P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
2013,(7), (1) :158-163

New- Intuitionistic Generalized - Closed Sets in Intuitionistic Topological Spaces

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ARTICLE INFO

Received: 5 / 1 /2012
Accepted: 15 / 10 /2012
Available online: 19/7/2022
DOI:

Keywords:
T^* - intuitionistic generalized closed sets in intuitionistic topological spaces,
T^* - intuitionistic generalized open sets in intuitionistic topological spaces.

ABSTRACT

The aim of this paper is to introduce a new class of sets called T^*- intuitionistic generalized closed sets and T^*- intuitionistic generalized open sets in intuitionistic topological spaces and study some of their properties.And it is topology defined by $T^* = \{U: Icl^*(\overline{U}) = (\overline{U})\}$. This paper had been generalized of Pushpalatha, A. Eswaran, S. and Rajar, P. (2009) " T^* - generalized closed sets in topological spaces ".

I . Introduction

The concept of generalized closed set is given by Levine, N. [7] using generalized closed sets. Dunham, W. [4] introduced the concept of the closure operator cl* and a new topology T^* and studied some of their properties. Bhattacharyya, P. and Lahiri, B.K. [1]. Maki, H. Devi, R. and Balachandran, K. [9], Dontchev, J. and Maki, H. [5], Gnanambal, Y. [6] studied generalized semi-closed sets, semi generalized-closed sets, generalized semipre-closed sets, presemi generalized-closed sets, generalized presemi-closed sets, pre-closed sets, semipre-closed sets and presemi-closed sets respectively. Raouf, G. A. [12] studied intuitionistic generalized closed sets and some kinds in intuitionistic topological spaces. In this paper, we obtain a new generalization of closed sets in the weaker intuitionistic topological space (X, T^*).

Throughout this paper X denotes intuitionistic topological space on which no separation axioms are assumed unless otherwise explicitly stated For a subset A of an intuitionistic topologiacal space (ITS) X, $I\,int(A), I\,cl(A)$, $cl^*(A), I\,scl(A), I\,spcl(A), Ipscl(A)$ denote the intuitionistic interior, intuitionistic closure, intuitionistic closure*, intuitionistic semi-closure, intuitionistic semipre-closure, intuitionistic presemi-closure, and complement of A respectively.We denote the closure of A (respectively the generalized closure operator is defined by the intersection of all Ig-closed contining A , and A^c represent closure of A and complement of A to an intuitionistic topological spaces (ITS) on T^* by cl* (A))

II . PRELIMINARIES

We recall the following definitions which are needed in our work

- Let X be a non-empty set, and let A and B be IS having the form $A = \langle x, A_1, A_2\rangle$; $B = \langle x, B_1, B_2\rangle$ respectively. Furthermore, let $\{A_i : i \in I\}$ be an arbitrary family of IS in X, where $A_i = \langle x, A_i^{(1)}, A_i^{(2)}\rangle$, then:
  1) $\tilde{\emptyset} = \langle x, \emptyset, X\rangle$; $\tilde{X} = \langle x, X, \emptyset\rangle$.
  2) $A \subseteq B, iff\ A_1 \subseteq B_1\ and\ A_2 \supseteq B_2$.
  3) The complement of A is denoted by $\bar{A}$ and defined by $\bar{A} = \langle x, A_2, A_1\rangle$.

  $\cup A_i = \langle x, \cup A_i^{(1)}, \cap A_i^{(2)}\rangle, \cap A_i = \langle x, \cap A_i^{(1)}, \cup A_i^{(2)}\rangle$ [2]. An intuitionistic topology (IT, for short) on a non-empty set X, is a family T of IS in X containing $\tilde{\emptyset}, \tilde{X}$ and closed under arbitrary unions and finitely intersections. The pair (X, T) is called an intuitionistic topological space (ITS, for short) [3]. A subset A of intuitionistic topological spaces (ITS, for short) (X, T) is said to be generalized closed (g-closed) if $I\,cl(A) \subseteq U$ whenever $A \subseteq U$ and U is I-open in X [7]. A subset A of ITS (X, T) is said

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III. T^*- Intuitionistic Generalized Closed Sets in Intuitionistic Topological Spaces.

In this section, we introduce the concept of T^*-intutionistic generalized closed sets in intuitionistic topological spaces.

Proposition 3.1 Let (X, T) be ITS and let A be IS in X then $T^* = \{U : Icl^*(\bar{U}) = (\bar{U})\}$ is intuitionistic topology on X.

Proof 1. $\emptyset \in T^*$ since $I\,cl^*(X)=X$.Moreover $X \in T^*$ since $I\,cl^*(\emptyset) = \emptyset$

∴ Therefore $\emptyset, X \in T^*$.

2. Let $A, B \in T^*$ and let $A, B \subseteq U$ where U is any T^*-I open set in X and $A \cup B \subseteq U$ then

$I\,cl^*(\overline{A \cup B}) = I\,cl^*(\bar{A}) \cap I\,cl^*(\bar{B}) \subseteq I\,cl^*(\overline{A \cap B}) = \overline{A \cap B}$ to prove that $I\,cl^*(\overline{A \cup B}) = (\overline{A \cup B})$

$(\overline{A \cup B}) \subseteq I\,cl^*(\overline{A \cup B}) \subseteq cl^*(\bar{A}) \cap cl^*(\bar{B}) = (\overline{A \cup B})$

∴ Therefore $A \cup B \in T^*$.

3. Let $A, B \in T^*$ and $A \cup B \subseteq U$ where U is any T^*-I open set in X, then

$I\,cl^*(\overline{A \cap B}) = I\,cl^*(\bar{A}) \cup I\,cl^*(\bar{B}) = cl^*(\overline{A \cup B}) = \overline{A \cap B}$

∴ Therefore $A \cap B \in T^*$.

Now, we are ready to give a definition of T^*-topology

Definition 3.2 [11] A subset A of ITS(X, T), the topology T^* is defined by $T^* = \{U : I\,cl^*(\bar{U}) = \bar{U}\}$.

Definition 3.3 A subset A of ITS (X, T) is called T^*-intuitionistic generalized-closed sets (T^*-Ig-closed) if $I\,cl^*(A) \subseteq U$ whenever $A \subseteq U$ and U is T^*-I open. The complement of T^*-generalized-closed set is called the T^*-generalized open set (T^*-Ig-open).

Proposition 3.4 Every I closed set in X is T^*-Ig-closed.

Proof Let A be an I closed set in X. Let $A \subseteq U$ where U is any T^*-I open set in X. Since A is an I closed then $I\,cl(A) = A \subseteq U$ but $I\,cl^*(A) \subseteq I\,cl(A)$. Thus, we have $I\,cl^*(A) \subseteq U$ whenever $A \subseteq U$. Therefore A is T^*-Ig-closed.

The converse of the above proposition need not be true as the following example shows.

to be semi generalized-closed (sg-closed) if $I\,scl(A) \subseteq U$ whenever $A \subseteq U$ and U is I semi-open in X [1]. A subset A of ITS(X, T) is said to be generalized semi-closed (gs-closed) if $I\,scl(A) \subseteq U$ whenever $A \subseteq U$ and U is I-open in X [12]. A subset A of ITS(X, T) is said to be presemi generalized-closed (ps$g$-closed) if $I\,pscl(A) \subseteq U$ whenever $A \subseteq U$ and U is I-open in X [9] . A subset A of ITS(X, T) is said to be generalized presemi-closed ($g$ps-closed) if $I\,pscl(A) \subseteq U$ whenever $A \subseteq U$ and U is I preseme-open in X [12]. A subset A of ITS(X, T) is said to be generalized semipre-closed ($g$sp-closed) if $I\,spcl(A) \subseteq U$ whenever $A \subseteq U$ and U is I-open in X [6]. A subset A of ITS(X, T) is said to be presemi-closed (ps-closed) if $I\,cl\left(I\,int(I\,cl(A))\right) \subseteq A$ [1]. A subset A of ITS(X, T) is said to be pre-closed (p-closed) if $I\,cl(I\,int(A)) \subseteq A$ [6]. A subset A of ITS (X, T) is said to be semi-closed if $I\,int(I\,cl(A)) \subseteq A$ [6]. A subset A of ITS(X, T) is said to be semipre-closed if $I\,int\left(I\,cl(I\,int(A))\right) \subseteq A$ [1] . The complement of the above mentioned sets are called their I-open sets, g-closed sets, sg-closed sets , gs-closed sets , ps$g$-closed sets ,$g$ps-closed sets, $g$sp-closed sets, ps-closed sets, p-closed sets ,s-closed sets, sp- closed sets respectively. For the subset A of ITS (X, T), the intuitionistic generalized closure operator $I\,cl^*$ is defined by the intersection of all I g-closed sets containing A [4]. For the subset A of ITS (X, T), the semi-closure of $A(I\,scl(A))$ is defined as the intersection of all I semi-closed sets containing A [6]. For the subset A of ITS (X, T), the semipre-closure of $A(I\,spcl(A))$ is defined as the intersection of all I semipre-closed sets containing A [5]. For the subset A of ITS (X, T), the presemi-closure of $A(I\,pscl(A))$ is defined as the intersection of all I presemi-closed sets containing A [1].

P- ISSN 1991-8941 E-ISSN 2706-6703 Journal of University of Anbar for Pure Science (JUAPS) Open Access
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and $\alpha_{20} = \langle x, \emptyset, \{b, c\}\rangle$. then A is I sp-closed set, since $I\ \mathrm{int}\left(I\ \mathrm{cl}(I\ \mathrm{int}(A))\right) = A$ is subset of A. But A is not T^*-Ig-closed, since for all T^*-I open set in X are A, C and

α_7 $I\ \mathrm{cl}^*(A) = \alpha_1 \nsubseteq A, C$ or α_7.

The next example gives a T^*-Ig-closed set, which is not I sp-closed set.

Example 3.10 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a\}, \{b, c\}\rangle$.

$spoX = T \cup \{B, C, D, E, F, R, H, I, J, K, L, M, N, P, Q\}$, where

$B = \langle x, \{a\}, \emptyset\rangle$; $C = \langle x, \{a\}, \{b\}\rangle$; $D = \langle x, \{a\}, \{c\}\rangle$; $E = \langle x, \{a, c\}, \emptyset\rangle$; $F = \langle x, \{a, c\}, \{b\}\rangle$; $R = \langle x, \{a, b\}, \emptyset\rangle$; $H = \langle x, \{b\}, \emptyset\rangle$; $I = \langle x, \{c\}, \emptyset\rangle$; $J = \langle x, \emptyset, \{b, c\}\rangle$; $K = \langle x, \emptyset, \{b\}\rangle$; $L = \langle x, \{b\}, \{c\}\rangle$; $M = \langle x, \emptyset, \{c\}\rangle$; $N = \langle x, \{c\}, \{b\}\rangle$; $P = \langle x, \{b, c\}, \emptyset\rangle$

and $Q = \langle x, \{a, b\}, \{c\}\rangle$. A is T^*-Ig-closed, since for all T^*-I open set in X that contain A are B, C and D then $I\ \mathrm{cl}^*(A) = A$ is subset of B, C and D. But A is not I sp-closed.

Example 3.11 Recall example 3.7 we can get the following that T^*-Ig-closed set, but not I presemi-closed. $psoX = T \cup \{Z_3, Z_4\}$ Let $Z_1 = \langle x, \{a\}, \{b\}\rangle$ is T^*-Ig-closed. Since for all T^*-I open set in X is A, Z_3 and Z_4 then $I\ \mathrm{cl}^*(Z_1) = Z_1$ are subset of A, Z_3 or Z_4. But Z_1 is not I ps-closed.

We show in this example that I presemi-closed set, need not be T^*-Ig-closed.

Example 3.12 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \emptyset\rangle$ and $B = \langle x, \{a, b\}, \emptyset\rangle$ $psoX = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $C = \langle x, \{a, c\}, \emptyset\rangle$. Let $\bar{A} = \langle x, \emptyset, \{a\}\rangle$ is I presemi-closed set. But $\bar{A}$ is not T^*-Ig-closed set, since the only T^*-I open set that contain $\bar{A}$ are A and B then $I\ \mathrm{cl}^*(\bar{A}) = \langle x, \{c\}, \{a\}\rangle \nsubseteq A$ or B.

Example 3.13 Recall example 3.9 it is clear that Isg-closed set, but not T^*-Ig-closed set. Since for all I semi-open sets $A \subseteq A, C, \alpha_1$ and α_2 then $I\ \mathrm{scl}(A) = A$ are subsets of A, C, α_1 and α_2. But is not T^*-Ig-closed.

Example 3.5 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{b\}, \{a, c\}\rangle$, $B = \langle x, \{b\}, \emptyset\rangle$. Then B is T^*-Ig-closed set in X, since for all T^*-I open in X that contained B is Ig-closed set C and D where $C = \langle x, \{a, b\}, \emptyset\rangle$, $D = \langle x, \{b, c\}, \emptyset\rangle$ then $I\ \mathrm{cl}^*(B) = B$ are subsets of C or D, but B is not I closed set.

Proposition 3.6 Every Ig-closed set in X is T^*-Ig-closed set.

Proof Let A be Ig-closed set in X. Let $A \subseteq U$ where U is T^*-I open in X. Then $I\ \mathrm{cl}(A) \subseteq U$, since A is Ig-closed, $I\ \mathrm{cl}^*(A) \subseteq I\ \mathrm{cl}(A)$.

Therefore $I\ \mathrm{cl}^*(A) \subseteq U$.

Hence A is T^*-Ig-closed.

The converse of the above proposition need not be true as the following example shows.

Example 3.7 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{a\}, \emptyset\rangle$. A is T^*-Ig-closed set in X, since the only T^*-I open in X that contain A are Z_3 and Z_4 where $Z_3 = \langle x, \{a, c\}, \emptyset\rangle$, $Z_4 = \langle x, \{a, b\}, \emptyset\rangle$, and $I\ \mathrm{cl}^*(A) = A$ which are subset of Z_3 or Z_4, but A is not Ig-closed, since the only I open in X that contain A is A only, but $\mathrm{cl}(A) = X$ is not subset of A.

Remark 3.8 The notion T^*-Ig-closed set in ITS is independent from I sp-closed set, Isg-closed set, I ps-closed set, I pre-closed set, Igs-closed set, Igsp-closed set, Ips$g$-closed set and Igps-closed set.

The following example shows that there is an I sp-closed set, which is not T^*-Ig-closed set.

Example 3.9 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B, C\}$ where $A = \langle x, \{c\}, \{a, b\}\rangle$; $B = \langle x, \{a\}, \{b, c\}\rangle$. and $C = \langle x, \{a, c\}, \{b\}\rangle$.

$spoX = T \cup \{\alpha_i\}_{i=1}^{20}$, where

$\alpha_1 = \langle x, \{c\}, \{a\}\rangle$; $\alpha_2 = \langle x, \{a, c\}, \emptyset\rangle$; $\alpha_3 = \langle x, \{b, c\}, \{a\}\rangle$; $\alpha_4 = \langle x, \{a\}, \{c\}\rangle$; $\alpha_5 = \langle x, \{a, b\}, \{c\}\rangle$; $\alpha_6 = \langle x, \{c\}, \emptyset\rangle$; $\alpha_7 = \langle x, \{c\}, \{b\}\rangle$; $\alpha_8 = \langle x, \{a\}, \{b\}\rangle$; $\alpha_9 = \langle x, \{a\}, \emptyset\rangle$; $\alpha_{10} = \langle x, \{b, c\}, \emptyset\rangle$; $\alpha_{11} = \langle x, \{a, b\}, \emptyset\rangle$; $\alpha_{12} = \langle x, \{b\}, \emptyset\rangle$; $\alpha_{13} = \langle x, \{b\}, \{a\}\rangle$; $\alpha_{14} = \langle x, \{b\}, \{c\}\rangle$; $\alpha_{15} = \langle x, \{b\}, \{a, c\}\rangle$; $\alpha_{16} = \langle x, \emptyset, \{a\}\rangle$; $\alpha_{17} = \langle x, \emptyset, \{b\}\rangle$; $\alpha_{18} = \langle x, \emptyset, \{c\}\rangle$; $\alpha_{19} = \langle x, \emptyset, \{a, b\}\rangle$

Example 3.17 Recall example 3.10 it is clear that A is T^*-Ig-closed set, but not Igs-closed set .

Example 3.18 Recall example 3.9 we see that $A = \langle x, \{c\}, \{a, b\} \rangle$ is I$g$sp-closed set, but not T^*-Ig-closed set .

Recall example 3.15 show that A is T^*-Ig-closed set, but not I$g$sp-closed set.

In the following example we show:

1- Ips$g$-closed set, but not T^*-Ig-closed set.

2- I$g$ps-closed set, but not T^*-Ig-closed set.

Example 3.19 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{c\}, \{a, b\} \rangle$ and $B = \langle x, \{b, c\}, \{a\} \rangle$

$psoX = T \cup \{\beta_i\}_{i=1}^{6}$, where

$\beta_1 = \langle x, \{c\}, \emptyset \rangle$; $\beta_2 = \langle x, \{c\}, \{a\} \rangle$; $\beta_3 = \langle x, \{c\}, \{b\} \rangle$; $\beta_4 = \langle x, \{a, c\}, \emptyset \rangle$; $\beta_5 = \langle x, \{b, c\}, \emptyset \rangle$

and $\beta_6 = \langle x, \{a, c\}, \{b\} \rangle$. we can see that A is I ps$g$-closed and I$g$ps-closed. But not T^*-Ig-closed because the only T^*-I open in X that contain β_7 is B $I\ cl^*(\beta_7) = \langle x, \{b\}, \{c\} \rangle \not\subseteq B$.

Example 3.20 Recall example 3.7 it is clear that A is T^*-Ig-closed set, but not Ips$g$-closed set .

Recall example 3.7

$psoX = T \cup \{Z_2, Z_3\}$, where $Z_2 = \langle x, \{a\}, \{c\} \rangle$ and $Z_3 = \langle x, \{a, c\}, \emptyset \rangle$. we can see that $Z_1 = \langle x, \{a\}, \{b\} \rangle$ is T^*-Ig-closed set but not I$g$ps-closed set .

Proposition 3.21 Let (X, T) be ITS, let A and B be IS in X then $I\ cl^*(A \cup B) = I\ cl^*(A) \cup I\ cl^*(B)$.

Proof Since $A \subseteq A \cup B$, then $I\ cl^*(A) \subseteq I\ cl^*(A \cup B)$ and since $B \subseteq A \cup B$, then $I\ cl^*(B) \subseteq I\ cl^*(A \cup B)$. Therefore $I\ cl^*(A) \cup I\ cl^*(B) \subseteq I\ cl^*(A \cup B)$.

Also $I\ cl^*(A)$ and $I\ cl^*(B)$ are I closed. Therefore $I\ cl^*(A) \cup I\ cl^*(B)$ is also a I closed, $A \subseteq I\ cl^*(A)$ and $B \subseteq I\ cl^*(B)$ implies $A \cup B \subseteq I\ cl^*(A) \cup I\ cl^*(B)$, Thus, $I\ cl^*(A) \cup I\ cl^*(B)$is I closed set containing $A \cup B$. Since $I\ cl^*(A \cup B)$ is the smallest I closed set containing $A \cup B$ we have $I\ cl^*(A \cup B) \subseteq I\ cl^*(A) \cup I\ cl^*(B)$. Thus, $I\ cl^*(A \cup B) = I\ cl^*(A) \cup I\ cl^*(B)$.

Recall example 3.10 we see that A is T^*-Ig-closed set, but not Isg-closed.

$SoX = T \cup \{B, C, D, E, F, R, Q\}$

In the next example we show that I pre-closed set need not be T^*-Ig-closed set.

Example 3.14 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\} \rangle$, $B = \langle x, \{a, b\}, \emptyset \rangle$.

$poX = T \cup \{C, D, E, F, G, H, I, J, K, L, M, O, S, W, N, P, Y\}$

where

$C = \langle x, \{a\}, \emptyset \rangle$; $D = \langle x, \{a\}, \{c\} \rangle$; $E = \langle x, \{a, b\}, \{c\} \rangle$; $F = \langle x, \{a, c\}, \emptyset \rangle$; $G = \langle x, \{a, c\}, \{b\} \rangle$; $H = \langle x, \{a\}, \{b, c\} \rangle$; $I = \langle x, \{b, c\}, \emptyset \rangle$; $J = \langle x, \{b, c\}, \emptyset \rangle$; $K = \langle x, \{c\}, \emptyset \rangle$; $L = \langle x, \{c\}, \{a\} \rangle$; $M = \langle x, \{c\}, \{b\} \rangle$; $O = \langle x, \emptyset, \{b, c\} \rangle$; $S = \langle x, \emptyset, \{b\} \rangle$; $W = \langle x, \emptyset, \{c\} \rangle$; $N = \langle x, \{c\}, \{a, b\} \rangle$; $P = \langle x, \{b\}, \emptyset \rangle$

and $Y = \langle x, \{b\}, \{c\} \rangle$. D is I pre-closed set in X. But not T^*-Ig-closed, since the only T^*-I open set that contains D is B then $I\ cl^*(D) = F \not\subseteq B$.

we show in the next example that T^*-Ig-closed set, need not be I pre-closed set.

Example 3.15 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A\}$ where $A = \langle x, \{b\}, \{a, c\} \rangle$. $poX = T \cup \{Q_i\}_{i=1}^{20}$, where

$Q_1 = \langle x, \{b\}, \emptyset \rangle$; $Q_2 = \langle x, \{b\}, \{a\} \rangle$; $Q_3 = \langle x, \{b\}, \{c\} \rangle$; $Q_4 = \langle x, \{a, b\}, \emptyset \rangle$; $Q_5 = \langle x, \{a, b\}, \{c\} \rangle$; $Q_6 = \langle x, \{b, c\}, \{a\} \rangle$; $Q_7 = \langle x, \{b, c\}, \emptyset \rangle$; $Q_8 = \langle x, \{a\}, \emptyset \rangle$; $Q_9 = \langle x, \{a\}, \{c\} \rangle$; $Q_{10} = \langle x, \{a, c\}, \emptyset \rangle$; $Q_{11} = \langle x, \{c\}, \emptyset \rangle$; $Q_{12} = \langle x, \{c\}, \{a\} \rangle$; $Q_{13} = \langle x, \emptyset, \{a\} \rangle$; $Q_{14} = \langle x, \emptyset, \{c\} \rangle$

and $Q_{15} = \langle x, \emptyset, \{a, c\} \rangle$. A is T^*-Ig-closed set, since the only T^*-I open in X that contained A are A, Q_1, Q_2 or Q_3. But not I pre-closed.

In the following example we give an Igs-closed set, which is not T^*-Ig-closed set.

Example 3.16 Let $X = \{a, b, c\}$; $T = \{\tilde{\emptyset}, \tilde{X}, A, B\}$ where $A = \langle x, \{a\}, \{b\} \rangle$ and $B = \langle x, \{a, b\}, \emptyset \rangle$. Let $Z_1 = \langle x, \{a\}, \{b\} \rangle$ is Igs-closed set but not T^*-Ig-closed since the only T^*-I open set in X that contain Z_1 are A and B then $I\ cl^*(Z_1) = \langle x, \{b, c\}, \emptyset \rangle$ are not subset of A or B.

Proposition 3.26 Let (X, T) be ITS and let C be IS in X then C is T^*-Ig-closed and $C \subseteq D \subseteq I\ cl^*(C)$, then D is T^*-Ig-closed set in X.

Proof Let C be a T^*-Ig-closed such that $C \subseteq D \subseteq I\ cl^*(C)$. Let U be a T^*-I open set of X such that $D \subseteq U$. since C is T^*-Ig-closed, we have $I\ cl^*(C) \subseteq U$.
$I\ cl^*(C) \subseteq I\ cl^*(D) \subseteq I\ cl^*(I\ cl^*(C)) = I\ cl^*(C) \subseteq U$. That is $I\ cl^*(D) \subseteq U$, and U is T^*-I open. Therefore D is T^*-Ig-closed set in X.

The converse of the above proposition is not true in general as the following example shows.

Example 3.20 Recall example 3.15 let $C = \langle x, \{a, c\}, \{b\} \rangle$ and $D = \langle x, \{a, c\}, \emptyset \rangle$. Then C and D are T^*-Ig-closed set in X. Since for all T^*-I open in X $C, D \subseteq D$ then $I\ cl^*(C) = C$ and $I\ cl^*(D) = D$ is a subset of D. But $C \subseteq D$ which is not a subset of $I\ cl^*(C)$.

We end this paper by the following characterization.

Theorem 3.23 Let (X, T) be ITS, let A be IS in X then A is Ig-closed if and only if $I\ cl^*(A) - A$ is T^*-I open.

Proof Let A be Ig-closed in X, then $cl^*(A) = A$ and $cl^*(A) - A = \emptyset$ which is T^*-I open in X.

Conversely

Let $I\ cl^*(A) - A$ is T^*-I open in X. Since A is T^*-Ig-closed so, by the theorem 3.23, $I\ cl^*(A) - A$ contains no non-empty T^*-I closed set in X. Then $cl^*(A) - A = \emptyset$ hence A is Ig-closed.

References

[1]. Bhattacharyya, P. and Lahiri, B.K. "Semi generalized closed sets in topology" Indian J. Math. 29 (1987), pp. 376-382.

[2]. Gao, J. Ganster, M. and Reilly, I. "Submaximality, Extremal disconnectedness and generalized closed sets" Houston, J. of Math. Vol. 24, No. 4 (1998), pp. 681-688.

[3]. Cueva, M.C. " Semi-generalized continuous maps in topological spaces" Portugalie, Math. Vol. 52 Fasc. 4. (1995).

Proposition 3.22 Let (X, T) be ITS, then the union of T^*-Ig-closed sets in X is a T^*-Ig-closed set in X.

Proof Let A and B be two T^*-Ig-closed sets. Let $A \cup B \subseteq U$, where U is T^*-I open. Since A and B are T^*-Ig-closed sets, $I\ cl^*(A) \cup I\ cl^*(B) \subseteq U$. But by Proposition 3.21 $I\ cl^*(A \cup B) = I\ cl^*(A) \cup I\ cl^*(B)$. Therefore $I\ cl^*(A \cup B) \subseteq U$. Hence $A \cup B$ is a T^*-Ig-closed sets.

The next theorem is a characterization of T^*- Ig-closed set.

Theorem 3.23 Let (X, T) be ITS and let A be IS in X then A is T^*-Ig-closed if and only if $I\ cl^*(A) - A$ does not contain any non-empty T^*-I closed set in X.

Proof Let A be a T^*-Ig-closed set Suppose that F is a non-empty T^*-I closed subset of X such that $F \subseteq I\ cl^*(A) - A$ then $F \subseteq Icl^*(A) \cap A^c$, since $Icl^*(A) - A = Icl^*(A) \cap A^c$. Therefore $F \subseteq I\ cl^*(A)$ and $F \subseteq A^c$ Since $\bar{F}$ is a T^*-I open set and A is a T^*-Ig-closed, $I\ cl^*(A) \subseteq \bar{F}$. That is then $F \subseteq (Icl^*(A))^c$. Hence $F \subseteq Icl^*(A) \cap (Icl^*(A))^c = \emptyset$ That is $F = \emptyset$, Thus $I\ cl^*(A) - A$ contain no non-empty T^*-I closed set in X.

Conversely A assume that $cl^*(A) - A$ contains no nonempty T^*-Iclosed set. Let $A \subseteq U$ and U is T^*-I open. Let $I\ cl^*(A)$ be not contained in U, then $I\ cl^*(A) \cap \bar{U}$ is a non-empty T^*-Iclosed set of $I\ cl^*(A) - A$ which is a contradiction. Therefore $I\ cl^*(A) \subseteq U$ and hence A is T^*-Ig-closed.

Corollary 3.24 Let (X, T) be ITS and let A be IS in X then A is T^*-Ig-closed if and only if $I\ cl^*(A) - A$ contains no non-empty I closed set in X.

Proof By Theorem 3.23 and the fact that every I closed set is T^*-I closed set in X.

Corollary 3.25 Let (X, T) be ITS and let A be IS in X then A is T^*-Ig-closed if and only if $I\ cl^*(A) - A$ contains no non-empty I open set in X.

Proof By Theorem 3.23 and the fact that every I open set is T^*-I open set in X.

Mem. Fac. Sci. Kochi Univ. Math. 15 (1994), pp. 51-63.

[10].Navalagi, G.B. "Semipre-continuous function and properties of generalized semipro-closed sets in topological spaces" Int. J. MMS 29,2 (2002), pp. 85-98.

[11].Pushpalatha, A. Eswaran, S. and Rajar, P. "T^*-generalized closed sets in topological spaces" pro. of W. con. on Engineering (2009), ISBN. 978-988.

[12].Raouf, G. A. "On generalized homomeorphism between ITS" MSc. Tikrit uni. Thesis coll. of Education (2008).

[4]. Dunham, W. "A new closure operator for non-T_1topologies" Kyungpook Math. J. 22 (1982), pp. 55-60.

[5]. Dontcher, J. and Maki, H. "On θ-generalized closed set" Int. J. Math. And Math. Sci. Vol. 22, No. 2 (1999), pp. 239-249.

[6]. Gananambal, Y. "On generalized closed sets in topological spaces" Indian J. prue apple Math. 28, No. 3 (1997), pp. 351-360.

[7]. Levine, N. "Generalized closed sets in topology" Rend. Circ. Mat. Palermo, 19, 2 (1970), pp. 89-96.

[8]. Levine, N. "Semi-open sets and semi-continuity in topological spaces" Amer. Math. Monthly, 70 (1963), pp. 36-41.

[9]. Maki, H. Devi, R. and Balachandran, K. "Assiciated topologies of generalized presemi-closed sets and presemi-generalized closed sets"

تعميم حدسي جديد للمجموعات المغلقة في الفضاءات التبولوجية الحدسية

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الخلاصة

ان الهدف من هذا البحث هو اعطاء صنفين جديدين من المجموعات الحدسية وأسميناها تعميم المجموعات المغلقة الحدسية – T^* وتعميم المجموعات المفتوحة الحدسية – T^* في الفضاءات التبولوجية الحدسية ودرسنا بعض صفاتهما.