname{plim}_{n\to\infty} \frac{X_1^T M_2 \varepsilon}{n} = 0, \ \lim_{n\to\infty} \frac{\hat{K}^*}{n} = 0$$

where, Ψ is a constant. From the central limit theorem, it can be shown that the limiting distribution of $\frac{X_1^T M_2 \varepsilon}{\sqrt{n}}$ is normal with zero and variance $\sigma^2\Psi$. Rewriting

$$\hat{B}_1^J\left(\hat{K}^*\right)=\left(1+\frac{\hat{K}^*}{n}\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\right)\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\left(\frac{X_1{}^TM_2X_1}{n}\right)B_1$$
$$+\left(1+\frac{\hat{K}^*}{n}\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\right)\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\left(\frac{X_1{}^TM_2\varepsilon}{n}\right)$$

where $M_2=I-X_2\left(X_2{}^TX_2\right)^{-1}X_2{}^T$ and $X_1{}^TM_2X_1>0$. It can be shown that $\hat{B}_1^J\left(\hat{K}^*\right)$ is a consistent estimator of B_1. Further, the limiting distribution of (see Dwivedi et al. [2]) $\left(1+\frac{\hat{K}^*}{n}\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\right)\left(\frac{X_1{}^TM_2X_1}{n}+\frac{\hat{K}^*}{n}\right)^{-1}\left(\frac{X_1{}^TM_2\varepsilon}{\sqrt{n}}\right)$ is normal with 0 and variance $\sigma^2\Psi^{-1}$. Asymptotically, the distribution of $\hat{B}_1^J\left(\hat{K}^*\right)$ can be written as follows. $\hat{B}_1^J\left(\hat{K}^*\right)\overset{asy}{\sim}N\left(B_1,\frac{\sigma^2\Psi^{-1}}{n}\right)$.

The above asymptotic distribution of $\hat{B}_1^J\left(\hat{K}^*\right)$ can be used to conduct hypothesis tests and interval estimation for B_1 when sample size is large. The sampling properties of $\hat{B}_1^J\left(\hat{K}^*\right)$ can be examined by rewriting $\hat{B}_1^J\left(\hat{K}^*\right)$ in terms of the OLS estimator $\hat{B}_{1LS}$.

$$\hat{B}_1^J\left(\hat{K}^*\right)=\frac{t^4+2t^2}{\left(t^2+1\right)^2}\hat{B}_{1LS}$$

The above expression of $\hat{B}_1^J\left(\hat{K}^*\right)$ is parallel to Dwivedi et al. [2] expression of the adaptive generalized ridge estimator for a canonical form of a linear regression model proposed by Hoerl and Kennard [5]. The exact first two moments of $\hat{B}_1^J\left(\hat{K}^*\right)$ can be expressed applying Dwivedi et al. [2] results. Rewriting the estimator, $\hat{B}_1^J\left(\hat{K}^*\right)$ as a function of two random variables as follows:

$$\hat{B}_1^J\left(\hat{K}^*\right)=\frac{\left(1+2\frac{\chi^2/\nu}{Z^2}\right)\sqrt{\hat{\sigma}^2\left(X_1{}^TM_2X_1\right)^{-1}}\cdot Z}{\left(1+2\frac{\chi^2/\nu}{Z^2}\right)^2}=\sqrt{\hat{\sigma}^2\left(X_1{}^TM_2X_1\right)^{-1}}\left[\left(\frac{Z^3}{Z^3+\chi^2}\right)\left(1-\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{-1}+\left(\frac{Z^3}{\left(Z^3+\chi^2\right)^2}\right)\left(1-\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{-2}\right]$$

where Z is normally distributed with mean $\frac{\hat{B}_{1LS}}{\sqrt{\hat{\sigma}^2\left(X_1{}^TM_2X_1\right)^{-1}}}$ and variance 1. χ^2 is a Chi- Square variable with degree of freedom ν . Since $\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)<1$, $\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)$ and $\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)^2$ can be expressed using Taylor series expression as follows.

$$\hat{B}_1^J\left(\hat{K}^*\right)=\sqrt{\hat{\sigma}^2\left(X_1{}^TM_2X_1\right)^{-1}}\left[\left(\frac{Z^3}{Z^3+\chi^2}\right)\sum_{\alpha=0}^{\infty}\left(\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{\alpha}+\left(\frac{Z^3}{\left(Z^3+\chi^2\right)^2}\right)\sum_{\alpha=0}^{\infty}(\alpha+1)\left(\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{\alpha}\right],$$

$$\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)^2 = \hat{\sigma}^2\left(X_1{}^T M_2 X_1\right)^{-1}\left[\begin{array}{l}\left(\dfrac{Z^6}{\left(Z^6+\chi^2\right)^2}\right)\displaystyle\sum_{\alpha=0}^{\infty}(\alpha+1)\left(\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{\alpha}\\ +\left(\dfrac{Z^6}{\left(Z^6+\chi^2\right)^3}\right)\displaystyle\sum_{\alpha=0}^{\infty}(\alpha+1)(\alpha+2)\left(\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{\alpha}\\ +\left(\dfrac{Z^6}{\left(Z^6+\chi^2\right)^4}\right)\displaystyle\sum_{\alpha=0}^{\infty}\frac{(\alpha+1)(\alpha+2)(\alpha+3)}{6}\left(\left(\frac{\nu-1}{\nu}\right)\left(\frac{\chi^2}{Z^3+\chi^2}\right)\right)^{\alpha}\end{array}\right].$$

Where ν is the degree of freedom associated with $S_{\hat{B}_{1LS}}$. The expected value of $\hat{B}_1^J\left(\hat{K}^*\right)$ and $\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)^2$ can be expressed in relation to a particular type of expectations that involves the two random variables, Z and χ^2. Let θ be the mean of Z. Dwivedi et al.[2] proved (in their appendix), Firinguetti [3] and Ohtani [10] that

$$E\left(\frac{Z^m\left(\chi^2\right)^q}{\left(Z^2+\chi^2\right)^r}\right)=\frac{2^{q-r+\frac{m}{2}}\,\Gamma\left(q+\frac{\nu}{2}\right)\,e^{-\frac{\theta^2}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\sum_{j=0}^{\infty}\frac{\Gamma\left(q-r+j+\frac{m+\nu+1}{2}\right)\Gamma\left(j+\frac{m+1}{2}\right)}{\Gamma\left(q+j+\frac{m+\nu+1}{2}\right)\Gamma\left(j+\frac{1}{2}\right)}\cdot\frac{\left(\frac{\theta^2}{2}\right)^j}{j!}$$

when m is an even integer and

$$E\left(\frac{Z^m\left(\chi^2\right)^q}{\left(Z^2+\chi^2\right)^r}\right)=\frac{2^{q-r+m-\frac{1}{2}}\,\theta\,\Gamma\left(q+\frac{\nu}{2}\right)\,e^{-\frac{\theta^2}{2}}}{\Gamma\left(\frac{\nu}{2}\right)}\sum_{j=0}^{\infty}\frac{\Gamma\left(q-r+j+\frac{m+\nu}{2}+1\right)\Gamma\left(j+\frac{m}{2}+1\right)}{\Gamma\left(q+j+\frac{m+\nu}{2}+1\right)\Gamma\left(j+\frac{3}{2}\right)}\cdot\frac{\left(\frac{\theta^2}{2}\right)^j}{j!}$$

when m is an odd integer. Using their result of $m=3$ for the expected value of $\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)$ and $m=6$ for the expected value of $\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)^2$. The first two moments of jackknifed ridge estimator can be expressed as follows.

$$E\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)=B_1\;e^{-\frac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}}\times\sum_{\alpha=0}^{\infty}\sum_{j=0}^{\infty}\left(\frac{\nu-1}{\nu}\right)^{\alpha}\frac{\Gamma\left(\alpha+\frac{\nu}{2}\right)\Gamma\left(j+\frac{5}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\alpha+j+\frac{\nu+5}{2}\right)\Gamma\left(j+\frac{3}{2}\right)\Gamma(j+1)}$$

$$\times\left[\Gamma\left(j+\frac{\nu+3}{2}\right)\left(\frac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}\right)^{j}+(\alpha+1)\Gamma\left(j+\frac{\nu+1}{2}\right)\left(\frac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}\right)^{j-1}\right]$$

$$E\left(\hat{B}_1^J\left(\hat{K}^*\right)\right)^2=B_1^2\;e^{-\frac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}}\sum_{\alpha=0}^{\infty}\sum_{j=0}^{\infty}\left(\frac{\nu-1}{\nu}\right)^{\alpha}\frac{\Gamma\left(\alpha+\frac{\nu}{2}\right)\Gamma\left(j+\frac{7}{2}\right)}{\Gamma\left(\frac{\nu}{2}\right)\Gamma\left(\alpha+j+\frac{\nu+7}{2}\right)\Gamma\left(j+\frac{1}{2}\right)\Gamma(j+1)}$$

$$\times\left[\begin{array}{l}(\alpha+1)\Gamma\left(j+\frac{\nu+3}{2}\right)\left(\dfrac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}\right)^{j-1}\\ +(\alpha+1)(\alpha+2)\Gamma\left(j+\frac{\nu+1}{2}\right)\left(\dfrac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}\right)^{j-2}\\ +\dfrac{(\alpha+1)(\alpha+2)(\alpha+3)}{6}\Gamma\left(j+\frac{\nu-1}{2}\right)\left(\dfrac{B_1^2}{2\sigma^2\left(X_1^TM_2X_1\right)^{-1}}\right)^{j-3}\end{array}\right]$$

The exact moments of adaptive ridge estimators are complex because these estimators are nonlinear in Y.

4. Numerical Example

Expressions in (3.2) and (3.3) enable us to recalculate the coefficient estimates of the air quality variable. The Twelve property value studies selected for recalculation all report OLS results. The complete citations of the Twelve studies are given in the Huang [7] and Smith and Huang [12] and all these studies have the problem of Multicollinearity. These studies have the OLS coefficients and their variance. We recalculate the HKB and JRR estimator and their MSE by using OLS results using the SPSS package. The OLS and their corresponding jackknife ridge estimates of the Twelve studies are reported in Tables 1 and 2.

Table 1 compares the jackknife estimators $\hat{B}_1^J(\hat{K}^*)$ and $MSE(\hat{B}_1^J(\hat{K}^*))$ to OLS estimator $\hat{B}_{1LS}$ and $MSE(\hat{B}_{1LS}) = S^2_{\hat{B}_{1LS}}$ for each of the selected studies. By examining the ratio of $MSE(\hat{B}_1(\hat{K}^*))$ and $MSE(\hat{B}_1^J(\hat{K}^*))$ to $S^2_{\hat{B}_{1LS}}$, it is seen that the mean square errors of the jackknife ridge estimates are universally smaller than the variance of the original OLS estimates in all studies. Further, the smaller the *t* value, the more is the $\hat{B}_1^J(\hat{K}^*)$ shrunk toward zero and the larger is $MSE(\hat{B}_1^J(\hat{K}^*))$. That is, the precision of the estimated coefficient improves less when the t value under OLS is small. In general, $\hat{B}_1^J(\hat{K}^*)$ gives more reliable estimates because the estimated mean square error is uniformly reduced. Table 2 compares the HKB's jackknife ridge estimator $\hat{B}_1^J(\hat{K}_h^*)$ and $MSE(\hat{B}_1^J(\hat{K}_h^*))$ to OLS results. The improvement of $\hat{B}_1^J(\hat{K}_h^*)$ over $\hat{B}_{1LS}$ appears to be small. By examining the formula of $\hat{B}_1^J(\hat{K}_h^*)$ in (3.3), the possible reason is that $\hat{B}_1^J(\hat{K}_h^*)$ depends on all coefficient estimates; hence, it is affected by measurement units of the explanatory variables and the model size. Given that the focus is to recover the marginal effect of one particular regressor, the jackknife estimator $\hat{B}_1^J(\hat{K}^*)$ appears to outperform the HKB and the OLS estimators in terms of the MSE criterion in all Twelve studies.

5. Conclusion

This paper has proposed to apply adaptive Jackknife ridge estimators to calculating a set of new estimates associated with marginal air quality benefits and compared them with original OLS results. The comparison of the proposed jackknife ridge estimates to the OLS results shows an improvement in mean square error in all studies. This implies that the benefit of improved air quality is measured more conservatively when applying jackknife ridge estimates and thus may affect policy decisions.

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Table 1. OLS and Jackknife $\hat{B}_1^J(K)$ estimators

	OLS RESULTS		JACKKNIFE ESTIMTATES		MSE1/MSE0
STUDIES	Coef.	Var.(MSE0)	Coefficient	MSE1	
Deyak &Smith 74	-0.0870	0.001964	-0.069076	0.001814	0.923855
"	-0.0880	0.001874	-0.070854	0.001746	0.931469
"	-0.0870	0.001842	-0.069972	0.001715	0.930879
"	-0.0830	0.001953	-0.064667	0.001783	0.913202
"	-0.0850	0.001804	-0.068017	0.001674	0.928136
Smith & Deyak 75	-0.1150	0.004221	-0.087176	0.003787	0.897087

=	Berry, B.J.L. 76	Harrison & Rubin. 78	Smith, B.A.78	=	Brookshire 79	=	Jackson, J.R. 79	Mc Donld, J.F. 80	=	=	Brookshire, et al.82	Plamquist, R.B. 84	=	=
-0.0480	0.4150	-0.0510	-42.9100	-51.0900	-316.8900	0.0000	-1.4730	0.0010	0.0030	-0.0020	-0.2218	-45.4700	-26.0400	-11.8600
0.001574	0.058898	0.000041	85.522634	282.438980	12993.5397	0.000000	4.557297	0.000016	0.000070	0.000090	0.003360	80.751180	13.109394	13.566182
-0.028518	0.309244	-0.050209	-41.005395	-46.101522	-280.584377	0.000000	-0.475100	0.000059	0.000342	-0.000085	-0.207620	-43.760833	-25.546115	-10.816757
0.001161	0.052223	0.000041	85.193133	277.316486	12671.974570	0.000000	1.791092	0.000001	0.000009	0.000004	0.003333	80.527278	13.100052	13.365478
0.737388	0.886668	0.999522	0.996147	0.981863	0.975252	0.94600	0.393016	0.062080	0.125424	0.044287	0.992087	0.997227	0.999287	0.985206

=	=	=	=	Atkinson& Crocker87	Blomquist, et al. 88
-1.6000	-0.0030	-0.2200	-8.5900	-97.0270	-0.5344
11.111111	0.000000	3.361111	33.687043	123.721129	0.003367
-0.299610	0.000000	-0.003123	-5.897548	-95.768420	-0.528173
2.397277	0.000009	0.048381	28.105242	123.679765	0.003366
0.215755	0.99200	0.014394	0.834304	0.999666	0.999730

Table 2. OLS and HKB $\hat{B}_1^J(K_h)$ estimators

	STUDIES	Deyak &Smith 74	=	=	=	=	Smith & Deyak 75
OLS RESULTS	Coef.	-0.0870	-0.0880	-0.0870	-0.0830	-0.0850	-0.1150
	Var.(MSE0)	0.001964	0.001874	0.001842	0.001953	0.001804	0.004221
HKB JACK ESTIMTATES	Coefficient	-0.0866	-0.0875	-0.0866	-0.0827	-0.0847	-0.1149
	MSE1	1.95E-03	1.85E-03	1.82E-03	1.94E-03	1.79E-03	4.19E-03
MSE1/MSE0		0.9911	0.9883	0.9894	0.9930	0.9935	0.9929

=	=	Mc Donld, J.F. 80	Jackson, J.R. 79	=	Brookshire 79	=	Smith, B.A.78	Harrison & Rubin. 78	Berry, B.J.L. 76	=
-0.0020	0.0030	0.0010	-1.4730	0.0000	-316.8900	-51.0900	-42.9100	-0.0510	0.4150	-0.0480
0.000090	0.000070	0.000016	4.557297	0.000000	12993.5397	282.438980	85.522634	0.000041	0.058898	0.001574
-0.0020	0.0030	0.0010	-1.4730	0.0000	-316.8900	-51.0768	-42.6069	-0.0510	0.3362	-0.0480
8.40E-06	6.80E-06	1.60E-05	4.56E+00	0.00E+00	1.30E+04	2.82E+02	8.55E+01	4.10E-05	4.46E-02	1.57E-03
0.0934	0.0966	0.9999	1.0000	1.0000	1.0000	0.9995	0.9998	0.9990	0.7580	0.9990

Blomquist, et al. 88	Atkinson& Crocker87	=	=	=	=	=	=	Plamquist, R.B. 84	Brookshire, et al.82
-0.5344	-97.0270	-8.5900	-0.2200	-0.0030	-1.6000	-11.8600	-26.0400	-45.4700	-0.2218
0.003367	123.721129	33.687043	3.361111	0.000000	11.111111	13.566182	13.109394	80.751180	0.003360
-0.5344	-97.0267	-8.5900	-0.2200	-0.0030	-1.6000	-11.8600	-26.0400	-45.4690	-0.2136
3.36E-03	1.24E+02	3.37E+01	3.36E+00	1.00E-07	1.11E+01	1.36E+01	1.31E+01	8.07E+01	3.19E-03
0.9991	1.0000	1.0000	1.0000	0.9998	1.0000	1.0000	1.0000	1.0000	0.9479

معالجة مقدرات انحدار الحرف لـ Jackknife بواسطة المربعات الصغرى

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الخلاصة:

ان الهدف من هذا البحث هو معالجة او اعادة حساب طرق التقدير في نموذج الانحدار الخطي المتعدد عند وجود مشكلة التداخل الخطي في النموذج مثل مقدر انحدار الحرف لـ Hoerl و Kannard و Baldwin (HKB) ومقدر انحدار الحرف لـ Jackknife (JRR) باستخدام مقدرات المربعات الصغرى التي هي افضل مقدرات غير متحيزة ومتسقة وخطية. في هذا البحث اقترحنا صيغة لحساب المقدرات اعلاه بطريقة اسهل اعتمادا على مقدر المربعات الصغرى وهذه المعالجة كصيغة رياضية اسرع من مقدر HKB التي تعتمد على تقليل التباين ومقدر JRR التي تعتمد على تقليل التحيز. قمنا باستخدام امثلة عددية لاسلوب التسعير في الجودة الشاملة وخاصة الجودة البيئية كتلوث الهواء في تسعير الاماكن حسب البيئة. بعد المقارنة لوحظ ان مقدر HKB ومقدر JRR متفوقة على مقدرات المربعات الصغرى باستخدام معيار متوسط مربعات الخطأ.