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Collective Excitations of ^{14}N and ^{10}B Nuclei

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ABSTRACT

The nuclear structure of ^{14}N and ^{10}B nuclei are studied in framework two collective excitation approximations. The first approximation is hole-hole Tamm-Dancoff Approximation (hh TDA) and the second is hole-hole Random Phase Approximation (hh RPA). All possible single body states of the allowed angular momenta are considered in the 1s, 1p, 2s-1d, and 1f shells. The Hamiltonian is diagonalized in the presence of Warburton and Brown interaction (WBP). The calculated compared with experimental data for the energy levels and form factors of electron scattering.

Keywords: *Nuclear structure, collective excitations, hole-hole Tamm-Dancoff Approximation hole-hole Random Phase Approximation, electron scattering.*

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الإستثارة التجمُعيّة للانوية ^{14}N و ^{10}B

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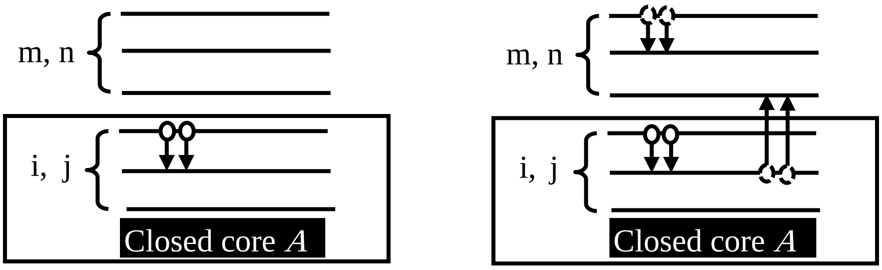
الملخص

التركيب النووي للانوية ^{14}N و ^{10}B تم دراسته في اطار تقريبين للاستثارة التجمُعيّة. التقريب الأول هو تقريب تام دانكوف فجوة – فجوة (*hh* TDA) والثاني هو تقريب الطور العشوائي فجوة–فجوة (*hh* RPA). كل مستويات الجسيم المنفرد المحتملة والمسموحة للزخم الزاوي اخذت بنظر الاعتبار في القشرات 1s، 1p، 2s-1d و 1f. الهاملتوني تم تحويله الى مصفوفة قطرية بوجود تفاعل (WBP). نتائج الحسابات لمستويات الطاقة و عوامل التشكل للأستطارة الإلكترونية تم مقارنتها مع البيانات العملية.

الكلمات الدالة: التركيب النووي، الاستثارة التجمُعيّة، تقريب تام دانكوف فجوة–فجوة، تقريب الطور العشوائي فجوة–فجوة، الأستطارة الالكترونية.

1. INTRODUCTION

A nuclear many-body problem is generally difficult to solve exactly. Various approximate approaches exist to deal with such systems. The collective excited states of closed shell nuclei can be described as a linear combination of particle-hole (*ph*) states for a truncated model space [1-7]. The nuclear charge-changing *ph* excitations correspond to the transition of nucleon from the ground-state of the nucleus (N, Z) to the final states in the neighboring nuclei $(N \mp 1, Z \pm 1)$ [8]. The generalized density random phase approximation GDRPA Hamiltonian are expressed in terms of the one-body particle-particle (*pp*) and hole-hole (*hh*) density matrices, and the nuclear force contributes not only in the *ph* channel, as in normal RPA [9]. According to the collective model, the excited states of A-2 nuclei can be described as a linear combinations of hole-hole pairs. Such an approximation is called hole-hole Tamm-Dancoff Approximation *hh* TDA [1, 3]. A system of state more general than that considered in the TDA appears when treating the ground states and the excited states more symmetrically. In that case, the ground state as well as the excited states are treated on the same footing, both the ground states and the excited states can be described as a linear combination of hole-hole states. Such an approximation is referred as the hole-hole Random Phase Approximation *hh* RPA [10-12], as shown in Fig. (1).



(a) **(b)**

Fig. (1): Collective excited hole-hole (*hh*) model space, (a) TDA and (b) RPA

In the present work, the structure of ^{14}N and ^{10}B to be studied in the framework of the *hh* TDA and *hh* RPA. The calculations within a model space with particle orbits {$1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $1f_{7/2}$} for ^{14}N and{$1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$ and $1f_{7/2}$} for ^{10}B and the hole orbits {$1s_{1/2}$, $1p_{3/2}$ and $1p_{1/2}$} for ^{14}N and {$1s_{1/2}$ and $1p_{3/2}$} for ^{10}B using Warburton and Brown interaction (WBP) [13] are performed. The 1s, 1p, 2s1d, 2p1f - shells WBP interaction is determined by least square fitting with experimental single particle energies SPE and two-body matrix element TBME. The TDA and RPA amplitudes are used to calculate the longitudinal and transverse electron scattering form factors. The results for $J_f^{\pi} T_f\ (E_x\ \text{MeV}) = 1^+0(0.0)$ state for ^{14}N and $J_f^{\pi} T_f\ (E_x\ \text{MeV}) = 1^+0(0.718)$ state for ^{10}B are interpreted in terms of the harmonic-oscillator (HO) wave functions of size parameter *b*. A comparison with the available electron scattering form factors is presented.

2. THEORY

The RPA is a generalization of TDA. This method was originally introduced by Bohm and Pines for studying the plasma oscillations of the electron gas [11]. Collective excited states of $A-2$ systems of multipolarity *J* and isospin *T* are generated by operating on the ground state of A nucleons system with annihilation operators [10]:

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$$|A-2,\lambda\rangle=\left(\sum_{i\le j}X_{ij}a_ia_j-\sum_{m\le n}Y_{mn}a_ma_n\right)|A,0\rangle \qquad \text{.....(1)}$$

Here $|A,0\rangle$ is the ground state in the close shell system of A nucleons, a is annihilation operator, indices (ij) represent the quantum numbers of orbits below the Fermi sea and we shall call refer to these states as hole states. Indices (mn) represent orbits above the Fermi sea and we call such states particle states. X and Y are amplitudes.

The compact matrix form of *hh* RPA equations has the following form:

$$\begin{pmatrix} A & B \\ B^{\dagger} & C \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix}=E_x\begin{pmatrix} 1 & 0 \\ 0 & -1 \end{pmatrix}\begin{pmatrix} X \\ Y \end{pmatrix} \qquad \text{.....(2)}$$

with

$$A_{iji'j'}=-(\varepsilon_i+\varepsilon_j)\delta_{ii'}\delta_{jj'}+V^{JT}_{iji'j'} \qquad \text{.....(3)}$$

$$B_{ijmn}=-V^{JT}_{ijmn} \qquad \text{.....(4)}$$

$$C_{mnm'n'}=(\varepsilon_m+\varepsilon_n)\delta_{mm'}\delta_{nn'}+V^{JT}_{mnm'n'} \qquad \text{.....(5)}$$

where E_x is the excitation energy, ε_i is single particle energy and $V_{iji'j'}$ is antisymmetrized two-body matrix elements. A, B and C are submatrices of dimensions $n_h\times n_h$, $n_h\times n_p$ and $n_p\times n_p$, respectively. If the sub-matrices C and B are vanished, the RPA equations will be reduced to TDA equation.

The total form factor $F(q)$ of a given multipolarity *J* and a given momentum transfer *q*, is the sum of the longitudinal (coulomb) $F_L(C)$ and transverse F_T terms [14],

$$|F(q,\theta)|^2=\frac{q_\mu^4}{q^4}|F_L(q)|^2+\left[\frac{q_\mu^4}{2q^2}+\tan^2(\frac{\theta}{2})\right]|F_T(q)|^2 \qquad \text{.....(6)}$$

where q_μ is the four momentum transfer, given by:

$$q_\mu^2=q^2-(E_i-E_f)^2 \qquad \text{.....(7)}$$

where

$$q^2=4E_iE_f\sin^2(\theta/2)+(E_i-E_f)^2 \qquad \text{.....(8)}$$

and E_i and E_f denote, respectively the initial and final total energies of the incident and scattered electron. In conventional unit the momentum transfer contains the factor $(\hbar c)^{-1}$ and

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will be given in units fm^{-1}. The transverse form factor F_T is either the sum of the electric form factors F_E or magnetic form factors F_M . The total L and T form factors are given by

$$|F_L(q)|^2 = \sum_{J\geq 0} |F_{CJ}(q)|^2 \quad \text{.......(9)}$$

$$|F_M(q)|^2 = \sum_{J>0} \left[|F_{EJ}(q)|^2 + |F_{MJ}(q)|^2 \right] \quad \text{.......(10)}$$

The form factor can be written in terms of the matrix elements reduced in spin and isospin space as [15]:

$$|F_{\Lambda J}(q)|^2 = \frac{4\pi}{Z^2(2J_i+1)} \left| (-1)^{T_f - T_{z_f}} \sum_{T=0}^{1} \begin{pmatrix} T_f & T & T_i \\ -T_{z_f} & 0 & T_{z_i} \end{pmatrix} \left\langle J_f T_f \left\| \hat{O}^{\Lambda}_{JT}(q) \right\| J_i T_i \right\rangle F_{cm}(q) F_{fs}(q) \right|^2 \text{.(11)}$$

with Λ selecting the longitudinal (L), Coulomb (C), transverse electric (E) and transverse magnetic (M) form factors, respectively. $T_Z = (Z-N)/2$ is the projection of the total isospin. The nucleon finite size (fs) form factor is $F_{fs}(q) = \exp(-0.43q^2/4)$ and $F_{cm}(q) = \exp(q^2 b^2/4A)$ is the correction for the lack of translational invariance in the shell model (center of mass correction), where A is the mass number and b is the harmonic oscillator size parameter. The parity selection rules are:

$$\Delta\pi^{EJ} = (-1)^J, \text{ and } \Delta\pi^{MJ} = (-1)^{J+1} \quad \text{.......(12)}$$

The single-particle matrix elements $\left\langle J_f T_f \left\| \hat{O}^{\Lambda}_{JT}(q) \right\| J_i T_i \right\rangle$ for the required electron scattering operators used in this work are those of Brown et al. [16].

3. RESULTS AND CALCLATIONS

The nuclear structure of ^{14}N and ^{10}B nucleus are studied in framework of TDA and RPA. These nuclei have two holes in the closed shells of ^{16}O and ^{12}C nuclei respectively. By rearranging the holes within the hole orbits: $1s_{1/2}$, $1p_{3/2}$, $1p_{1/2}$ for ^{14}N and; $1s_{1/2}$, $1p_{3/2}$ for ^{10}B and the particle orbits; $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$ for ^{14}N and; $1p_{1/2}$, $1d_{5/2}$, $1d_{3/2}$, $2s_{1/2}$, $1f_{7/2}$ for ^{10}B, and coupling them to different angular momenta J and isospin T, the Hamiltonian is diagonalized in the presence of WBP interaction.

The spectra of ^{14}N and ^{10}B nuclei are presented in Fig. (2) and Fig. (3) respectively. RPA results plotted in first column and TDA results are plotted in second column and compared

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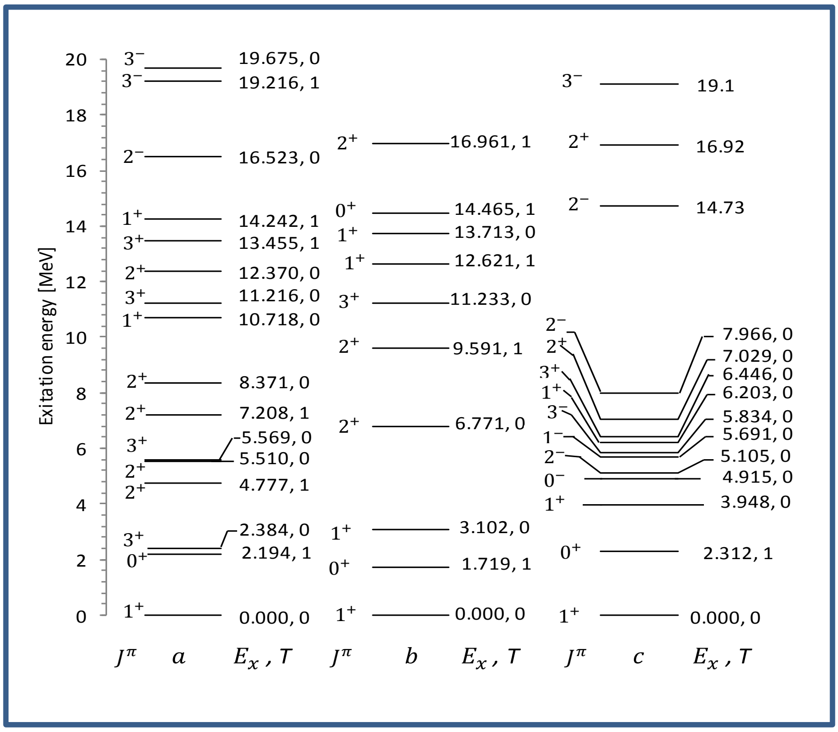
with experimental spectrum. The calculated results of RPA are obtained to be similar to that of TDA. Both calculations agree well with the available experimental data.

For ^{14}N, we get the ground state and low-lying excited states very nicely for both calculations TDA and RPA. The results of TDA cannot predict the first negative parity states of J = 3, 2. In both Figures (2) and (3) the calculations of RPA expected levels cannot be displayed by a TDA.



(a) (b) (c)

Fig. (2): Comparison between the experimental observed energy levels of ^{14}N with the results of the present work. *a*) RPA, *b*) TDA and *c*) Experiment.

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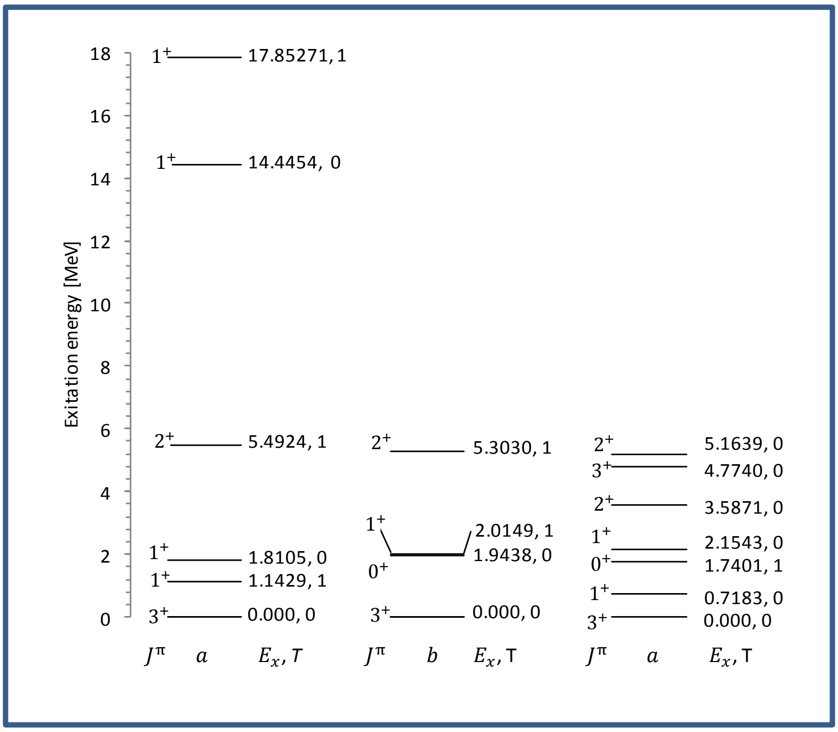

(a) **(b)** **(c)**

Fig. (3): Comparison between the experimental observed energy levels of ^{10}B with the results of the present work. *a*) RPA, *b*) TDA and *c*) Experiment.

The value of amplitudes X^{λ}_{JT} and Y^{λ}_{JT} are used to calculate the reduced matrix elements of the required multipole operators $\hat{O}_{JT}$ in terms of single-particle matrix elements. Calculations of the electron scattering form factors are presented for the $J^{\pi}_{f}T_{f}\,(E_x\ \mathrm{MeV}) = 1^{+}0(0.0)$ state for ^{14}N and $J^{\pi}_{f}T_{f}\,(E_x\ \mathrm{MeV}) = 1^{+}0(0.718)$ state for ^{10}B. The radial wave function for the single-particle matrix elements were calculated with the harmonic oscillator (HO) potential. The oscillator length parameter *b*=1.645 *fm* for ^{14}N and *b*=1.611 *fm* for ^{10}B was chosen to reproduce the measured root mean square charge radius.

Transverse electron scattering data [17] exist for the purely isoscaler elastic scattering from the $J^{\pi}=1^{+}$ (0.0 MeV), *T*=0 ground state. Calculations were made by Huffmman et al. [17], Genz et al. [18] and by Booten and van Hees [19] able to reproduce the elastic as well as the inelastic M1 form factor up to a momentum transfer of 2 fm^{-1}. At larger momentum transfer the calculated form factor fell off much more rapidly than the data.

Our calculated transvers magnetic form factor M1 for the $J_f^{\pi} T_f\,(E_x\ \text{MeV}) = 1^{+}0(0.0)$ based on TDA and RPA amplitudes are shown in Fig. (4). The results agree in shape and magnitude quite well with experimental data for all the momentum transfer values.

In Fig. (5), we compared the present RPA and TDA calculations of longitudinal form factor (C2+C4) with experimental data [20]. The RPA results displayed by dashed curve and the result of TDA displayed by solid curve. Good agreement has been obtained up to a momentum transfer of 1 fm^{-1}. At larger momentum transfer the calculated form factor is overestimate the experimental data.

Regarding to total transvers form factor, E2+M3 the present RPA and TDA calculations for the lowest 1^{+} (0.718 MeV) T=0 state are shown in Fig. (6) as solid and dashed line, respectively. For two models, good agreements with experimental data [20] have been obtained.

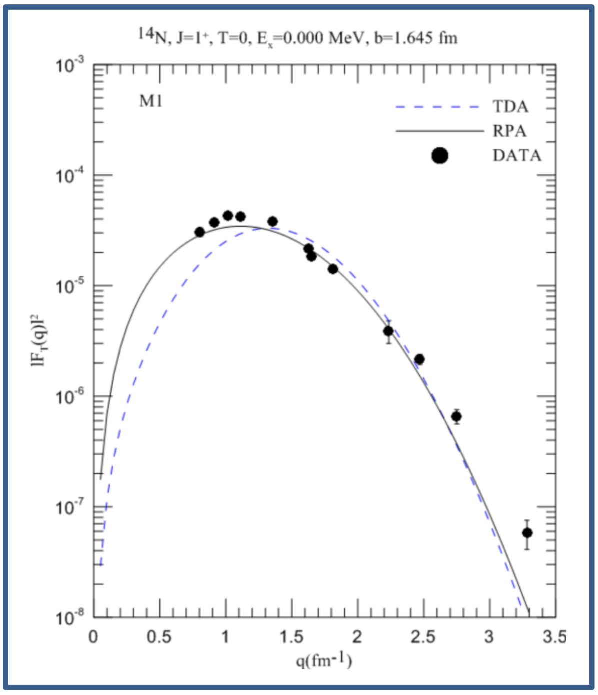

Fig. (4): The transverse magnetic form factor M1 data for the lowest 1^{+} (0.0MeV) T=0 state in ^{14}N taken from [17] in comparison with present TDA and RPA calculations.

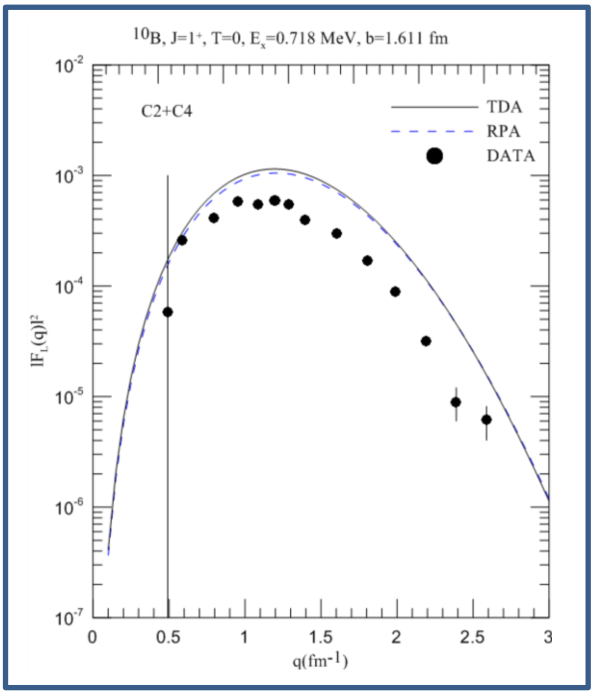

Fig. (5): The longitudinal form factor C2+C4 data for the lowest 1^{+} (0.718 MeV) T=0 state in ^{10}B taken from [20] in comparison with present RPA and TDA calculations.

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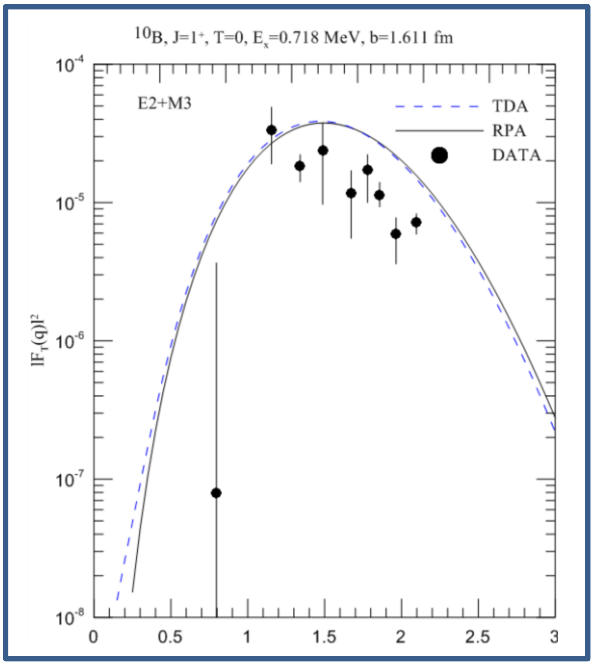

Fig. (6): The transverse total form factor E2+M3 data for the lowest 1^+ (0.718 MeV) T=0 state in ^{10}B taken from [20] in comparison with present RPA and TDA calculations.

4. CONCLUSION

When the Hamiltonian is diagonalized in the presence of WBP, the calculated results of *hh* RPA and are obtained to be similar to that of *hh* TDA for ^{14}N and ^{10}B nuclei. Both calculations agree well with the experimental data. But for ^{14}N, the results of TDA cannot predict the position of first 2^- and first 3^-. Generally, the calculations of RPA expected energy levels that cannot be displayed by a TDA.

At q >2.5 fm^{-1} the calculated electron scattering form factor for the $J_f^{\pi}T_f\ (E_x\ \text{MeV}) = 1^+0$ (0.0) state for ^{14}N, fell off much more rapidly than the data. In nucleus ^{10}B, for the excitation 1^+ (T=0, E_x =0.718 MeV), good agreement with the experimental has been obtained up to q = 1 fm^{-1} for the longitudinal form factor (C2+C4), but at larger momentum transfer the calculated form factor is overestimate the data. The transverse form factor E2+M3 has good agreements with the experimental data for both RPA and TDA.

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