LQR Controller for Kufasat

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Abstract

In this paper, Linear Quadratic Regulator (LQR)controller is applied to the attitude stabilization control of Kufasat. Using the linearized equations of motion for a rigid body in space, the linearized stability, effectiveness and robustness of a linear quadratic regulator (LQR) control design were compared with that of a Proportional-Integral-Derivative (PID) control design. The detailed design procedure of the LQR controller is presented. Simulation results show that precise attitude control is accomplished and the time of satellite maneuver is shortened in spite of the uncertainty in the system.

Key words: Linear Quadratic Regulator , Kufasat

نظام السيطرة (LQR) للقمر الصناعي كوفة- سات

محمد جساب مهدي محمد جعفر البيرماني

الخلاصة

كوفة سات هو برنامج قمر صناعي نانوي للطلبة , الهدف من هذا البرنامج هو تصميم واطلاق قمر صناعي مكعب ذو استقرارية المحاور الثلاثة وتأثير عزم انحدار جاذبية الارض ابعاده (10×10×10) سم وكتلته 1كغم تقريبا يدور في مدار ارضي منخفض على ارتفاع 600 كم وتكون مهمة هذا القمر هي تصوير سطح الارض باستخدام كاميره سي سي دي تعمل بالطيف المرئي والاشعه تحت الحمراء . الغرض من هذه الدراسة هو تصميم وتطوير نظام كفوء لتحديد الاتجاه والسيطرة عليه . ان الاستقراريه السلبيه بتاثير عزم انحدار جاذبية الارض تكون محدوده لذلك يتم دعمها باستقراريه فعاله وذلك باضافة ملفات مغناطيسيه حيث يوضع ثلاث ملفات مغناطيسيه على المحاور الثلاث لجسم القمر يتم تغذية هذه الملفات بتيار ثابت القيمه ذو اتجاهين فيولد مجالا مغناطيسيا يتفاعل مع المجال المغناطيسي الارضي فيولد عزما دورانيا يستخدم للسيطره على القمر لتدويره الى الوضع المطلوب .

تم استخدام طريقتين للسيطره على تيار الملفات المغناطيسية وهي السيطره باستخدام المسيطرالتناسبي- التكاملي- التفاضلي والسيطره المثلى باستخدام المقوم الخطي ذوالدرجةالثانية . تعتمد الطريقتين على النموذج الرياضي للقمر الصناعي الذي اشتقاقه .عند انفصال القمر عن المركبه الناقله يتعرض الى قوى بسبب تحرره من وصله الاطلاق مما يؤدي الى دحرجته مما يستدعي تفعيل طور ازالة الدحرجه وعندما تصبح حركة القمر قليله يتم نشر البوم المسيطرات يبدأ عملها بعد انتهاء طور ازالة الدحرجه وبعد انتشار البوم بشكل كامل . تم من خلال النتائج ملاحظة الاداء الجيد لطريقة السيطره المثلى باستخدام المقوم الخطي ذوالدرجةالثانية من خلال الاستجابه الجيده والسريعه وثبوت الحاله النهائيه اضافه الى قابليته في تنفيذ المناورات بالزوايا الكبيره والصغيره .

كلمات مفتاحية: قمر الكوفة الصناعي ، منظومة السيطرة التربيعية الخطية

1-Introduction

The Iraqi student satellite project kufasat was started at 2012. The launch of the satellite is planned for late 2016. The main tasks for kufasat will be to perform scientific measurements. The project is sponsored by the University of Kufa and it will be the first Iraqi satellite. Kufasat is a nano-satellite based on the cubesat concept. This means that its mass is restricted to 1 kg, and its size is restricted to a cube measuring 10×10×10 cm. It also contains 1.5m long gravity boom, which will be used for passive attitude stabilization. The satellite attitude control problem includes attitude stabilization and attitude maneuver. Attitude stabilization is the process of keeping original attitude and the attitude maneuver is the re-orientation process of changing one attitude to another [1] .In general, attitude stabilization systems are classified as active or passive. The simplicity and low cost of active magnetic control makes it an attractive option for small satellites in Low Earth Orbit (LEO).

A gravity gradient stabilized satellite has limited stability and pointing capabilities so, magnetic coils are added to improve both the three axis stabilization and the pointing properties. Magnetic coils around the satellite's XYZ axes can be fed with a constant current-switched in two directions-to generate a magnetic dipole moment M which will interact with the geomagnetic field vector B to generate a satellite torque N by taking the cross product[2]:

$$N = M \times B \qquad (1)$$

This torque is used to control the rotation of the satellite. The magnetic coils are controlled using LQR controller. Gravity gradient stabilization has been used in attitude control since the early sixties [3] , but accurate three-axis control has not been achieved using gravity gradient stabilization alone. Gravity gradient stabilization combined with magnetic torqueing, has gained increased attention as an attractive attitude control system (ACS) for small cheap satellites and is also proposed used in this satellite [4].

A problem is that both the direction and the strength of the geomagnetic field change and magnetic control become non-linear and time dependent. Attitude control with high accuracy cannot be achieved because the magnetic torques are constrained on a plane perpendicular to the local magnetic field .In this paper, a comparison between two attitude control laws that have been suggested used for kufasat. This paper proposes PID and LQR controller.

2- Dynamic model

The mathematical model of a satellite is described by the dynamic equations and kinematic equations of motion [5].The dynamic equation of motion for a satellite in low earth orbit is

$$^{b}T = I\,\dot{\omega}_{b/I} + {^{b}\omega_{b/I}} \times \left(I\ {^{b}\omega_{b/I}}\right) \qquad (2)$$

where

${^{b}\omega_{b/I}}$ is the angular velocity of body frame relative to an inertial frame . I is the moment of inertia matrix refer to body frame , $I = \mathrm{diag}[I_x \quad I_y \quad I_z]$. ${^{b}T}$ is total torque acting on satellite expressed in body frame components, which is consist of gravity gradient torque, magnetic torque and disturbance torque.

$$^{b}T = {^{b}T_G} + {^{b}T_m} + {^{b}T_D} \qquad (3)$$

Equation (1) can be expanded in components; we have three dynamic equations for the roll, pitch, yaw axes respectively as follows:

$$T_x = \dot{\omega_x} I_x + \left(I_z - I_y\right)\omega_y\omega_z \qquad (4a)$$
$$T_y = \dot{\omega_y} I_y + (I_x - I_z)\omega_z\omega_x \qquad (4b)$$
$$T_z = \dot{\omega_z} I_z + \left(I_y - I_x\right)\omega_x\omega_y \qquad (4c)$$

Where $\omega_x, \omega_y, \omega_z$ are angular velocities of body frame and I_x, I_y, I_z are the moment of inertia in body frame and T_x, T_y, T_z are the torques expressed in body frame components. These three equations are known as Euler's equations of motion for a rigid body [6]. If the Euler angles ϕ, θ, ψ are small in magnitude, the relationship between body angular velocities and Euler angular velocities may be approximated[7] as,

$$\begin{bmatrix}\omega_x\\ \omega_y\\ \omega_z\end{bmatrix} = \begin{bmatrix}\dot{\phi} + \omega_o\psi\\ \dot{\theta} - \omega_o\\ \dot{\psi} - \omega_o\phi\end{bmatrix} \qquad (5)$$

Gravity Gradient Torque:

The gravity gradient torque, using a small Euler angle approximation and taking principal axes as reference axis is given [3] by:

$$\left.\begin{array}{l} T_{Gx} = 3\omega_0^2(I_z - I_y)\\ T_{Gy} = 3\omega_0^2(I_z - I_x)\theta\\ T_{Gz} = 0 \end{array}\right] \qquad (6)$$

Where T_{Gx}, T_{Gy}, T_{Gz} are the gravity gradient torque about the Roll , Pitch , Yaw axis, respectively .

Magnetic Field Torque:

The magnetic coil produces a magnetic dipole when currents flow through its windings, which is proportional to the ampere-turns and the area enclosed by the coil. The torque generated by the magnetic coils can be modeled as:

$$T_m^b = m^b \times B^b \qquad (7)$$

Where m^b is the generated magnetic moment inside the body and $B^b = [B_x^b B_y^b B_z^b]^T$. is the local geomagnetic field vector

$$m^b = m_x^b + m_y^b + m_z^b = \begin{bmatrix}N_x i_x A_x\\ N_y i_y A_y\\ N_z i_z A_z\end{bmatrix} = \begin{bmatrix}m_x\\ m_y\\ m_z\end{bmatrix}$$

where N_k is number of windings in the magnetic coil, i_k is the coil current and A_k is the span area of the coil. The magnetic torque can be represented as:

$$\begin{bmatrix}T_{mx}\\ T_{my}\\ T_{mz}\end{bmatrix} = \begin{bmatrix}m_y B_\psi - m_z B_\theta\\ m_z B_\varphi - m_x B_\psi\\ m_x B_\theta - m_y B_\varphi\end{bmatrix} \qquad (8)$$

Where T_{mx} , T_{my} , T_{mz} are the magnetic torque about the Roll , Pitch , Yaw axes , respectively , and mx , my , mz are the corresponding of the magnetic moments and B_φ , B_θ , B_ψ is the earth's magnetic field affects the Roll , Pitch and Yaw axis respectively .After adding equation (6) and equation (8)to equation (4) the final form of linearized attitude dynamic model of the satellite including gravity gradient torque and magnetic coil torque written in body frame components becomes

$$\ddot{\phi} = \left(-\frac{4\omega_0^2(I_y - I_z)}{I_x}\right)\phi + \left(\frac{\omega_0(I_x - I_y + I_z)}{I_x}\right)\dot{\psi} + (m_y B_\psi - m_z B_\theta)/I_x \qquad \text{(Roll)} \quad (9a)$$

$$\ddot{\theta} = \left(-\frac{3\omega_0^2(I_x - I_z)}{I_y}\right)\theta + (m_z B_\varphi - m_x B_\psi)/I_y \qquad \text{(Pitch)} \quad (9b)$$

$$\ddot{\psi} = \left(-\frac{\omega_0^2(I_y - I_x)}{I_z}\right)\psi - \left(\frac{\omega_0(I_x - I_y + I_z)}{I_z}\right)\dot{\phi} + (m_x B_\theta - m_y B_\varphi)/I_z \qquad \text{(Yaw)} \quad (9c)$$

If the states are given as:

$x_1 = [\varphi]; x_4 = [\dot{\varphi}]$ (Roll)

$x_2 = [\theta]; x_5 = [\dot{\theta}]$ (Pitch)

$x_3 = [\psi]$; $x_6 = [\dot{\psi}]$ (Yaw)

, then the linear system can be expressed as a state space model taking the form:

$$\dot{x} = A\,x + B\,u \qquad (10)$$
$$y = Cx + Du \qquad (11)$$

then equations (9 a,b,c) can be represented as a state –space model as :

$$\begin{bmatrix}\dot{x}_1\\ \dot{x}_2\\ \dot{x}_3\\ \dot{x}_4\\ \dot{x}_5\\ \dot{x}_6\end{bmatrix} = \begin{bmatrix} 0 & 0 & 0 & 1 & 0 & 0\\ 0 & 0 & 0 & 0 & 1 & 0\\ 0 & 0 & 0 & 0 & 0 & 1\\ -\frac{4\omega_o^2(I_y-I_z)}{I_x} & 0 & 0 & 0 & 0 & \frac{\omega_o(I_x-I_y+I_z)}{I_x}\\ 0 & -\frac{3\omega_o^2(I_x-I_z)}{I_y} & 0 & 0 & 0 & 0\\ 0 & 0 & -\frac{\omega_o^2(I_y-I_x)}{I_z} & -\frac{\omega_o(I_x-I_y+I_z)}{I_z} & 0 & 0 \end{bmatrix} \begin{bmatrix}x_1\\ x_2\\ x_3\\ x_4\\ x_5\\ x_6\end{bmatrix} + \begin{bmatrix} 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & 0 & 0\\ 0 & B_\psi/I_x & -B_\theta/I_x\\ -B_\psi/I_y & 0 & B_\varphi/I_y\\ B_\theta/I_z & -B_\varphi/I_z & 0 \end{bmatrix} \cdot \begin{bmatrix}m_x\\ m_y\\ m_z\end{bmatrix} \qquad (12)$$

3- PID controller design

A Proportional-Integral-Derivative controller (PID controller) is the most widely used controller with feedback mechanism. It is one of the simplest control algorithms, and in the absence of knowledge of the underlying process, PID controller is often the best choice.

A typical structure of a PID control system is shown in Figure.1, where K_p is the proportional gain ,K_d is the derivative gain , and K_i is the integral gain. By appropriately adjusting theses gains , the desired output can be achieved .It can be seen that in a PID controller, the error signal e(t)is used to generate the proportional, integral, and derivative actions, with the resulting signals weighted and summed to form the control signal u(t)applied to the plant model.

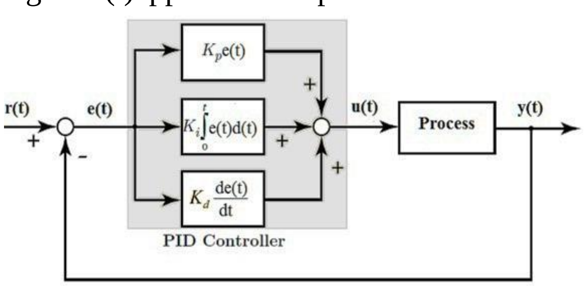


Fig (1) PID Controlled System

A mathematical description of the PID controller is [8] :

$$u(t) = K_p\left(e(t)dt + \frac{1}{T_i}\int e(t)\,dt + T_d\frac{de(t)}{dt}\right) \qquad (13)$$

where u(t)is the input signal to the plant model , the error signal e(t) is defined as e(t) = r(t) − y(t) , and r(t) is the reference input signal. In equation (13), the proportional action is related to the present error and it is used to reduce the rise time. The integral action is based on the past error and it is used to reduce the steady state error . Finally, the derivative action is related to the future behavior of error and it is used to increases the stability, reduces the overshoot and improves the transient response.

The PID controller is tuned by selecting parameters Kp, Ki , and Kd, that give an acceptable closed-loop response. A desirable response is often characterized by the measures of settling time, oscillation period, and overshoot, to mention a few. Many PID tuning methods have been proposed over the years, ranging from the simple, but most famous Ziegler-Nichols tuning method, to the more modern simple internal model control (SIMC) tuning rules by Skogestad . In this work all gains of PID controller tuned automatically in Simulink environment. The block diagram of the Simulink set up for the attitude control using PID controller is shown in Figure (2).

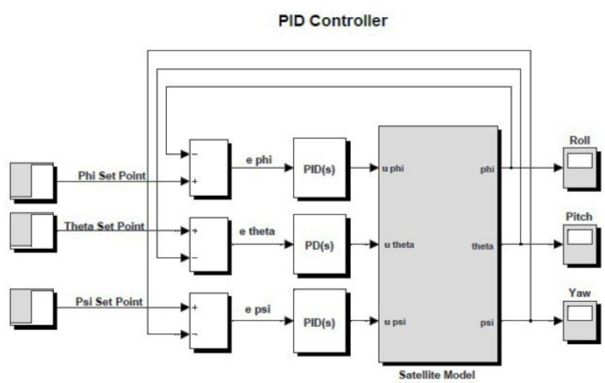


Fig (2) SIMULINK diagram of satellite model with PID controller

4- LQR controller design

The Linear Quadratic Regulator (LQR) is a powerful technique for designing controllers for complex systems that have stringent performance requirements. The standard theory of the optimal control is presented in [8,9,10]. Under the assumption that all state variables are available for feedback, the

LQR design method starts with a defined set of states which are to be controlled. In general, the system model can be written in state space equation as in equation (10)

$$\dot{x} = A\,x + B\,u \qquad (10)$$

Where: $x \in R^n$ and $u \in R^m$ denote the state variable, and control input vector, respectively. A is the state matrix of order $n \times n$; B is the control matrix of order $m \times m$.

Controllability:

The conditions of controllability may govern the existence of a complete solution to the control system design problem. The solution to this problem may not exist if the system considered is not controllable [8].The system described by Equation (10) is said to be state controllable at $t = t_o$ if it is possible to construct an unconstrained control signal that will transfer an initial state to any final state in a finite time interval $t_o \le t \le t_1$. If every state is controllable, then the system is said to be completely state controllable. The system given by equation (10) is completely state controllable if and only if the vectors B, AB, . . , A^{n-1} B are linearly independent, or the n ×n matrix[B, AB, . . . , A^{n-1} B]is of rank n [8] .

Weighting matrices Q and R determination:

The weighting matrices Q and R are important components of an LQR optimization process. The compositions of Q and R elements have great influences of system performance. The designer is free to choose the matrices Q and R, but the selection of matrices Q and R is normally based on an iterative procedure using experience and physical understanding of the problems involved. Commonly, a trial and error method has been used to construct the matrices Q and R elements. This method is very simple and very familiar in LQR application. However, it takes long time to choose the best values for matrices Q and R. The number of matrices Q and R elements are dependent on the number of state variable (n) and the number of input variable (m), respectively. The block diagram of the Simulink set up for the attitude control using LQR controller is shown in Figure (3).

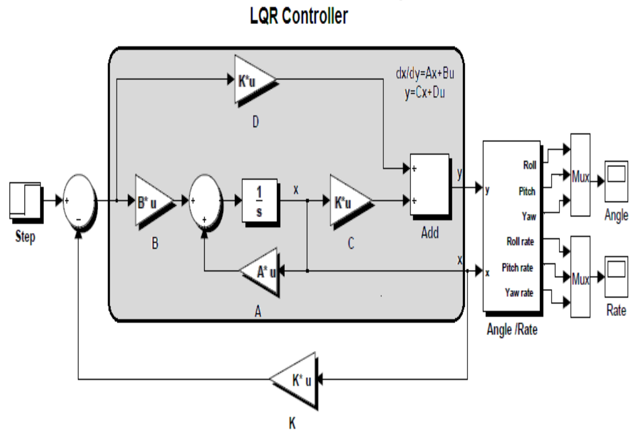


Fig (3) SIMULINK diagram of satellite model with LQR controller

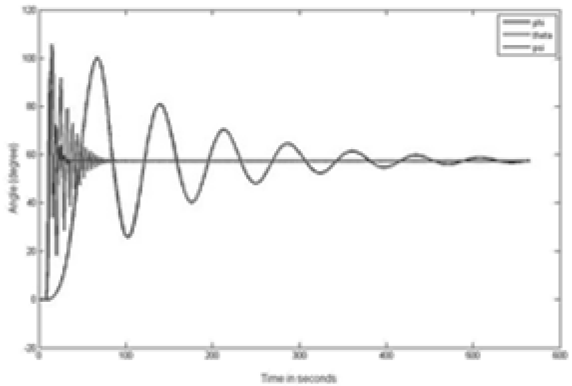
5- Simulation

In this paper, several simulations of the proposed controller have been done. The parameters values used for kufasat are listed in Table (1):

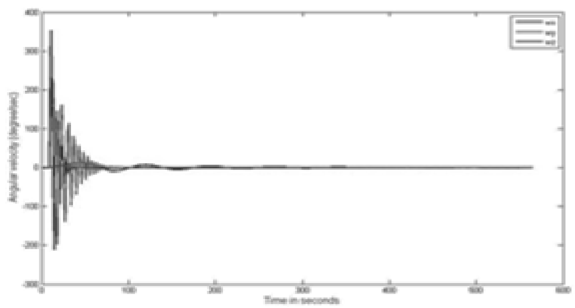
Table (1) kufasat parameters

Parameter	Value
Satellite height	600km
Weight	1 kg
Size	10 × 10 × 10 cm
Moments of inertia	Ix = 0.1043, Iy= 0.1020, Ix = 0.0031 kgm2
Boom length	1.5 m
Orbit angular velocity	1.083*10^-3
Maximum magnetic moment	0.1 Am2
Magneto-torquer	3 perpendicular magnetic coils
Desired Euler values [φ θ ψ]	[0 0 0]

A- Stabilization test

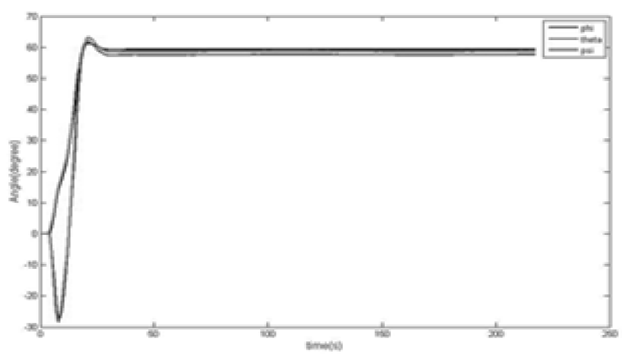


(a)

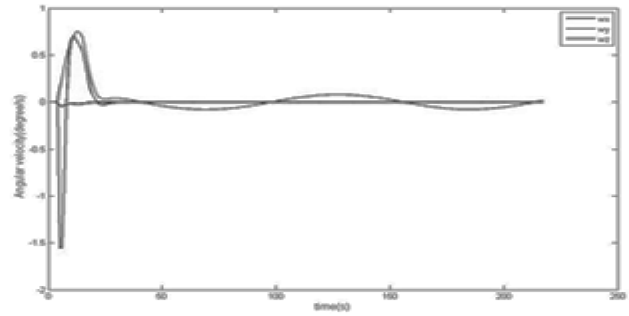


(b)

Fig(5) Attitude response for(1) rad step inputwith PID controller



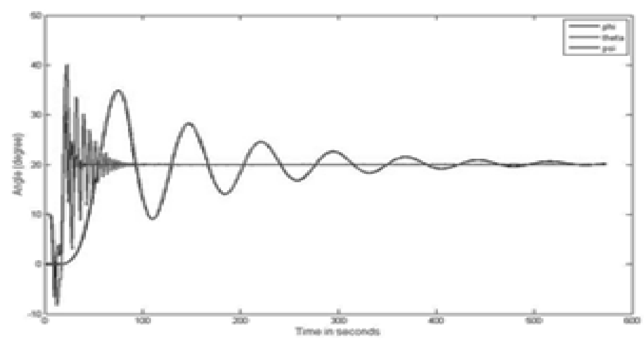
(a)



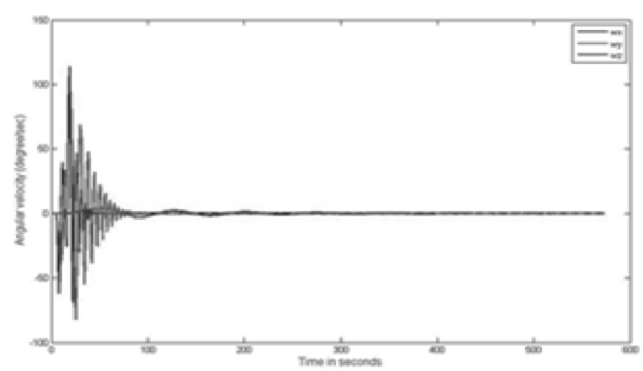
(b)

Fig(6) Attitude response for(1) rad step input with LQR controller

A- In this section PID and LQR controllers are tested to achieve different orientations. Figure (7, 8, 9,) illustrate kufasat attitude response to a small and a large ACM with PID and LQR controllers.

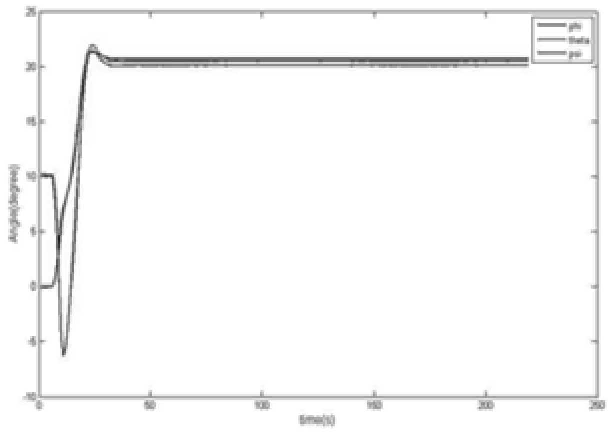


(a)

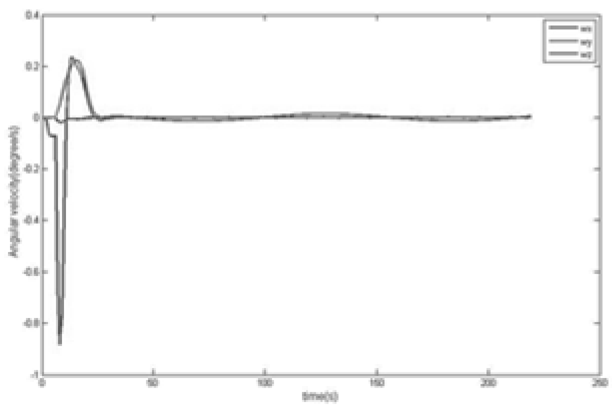


(b)

Fig (7) Response toa small ACM from[0° 10°10°] to [20° 20° 20°] with PID

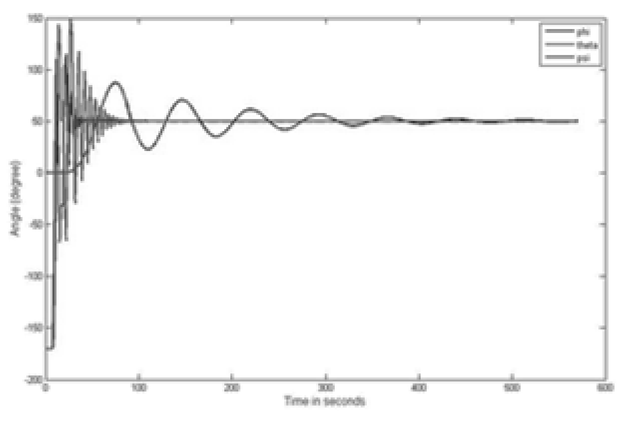


(a)

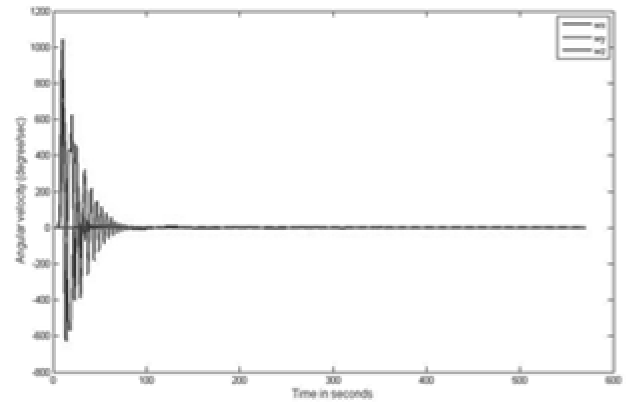


(b)

Fig (8) Response toa small ACM from[0° 10° 10°] to [20° 20° 20°] with LQR

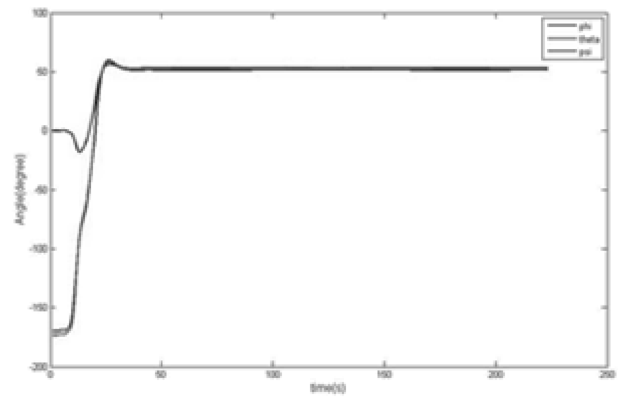


(a)

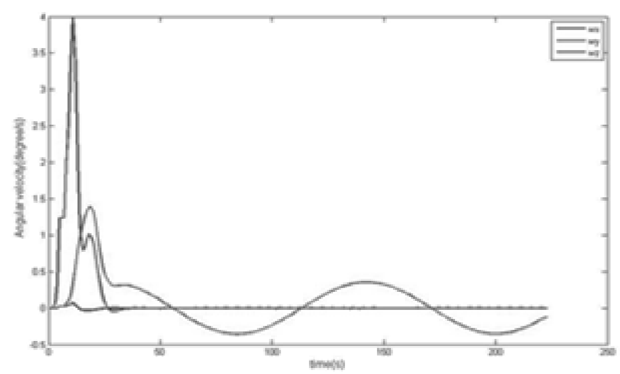


(b)

Fig (9) Response toa large ACM from [0° -170° -170°] to [50° 50° 50°] with PID



(a)



(b)

Fig (10) Response toa large ACM from [0° -170° -170°] to [50° 50° 50°] with LQR

6- Conclusion

In this paper, LQR controller for attitude control of kufasat is developed and its performance compared with the conventional PID controller. From the analysis it is observed that

1- The LQR controller was able to meet the design goals, minimum overshoot, minimum rise time and minimum steady state error.
2- The LQR has better performance in terms of percentage overshoot and rise time. It is observed that LQR is controllable and more stable than PID controller when the system is under effect of AMC. In addition to the time of satellite maneuver is shortened.
3- Even though, the PID controller produces the response with lower delay time and rise time, but it offers very high settling time due to the oscillatory behavior in transient period. It has severe oscillations with a very high peak overshoot which causes the damage in the system performance. The proposed LQR controller can effectively eliminate these dangerous oscillations and provides smooth operation in transient period.
4- Due to an onboard power limitation only one magneto-torquer coil can be switched on at a time. A control algorithm must be modified to allow for the choice of the coil that will achieve the best results, given the local geomagnetic field vector.

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