Comparison of the quadratic equations of best fit for total station and GPS

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Abstract

The adjustment of observation is an important applications in the remote sensing and the geodesy. for this purpose , nineteen control ground points at the University of Baghdad, navigated by Garmine GPS navigator (UTM projection, WGS-84 Datum) has accuracy about (15-100) m and measured by the Total Station Leica 1200. The configuration of set data or these control ground points were a full curve in the real world. In this paper, the none liner Least-Squares Estimation Method **(NLLSEM)** has been used to estimate parameters of the quadratic function and to test configuration of the points . The quadratic function for total station and GPS, have been compared and many mathematical processes have been written by **MATLAB** language. The quadratic function of best fit for total station : $y(x)=-0.38*x^2+3.4e+005*x-7.4e+010$ with accuracy 3.7422e+019 m^2 but for GPS :$y(x)=-0.24*x^2+2.1e+005*x-4.7e+010$ with accuracy 1.0336e+019 m^2.

Keywords— Total Station, The quadratic Function, Least-Squares Method, GPS, MATLAB

مقارنة معادلات من الدرجة الثانية للمحطة المتكاملة ونضام التموضع العالمي

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الخلاصة:

تعديل الرصد من التطبيقات المهمة في التحسس النائي وعلم الارض . لهذا الغرض تسعة عشر نقاط تحكم ارضي في حرم جامعة بغداد, ملحت بنظام التموضع العالمي كارمن الملاحي الذي له دقة حوالي m (15-100)و قيست بالمحطة المتكاملة لايكا 1200. شكل او هيئت مجموعة النقا ط او نقاط التحم الارضي كان منحني بالتمام في العالم الحقيقي. في هذه الورقة, استخدمت طريقة اقل المربعات الغير خطية لحساب معاملات معادلة من الدرجة الثانية لاختبار شكل النقاط . قرنت معادلة الدرجة الثانية للمحطة المتكاملة ونظام التموضع العالمي. هناك العديد من العمليات الرياضية استخدمت لاستخراج النتائج كتبت بلغة الماتلاب , بحيث اعطت دقة للمحطة المتكاملة (3.7422e+019) m^2 ولنظام التموضع العالمي(1.0336e+019) m^2.

كلمات مفتاحية-المحطة المتكاملة , الدالة التربيعية,طريقة اقل المربعات,نظام التموضع العالمي, ماتلاب

1.Introduction

The set data or control ground points have been taken at The Baghdad University as shown in ,fig.1and 2, where, configuration of points or set data were a full curve in real world. The control ground points ,navigated by the Garmine GPS navigator (UTM projection, WGS-84 Datum) has accuracy about (15-100)m [1], and measured by the Total Station Leica 1200[2]. From process of mathematical and graph the configuration of points or some data might not be on the curve exactly ,due to experimental errors. The configuration of points have been tested for GPS and total station by estimate errors using **(NLLSM**) then prediction of models quadratic function of best fit for total station and GPS .

In addition the above , the adjustment of observation important applications in the geodesy and remote sensing application , therefore some time go to **(NLLSEM**) to test values of errors or estimate best observation[3,4].

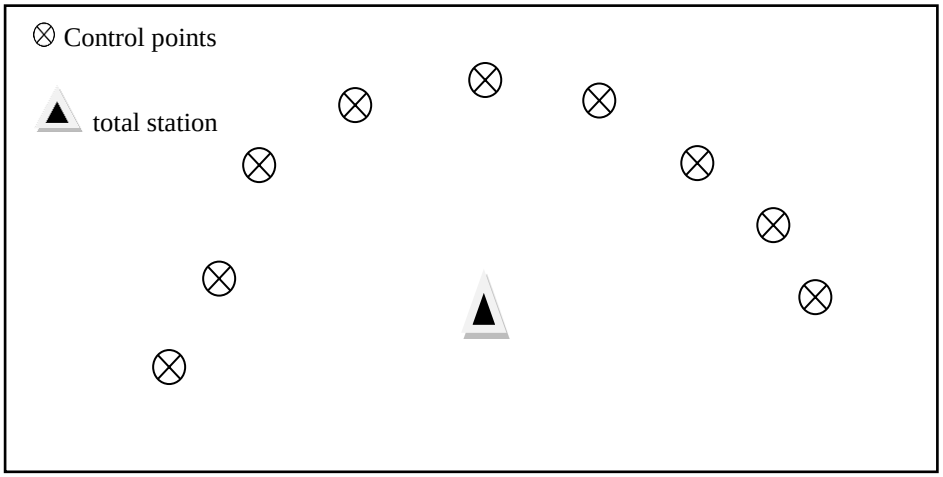

Fig. 1 Measurements of Total Station

1. **Fig. 2** Region of Interest

2. Material and Methods

The position of the sites were determined using a handheld Global Positioning System GPS Etrex vista Garmin Navigatorhas accuracy of 15 m to 100 m as shown in Fig. 3. Total Station Leica Power SearchTPS 1200 is

used for extract the coordinates of targets as shown in Fig. 4. The GPS was used to navigate the total station coordinates and to measure the point in the TS, where the TS used to estimate the target coordinates. Tables 1 and 2 show the GPS and TS specifications

Fig. 3 Global Positioning System

Fig. 4 The Total Station Leica TPS

Table 1 Specification of GPS [1]

Accuracy	15 m to 100 m
Unit dimensions, WxHxD	2.0" x 4.4" x 1.2" (5.1 x 11.2 x 3.0 cm)
Display size, WxH	1.1" x 2.1" (2.8 x 5.4 cm)
Display resolution, WxH	160 x 288 pixels
Display type	4 level gray LCD
Weight	5.3 oz (150 g) with batteries

Table 2 Specification of TS [2]

Accuracy Hz, V	1"
Display resolution (standard deviation, ISO 17123-3)	0.1" (0.1 mgon)
Compensator Working range	4' (0.07 gon)
Setting accuracy	0.5" (0.2 mgon)
Range Round prism (GPR1):	3000 m
Range 360° reflector (GRZ4):	1500 m
Range Mini prism (GMP101)	1200 m
Shortest measurable distance	1.5 m
Measurement Time	~ 1.5 s

According to configuration of observation points , the mathematical model used to predict quadratic function y(x) of best fit for total station and GPS is ;

$y(x)=ax^2+bx+c$ .....(1)

Where y(x) is northing coordinate , x is easting coordinate, a, b and c are the coefficients of quadratic function .

$$\varphi = \sum_{i=1}^{n} (y_i - ax_i^2 - bx_i - c)^2 \dots (2)$$

where φ stands for error which is the quantity to be minimized. The estimation of the parameters is obtained using basic results from calculus and, specifically, uses the property that a quadratic expression

$$\frac{\partial \varphi}{\partial a} = 2\sum_{i=1}^{n} (y_i - ax_i^2 - bx_i - c) * -x_i^2 = 0 \dots (3)$$

$$\frac{\partial \varphi}{\partial b} = 2\sum_{i=1}^{n} (y_i - ax_i^2 - bx_i - c) * -x_i = 0 \dots (4)$$

$$\frac{\partial \varphi}{\partial c} = 2\sum_{i=1}^{n} (y_i - ax_i^2 - bx_i - c) * -1 = 0 \dots (5)$$

Rearranging the above equations , to get;

$$\sum y_i x_i^2 = a\sum x^4_i + b\sum x^3_i + c\sum x_i^2 \dots (6)$$

$$\sum y_i x_i = a\sum x^3_i + b\sum x^2_i + c\sum x_i \dots (7)$$

$$\sum y_i = a\sum x^2_i + b\sum x_i + nc \dots (8)$$

Solving each system of above equations using Cramer's Rule , to get ;

The NLLSM defines the estimate of these parameters (a, b and c) as the values which minimize the sum of the squares (hence the name *least squares*) between the measurements and the model (*i.e.*, the predicted values). This amounts to minimizing the expression[5];

reaches its minimum value when its derivatives vanish. Taking the derivative of φ with respect to *a* ,*b* and *c* and setting them to zero gives the following set of equations (called the *normal* equations)[3]:

$$a = \frac{\begin{bmatrix} \sum y_i x_i^2 & \sum x^3_i & \sum x^2_i \\ \sum y_i x_i & \sum x^2_i & \sum x_i \\ \sum y_i & \sum x_i & n \end{bmatrix}}{\begin{bmatrix} \sum x^4_i & \sum x^3_i & \sum x^2_i \\ \sum x^3_i & \sum x^2_i & \sum x_i \\ \sum x^2_i & \sum x_i & n \end{bmatrix}} \dots (9)$$

$$b = \frac{\begin{bmatrix} \sum x^4_i & \sum y_i x_i^2 & \sum x^2_i \\ \sum x^3_i & \sum y_i x_i & \sum x_i \\ \sum x^2_i & \sum y_i & n \end{bmatrix}}{\begin{bmatrix} \sum x^4_i & \sum x^3_i & \sum x^2_i \\ \sum x^3_i & \sum x^2_i & \sum x_i \\ \sum x^2_i & \sum x_i & n \end{bmatrix}} \dots (10)$$

$$c = \frac{\begin{bmatrix} \sum x^4_i & \sum x^3_i & \sum y_i x_i^2 \\ \sum x^3_i & \sum x^2_i & \sum y_i x_i \\ \sum x^2_i & \sum x_i & \sum y_i \end{bmatrix}}{\begin{bmatrix} \sum x^4_i & \sum x^3_i & \sum x^2_i \\ \sum x^3_i & \sum x^2_i & \sum x_i \\ \sum x^2_i & \sum x_i & n \end{bmatrix}} \dots (11)$$

Where, index i represents no. of points, and n is the no. of points captured by total station and GPS.

3. Results

Table3 shows nineteen observations by the Garmine GPS navigator ,and TS.

Table 3 The Coordinates for TS and GPS

No. of points	TS \m		GPS\m	
1.	E	442273.497	E	442274
	N	3681944.304	N	3681944
2.	E	442278.405	E	442278
	N	3681942.384	N	3681942
3.	E	442279.743	E	442279
	N	3681947.432	N	3681939
4.	E	442280.598	E	442280
	N	3681937.517	N	3681938
5.	E	442280.471	E	442281
	N	3681934.582	N	3681933
6.	E	442281.336	E	442281
	N	3681931.562	N	3681931
7.	E	442282.201	E	442283
	N	3681928.561	N	3681928
8.	E	442283.585	E	442281
	N	3681924.48	N	3681925
9.	E	442281.479	E	442282
	N	3681921.291	N	3681922

The TS m column represents the estimated coordinates (UTM projection, WGS-84 Datum) by TS for each reflector point in that station. The GPS m column represents the navigated coordinates by the GPS in the reflector position. Which have configuration as a curve as shown in Fig. 5. Table 4 shows the results using MATLAB program and equations (1,2, 9, 10, and 11). The curve of best fit for data in TS and GPS is illustrated in Figure 6 and 7.

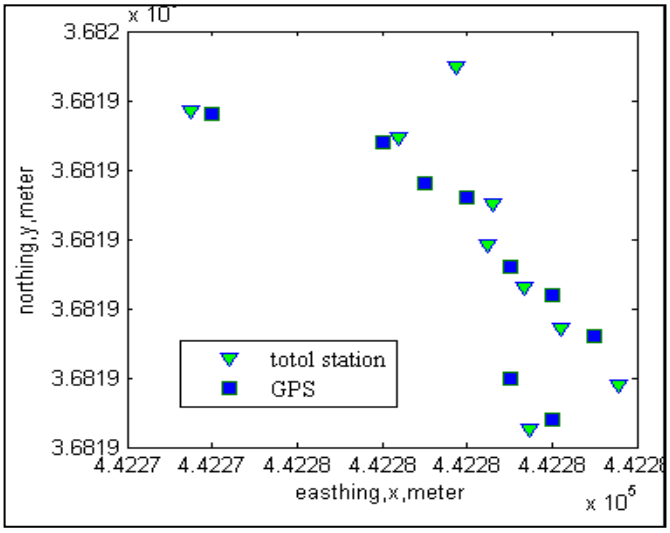

Fig. 5 Configuration of Points for total station and GPS

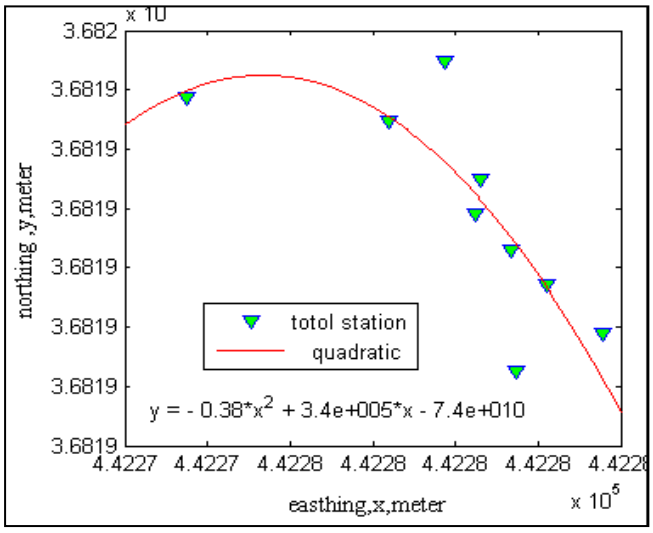

Fig. 6 The quadratic of best fit for TS

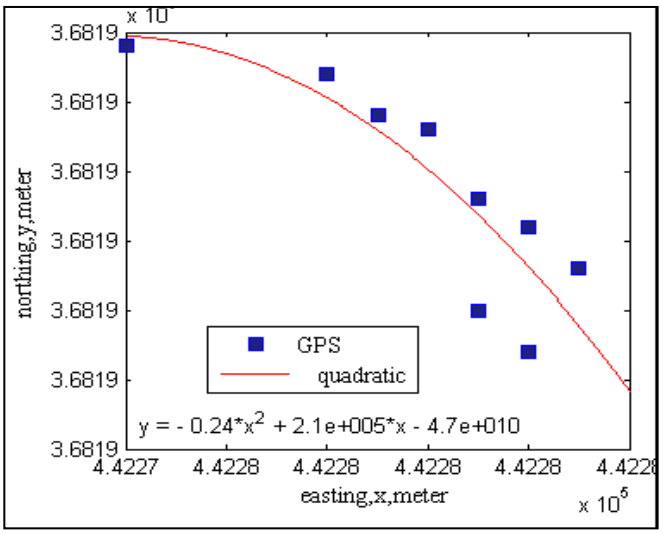

Fig. 7 The quadratic of best fit for GPS

Table 4 The Result for TS and GPS

	TS	GPS
Parameter of a	-0.38	-0.24
Parameter of b	3.4e+005	2.1e+005
Parameter of c	-7.4e+010	- 4.7e+010
The quadratic function	y(x)=-0.38*x^2+3.4e+005*x-7.4e+010	y(x)=-0.24*x2+2.1e+005*x-4.7e+010
Error (m^2)	3.7422e+019	1.0336e+019

The statistical data for coordinates in TS and GPS are compared as shown in Table 5.

Table 5 Comparison between TS and GPS

	TS		GPS	
	x	y	x	y
Mean	442280.146	3681934.679	442280	3681933.555
Standard deviation	2.893	9.035	2.738	7.699
Variation	8.369	81.639	7.5	59.277
Minimum	442273.497	3681921.291	442274	3681922
Maximum	442283.585	3681947.432	442283	3681944

4.Conclusion

The errors in GPS lesser than Total Station , then minimum error (φ) gives, The a , b and c. The data or ground points in GPS have configuration a curve better than form Total Station.

5.References

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