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On inductively Quasi –ĝs- closed functions

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A R T I C L E I N F O

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A B S T R A C T

The purpose of this paper is to give a new type of closed functions called inductively quasi – ĝs –closed functions, also we obtain, its characterization and its basic properties.

1.Introduction

Veerakuma . M.K.R.S.[1] introduced a new class of sets called ĝ-closed sets in topological spaces.

Also ,he introduced [2] that g^* -closed sets , g-closed sets , Between g^* -closed sets and g-closed sets .

After then Sundaram .P, Rajesh. N., Thivagar. M.L. and Dusynski,Z [5] ,they introduced ĝ semi– closed sets in topological spaces .

Also Rajesh. N. and Ekici. E, [8] had introduce anew function called quasi ĝs closed function .

Throughout this paper ,spaces means topological anew spaces and f:(X,T) → (Y,V) (or simply f: X →Y)denotes a function from a space (X,T) into a space (Y,V) , also f :X⇒Y denotes onto function ,and ĝs -closed means (generlization semi- closed for a subset A of a space X ,the closure of A denoted by cl (A),also in this paper I am introducing a new definition which is called (inductively quasi ĝs - closed function),and obtaining basic definitions and theorems for this definition.

2-Basic definitions and theorems

We would like to point out that all the definitions provided in this research has been formulated by researchers by adoption of their counterparts in topological spaces.

Definition(2-1):[3]

Let $A\subseteq X$ be a set ,we say that A is semi-open set(denoted by s-open) in X iff ∃ an open set G in X, ∍ $G\subset A\subset cl(G)$.

Definition(2-2): [3]

A subset of a space (X,T) is called semi- closure of a subset A of X,

Denoted by s cl(A),is defined to be the intersection of all s-closed sets containing A in X .

Definition(2-3): [1]

A subset A of space X is called ĝ -closed if $cl(A) \subset U$, whenever $A \subset U$ and U is s-open in X. the complement of ĝ -closed set is called ĝ -open set.

Definition(2-4): [2]

A subset A of space X is called g*- closed if $cl(A) \subset U$, whenever $A \subset U$ and U is ĝ -open in X. the complement of g* -closed set is called g* -open set.

Definition(2-5): [4]

A subset A of space X is called g#-semi-closed if $scl(A) \subset U$, whenever $A \subset U$ and U is g* -open in X. the complement of g#-closed set is called g#-open set.

Definition(2-6): [5]

A subset A of space X is called ĝ -semi-closed (briefly ĝs-closed) if $cl(A) \subset U$, whenever $A \subset U$ and U is g#-semi open in X. the complement of ĝs-closed set is called ĝs -open set.

The union(respectively intersection) ĝs-open (resp. ĝs -closed) sets , each contained in(resp. contaning)a set A

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in a spaces X is called the ĝs –interior (resp. ĝs -closure) of A and is denoted by ĝs- int (A) (resp. ĝs -cl(A))[6].

Definition(2-7): [7]

A function f:(X,T) → (Y,V) is said to be ĝs - closed if f(U)is ĝs- closed in Y for every closed subset U of X.

Definition(2-8) : [8]

A function f:(X,T) → (Y,V) is said to be quasi ĝs - closed if f(U)is closed in Y for every ĝs - closed subset U of X.

Definition(2-9): [9]

A function f:(X,T) → (Y,V) is said to be inductively closed function iff ∃ X1⊆X∍ f(X1)=f(X) and f│ X1: X1→ f(X) is closed .

Definition(2-10):

A function f:(X,T) → (Y,V) is said to be inductively quasi ĝs - closed function iff ∃ X1⊆X∍ f(X1)=f(X) and f│ X1: X1→ f(X) is quasi ĝs -. Closed

Definition(2-11) : [10]

Let f: X →Y be a function and A⊆X, a set A is said to be an inverse set if
A=f-1(f(A)).

Remark and example(2-12)

i- every quasi ĝs - closed function is closed as well as ĝs -. Closed.

ii- the converse of (i) is not true (every ĝs -. Closed (resp. Closed.) function may be not ĝs - closed function) by the following example

let X=Y={a ,b ,c }, T={Ø,X,{a ,b}} , and V={ Ø,Y,{a},{b ,c}}

define a function f:(X,T) → (Y,V) by :-

f(a)=b ,f(b)=c ,f(c)=a

f is ĝs - closed function as well as closed but not quasi ĝs - closed function .

3- Basic theorems:

Theorem(3-1)

If f: X ⇒Y is inductively quasi ĝs - closed 1-1function and A is an inverse subset of X ,then

f│ A: A→ Y is also an inductively quasi ĝs - closed function.

Proof:-

let f: X →Y be an inductively quasi ĝs - closed function,

then ∃ X1⊆X∍ f(X1)=f(X) and f│ X1: X1→ f(X) is quasi ĝs- closed

Now ,to show f│ A: A→ Y inductively quasi ĝs closed - function

Let A1⊆A ,and consider A1=A∩ X1

we need to show f(A1)= f(A) and f│ A1: A1→ f(A) is quasi ĝs - closed function.

f(A1)=f(A∩ X1)

=f(f-1(f(A)) ∩ X1)

=f(A) ∩ f(X1)

=f(A)

Now, let W be ĝs - closed in A1

So, ∃ ĝs- closed set W* in X1 ∍W= W*∩ A1

f(W)=f(W*∩A1)

=f(W*∩A∩X1)

= f(W*∩ A)

= f(W*)∩ f(A)

Since W* is ĝs- closed in X1 and f│ X1: X1→ f(X) is quasi ĝs- closed

f(W*) closed in f(X1)=f(X)

Therefore f(W*)∩ f(A) is closed in f(A)

Thus f│ A1: A1→ f(A) is quasi ĝs- closed function.

Therefore f │ A: A→ Y inductively quasi ĝs- closed function

Corollary: (3-2)

If f: X ⇒ Y is an inductively quasi ĝs- closed function and ∅≠ T ⊆ Y, then fT:f-1(T) →T is also an inductively quasi ĝs-closed function.

Proof:-

f: X ⇒ Y is inductively quasi ĝs-closed function

∃X 1 ⊆ X ∍ f(X1)=f(X) and f│ X1: X1→ f(X) is quasi ĝs-closed.

Now, to prove fT:f-1(T) →T inductively quasi ĝs-closed function

Let X* ⊆ f-1(T) ∍ X* = X 1∩ f-1(T)

Now, to show fT (X*) =T and fT│X* : X* →T is quasi ĝs-closed function.

fT (X*)=f (X*) = f(X 1∩ f-1(T))

= f(X 1)∩ T

=Y ∩ T

=T

Now ,let W be ĝs-closed in X*

Hence, ∃ ĝs-closed set W* in X1 ∍W= W*∩ X*

f(Wf(W*∩X*)

=f(W*∩X1∩f-1(T))

= f(W*∩ f-1(T))

$= f(W^*) \cap T$
Since W^* is ĝs-closed in X1 and $f|X1: X1\to f(X)$ is quasi ĝs- closed
Then $f(W^*)$ closed in Y
Therefore $f(W^*) \cap T$ is closed in T
Thus $fT|X^*: X^* \to T$ is quasi ĝs- closed function.
Therefore $fT: f\text{-}1(T) \to T$ inductively quasi ĝs- closed function.

Theorem(3-3)

Let $f: X \Rightarrow Y$ be a function ,$X=W1\cup W2$ with f(W1) and f(W2) are closed in f(X), iff $|W1: W1\to Y$ and $f|W2: W2\to Y$ are inductively quasi ĝs-closed functions .then $f: X\to Y$ is an inductively quasi ĝs-closed function.

Proof:-

$f|W1: W1\to Y$ is an inductively quasi ĝs-closed function then ,
$\exists X1\subseteq W1 \ni f(X1)=f(W1)$ and and $f|X1: X1\to f(W1)$ is quasi ĝs–closed function
also $f|W2: W2\to Y$ is an inductively quasi ĝs-closed function.
$\exists X2 \subseteq W2 \ni f(X2)\subseteq f(W2)$ and $f|X2: X2\to f(W2)$ is quasi ĝs-closed
Now to show $f: X\to Y$ is an inductively quasi ĝs-closed function.
Let $X^*= X1 \cup X2 \subseteq X$
$f(X^*)= f(X1 \cup X2)$
$= f(X1) \cup f(X2)$
$=f(W1) \cup f(W2)$
$=f(W1\cup W2)= f(X)$
So $f(X^*)= f(X)$ and to show $f|x^*: X^*\to f(X)$ is quasi ĝs –closed function
Let $\varnothing\neq T$ ĝs–closed in X^*
$T=T\cap X^*$
$=T\cap (X1 \cup X2)$
$=(T\cap X1) \cup (T\cap X2)$
$f(T)=f[(T\cap X1) \cup (T\cap X2)]$
$=f(T\cap X1) \cup f(T\cap X2)$
Since T ĝs–closed in X^*,so $T\cap X1$ ĝs –closed in X1
And $f|X1: X1\to f(W1)$ is quasi ĝs-closed function
So $f(T\cap X1)$ closed in f(W1) and f(W1) closed in f(X),
$f(T\cap X1)$ closed in f(X)
similarly $f(T\cap X2)$ closed in f(X)
$f|x^*: X^*\to f(X)$ is quasi ĝs- closed function
therefore $f: X\to Y$ is an inductively quasi ĝs-closed function.

Theorem(3-4)

If $f: X \to Y$ is an inductively quasi ĝs-closed function and $g: Y\to Z$ is a closed function ,then $g \circ f: X \to Z$ is an inductively quasi ĝs - closed function

Proof:-

since $f: X \to Y$ is an inductively quasi ĝs - closed function
then$\exists X^* \subseteq X \ni f(X^*)=f(X)$ and $f|x^*: X^*\to f(X)$ is quasi ĝs – closed function
now ,to prove $g \circ f: X \to Z$ is an inductively quasi ĝs - closed function
we need to show $gof(X^*)=gof(X)$ and $gof|x^*: X^*\to Z$ quasi ĝs -closed function
$gof(X^*)=g[f(X^*)]=g[f(X)]= gof(X)$
now ,let U be ĝs-closed set in X^* ,and since $f|x^*: X^*\to f(X)$ is quasi ĝs–closed function ,then f(U)is closed in Y
and since $g: Y\to Z$ is a closed function
then $g(f(U))=gof(U)$ is closed in Z
so $gof|x^*: X^*\to Z$ is a quasi ĝs - closed function
therefore $g \circ f: X \to Z$ is an inductively quasi ĝs- closed function.

Theorem(3-5)

Let $f: X\to Y$ and $g: Y\Rightarrow Z$ be two functions and g of $: X\to Z$ is an inductively quasi ĝs - closed function ,if g is continuous one to one function ,then f is inductively quasi ĝs-closed function .

Proof:-

to prove $f: X \to Y$ inductively quasi ĝs-closed function.
Since $gof: X\to Z$ is an inductively quasi ĝs–closed function
Then , $\exists X1 \subseteq X \ni gof(X1)=gof(X)$ and $gof|x1: X1 \to Z$ is quasi ĝs -closed function
Now, since $go f(X1)=go f(X)$
,then $g[f(X1)]=g[f(X)]$
$f(X1)=f(X)$[g is 1-1 and on to]
let G be ĝs–closed set in X 1
gof (G) is closed in Z[since and $gof: X\to Z$ is inductively quasi ĝs–closed function]
now $g: Y\Rightarrow Z$ is continuous one to one function
$f(G)=g\text{-}1(gof(G))= g\text{-}1[g[f(G)]]$ is closed in $Y=f(X)$
therefore $f: X \to Y$ is inductively quasi ĝs-closed function

Corolly (3-6)

Let f :X⇒Y and g :Y⇒Z be two inductively quasi ĝs– closed functions then

gof :X⇒Z is an inductively quasi ĝs - closed function .

Proof:-

since f: X ⇒ Y is an inductively quasi ĝs - closed function

then∃ X* ⊆ X ∋ f(X*)=f(X) and f | x*: X*⇒ f(X) is quasi ĝs – closed function

also g :Y⇒Z be two inductively quasi ĝs–closed function

then∃ Y*⊆ Y∋ f(Y*)=f(Y) and g | Y*: Y*⇒ f(Y) is quasi ĝs – closed function

to show g of :X⇒Z is inductively quasi ĝs - closed function .

we need to show g o f(X*)=g of(X) and gof| x*: X*→Z quasi ĝs - closed function

Now g[f(X*)]=g [f(X)]= g of(X)

Let W be ĝs - closed subset of X*

Since f | x*: X*⇒ f(X) is quasi ĝs – closed function

f(W) is closed in Y

and g | Y*: Y*⇒ f(Y) is quasi ĝs – closed function

so g is a closed function(by remark(2-7)

g(f(W)) is closed in Z

hence gof | x*: X*→Z is quasi ĝs - closed function

therefore g of :X⇒Z is an inductively quasi ĝs - closed function .

References

[1]- Veerakuma. M.K.R.S.,2003, "ĝ-closed sets in topological spaces ":Bull. Allahabad .Math .Soc., 18 ,99.

[2]- Veerakuma. M.K.R.S.,2006, " Between g* -closed sets and g-closed sets ." Antartica. J.Math 3 ,1,43-65.

[3]- Crossley.S.C. and Hildebrand.S.K.,1971, "Semi-closure": Texas .J.Sci.22, 99-112.

[4]- Veerakumar. M.K.R.S. 2005, "g#-semi-closed sets in topological spaces ":Antartica. J.Math 2 ,2.

[5]- Sundaram .P, Rajesh. N., Thivagar. M.L. and Dusynski,Z. , 2007, "ĝ semi– closed sets in topological spaces ": Math,Pannon,18 ,no. 1 ,51-61.

[6]- Veerakumar. M.K.R.S., " Between g* -closed sets and ĝ –sc** sets ": preprint

[7]- Rajesh. N. , Ekici. E . and Thiragar. M.L., 2006, "on ĝ-semi homeomorphisms in topological spaces " : Annals Univ. Craiova Math and Comp, Sci .Ser . 33, 208-218.

[8]- Rajesh. N. and Ekici. E, 2008, "on Quasi ĝs – open and Quasi ĝs – closed functions": Bull .Malays .Math .Sci .Soc ,(2),2-31

[9]- Levine N , 1963, "Semi open sets and semi continuity in topological spaces " : Amer. Math . Monthly ,70 , 36-41.

[10]- Ricceri ,B , 1984, "on inductively open real function" : Proc. Amer. Math. Soc. 90 ,3,485-487.

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الخلاصة

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