Finite Element Evoluation of Mode I Stress Intensity Factor of Composite Material Under Domain Loading

Dr. AMEEN A. NASSER, HAIDER HADI JASIM

Department of Mechanical Engineering, University of Basrah, Basrah, Iraq

Abstract

In this paper, a finite element method program under domain loading and plain strain conditions is developed and applied in evaluation of the stress intensity factor in opening mode (K_1) in two dimensions crack problems. Two types of crack problems are analyzed and verified: first, cracked rotating disc made from bi-directional fiber reinforced material composite, second crack blade made from bi-directional fiber reinforced metal matrix composite. It is found that the finite element method under domain loading is a good tool for the analysis of composite material. The simulation is accurate in comparison with that obtained from extrapolation method. The stress intensity factor for fiber reinforced metal matrix composite is larger when obtained from fiber-reinforced material under same condition.

تقييم معامل تركيز الإجهاد للطور الأول في المواد المركبة تحت أحمال مجاليه باستخدام طريقة العناصر المحددة

د. امين احمد نصار و حيدر هادي جاسم

الملخص

في هذا البحث، تم تطوير برنامج باستخدام طريقة العناصر المحددة تحت شروط الأحمال المجالية ومستوي الانفعال وطبق لحساب معامل شدة الإجهاد في حالة النمط المفتوح لمسائل الشقوق ثنائية البعد . نوعين من مسائل الشقوق تم تحليلها ودراستها: النوع الأول قرص من مادة مركبة من ألياف ثنائية الاتجاه مغمورة في قالب من مواد هندسية، النوع الثاني ريشة (Blade) من مادة مركبة مكونة من ألياف ثنائية الاتجاه مغمورة في معدن صلب. أن النتائج المستحصلة من البرنامج المطور تظهر توافق جيد مع النتائج المستحصلة من طرق الاستقراء الرياضي وهذا يعني أن طريقة العناصر المحددة هي طريقة فعالة في تحليل المواد المركبة وتعطي نتائج جيدة في حالة التحميل ألمجالي. أظهرت النتائج أن معامل شدة الإجهاد للنوع الثاني من المواد المركبة اكبر من معامل شدة الإجهاد للنوع الثاني من المواد المركبة.

Nomenclature

(X, Y): Are the component of the body forcers intensity (force per unit volume) at the i^{th} node.

N_i : Shape function.

t : Thickness of disc.

ρ : Material density (kg / m^3).

E: Modulus of elasticity.

ω_x, ω_y, and ω_z : Angular velocity in X, Y, and Z direction respectively in (rad/s)

x_o, y_o : The coordinates of the center of rotation.

$\underline{F}_i$ is a body force vector.

u_i : Displacement vector (in two-dimension problem $u_1 = u_x$ and $u_2 = u_y$).

T: Traction vector.

dx and dy: Infinitesimal element along the x-direction and y-direction.

ds : Infinitesimal distance on a path S.

$\underline{D}$: D- matrix (matrix contain the element properties such as modulus of elasticity and Poisson's ratio).

$\underline{K}_{(e)}$: Element stiffness matrix.

$\underline{P}_{(e)}$: Element force vector.

ne : Number of element.

[U]: Global displacement vector.

J_x , J_y : Jacobin parameter in x and y directions.

1-Introduction

In recent year, the use of the fast rotating component has increased dramatically in many of the engineering such as turbines disc and blade, turbo-compressors, generators ...etc. The cost of production of the fiber is decreased as a result of increasing fiber-processing technology; the manufacture of these parts from fiber reinforced composite become necessary to decrease cost and to improve the mechanical properties of these materials [1].

The authors [2-3] have reported on the rotational strength of rotating disc using finite difference method, analytically, laminated plate theory and experimental method. M. Umara and O. Byon [4] study the optimum lamination in fiber-reinforced flywheel. They had taken the affect of rotational stresses due to curing process in their analysis. Genta G. et al. [5] found the axisymmetrical stress distribution in orthotropic and central hole composite disc.

In this paper, the finite element method program under domain loading and plain strain conditions is developed and uses to analysis radial cracking rotating disc and single edge crack rotating blade to determine the stress intensity factor in opening mode.

2- Stress Intensity Factor

There are three types of stress intensity factor modes: opening mode (mode I) with K_I parameter, shear mode (mode II) with K_{II} parameter and mixed mode (mode III) with K_{III} parameter. Among them mode I with K_I parameter is the most important to know because K_I characterize the stress field in the neighborhood of a crack tip when the crack under tension and it can be obtained relatively easily with a rotating disc [6].

There are two method for calculating K_I, experimental method and numerical method by using finite element method (J-Integral method and extrapolation method).

3-Finite Element Simulations with Domain Loading

The procedure used can be simulating by using displacement method as followers [6, 7]:

Step 1:

The generalized equilibrium equation for linear situation can be expressed as follows:

$$\iiint B^t \sigma \, dv = F \quad \dots (1)$$

Step 2:

The whole domain is divided into finite element, which is connected together by specific nodes. Then the displacement vector $\underline{U}$ at point (x, y) can be expressed as follows:

$$\underline{U}(x,y) = \sum_{i=1}^{n} u_i N_i(x,y) \quad \dots (2)$$

Step 3:

The relation between the applied force acting on the nodes and the nodal displacement can be expressed by using which called element stiffness matrix as follows:

$$K_e = \iint \underline{B}^t \underline{D} \, B \, dxdy \quad \dots (3)$$

Step 4:

The nodal stiffness and nodal loads for each of the element sharing the same nodes are add to each other to obtain the net stiffness and the net load at the specific nodes, so the global matrix can be expressed as:

$$\underline{K} = \sum_{e=1}^{ne} \underline{K}_{(e)} \quad \dots (4)$$

$$P = \sum_{e=1}^{ne} \underline{P}_{(e)} \quad \dots (5)$$

Step 5:

The overall system of equation of the domain can be written as:

$$[K]\,[U] = [P] \quad \dots (6)$$

In order to solve the above system of equations, the following boundary conditions are applied:

1- At loaded nodes the displacement is unknown and the applied force is known.

2- At supported nodes the load is unknown and the displacement is known.

Step 6

In the context of body forces, the problems reduced to one of obtaining the nodal loading vector, which is equivalent to a given force field, represented by the following intensity vector:

$$\underline{X} = [X_1 \ Y_2 \ \ X_2 \ Y_2 \ ... \ X_n \ Y_n]$$

Now, if the equivalent nodal loading vector is:

$$\underline{\Omega} = [\ \Omega_{x1} \ \Omega_{y1} \ ... \ \Omega_{xn} \ \Omega_{yn}\ ] \quad ... (7)$$

Then,

$$\Omega_{xi} = \sum_{i=1}^{n} Q_{i,j} X_i \quad ... (8)$$

And

$$\Omega_{yi} = \sum_{i=1}^{n} Q_{i,j} Y_i \quad ... (9)$$

Where,

$$Q_{i,j} = \iint_{element} t \ N_i \ N_j \ dA \ ... (10)$$

And,

$$dA = dx\, dy = \det J \ d\xi \ d\eta$$

for the special case of a uniform body force i.e.

$$X_1 = X_2 = ... = X$$

$$Y_1 = Y_2 = ... = Y$$

It can be deduced that

$$\Omega_{xi} = Q_i X \quad ... (11)$$

$$\Omega_{yi} = Q_i Y \quad ... (12)$$

Where,

$$Q_i = \iint_{element} t \ N_i \ dA \quad ... (13)$$

To produce programmable equation, specific values for the body force terms X and Y have been considered. For the case of rotation inertia as following:

For a body rotate with a constant angular velocity ω, two cases are considered:

(i) Rotation about the Z-axis

$\omega_x = \omega_y = 0$

$$X = -\rho(x - x_o)\omega^2 \quad ... (14)$$

$$Y = -\rho(y - y_o)\omega^2 \quad ... (15)$$

(ii) Rotation about an axis in the x-y plane, $\omega_z = 0$

$$X = -\rho[(x - x_o)\omega_y^2 - (y - y_o)\omega_x\omega_y] \quad ... (16)$$

$$X = -\rho[(x - x_o)\omega_x^2 - (x - x_o)\omega_x\omega_y] \quad ... (17)$$

4- J-Integral Technique

In most fracture mechanics, J-Integral, a parameter is sought to represent the state of fracture process region and thereby a fracture criterion is given. In the case where body forces (centrifugal force) exist, the J-Integral equation is given as followers [7]:

$$J = \int_{\Gamma} w\,dy - \int_{\Gamma} \underline{T}^{t} \frac{\partial u_i}{\partial x} ds - \iint_{\Omega} \underline{F}_i \frac{\partial u_i}{\partial x} dxdy \qquad \dots (18)$$

The J-Integral related to the stress intensity factor in the opening mode (mode I) by the following equation:

$$J = \frac{K_I^2}{E'} \qquad \dots (19)$$

Where,

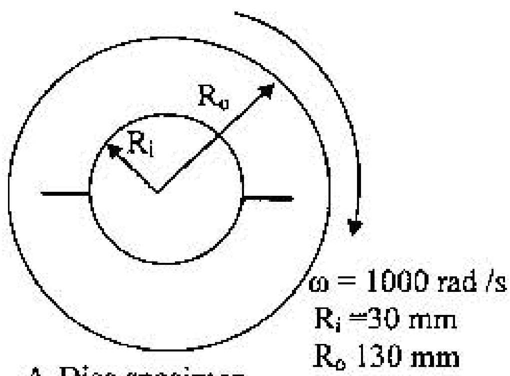
$E' = E$ for plain stress.

$E' = \dfrac{E}{1 - v^2}$ for plain strain.

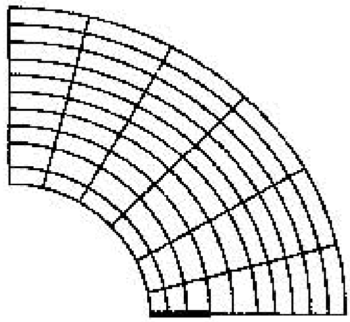
5-Finite element meshes and mechanical properties

Case Study (1) Double Edges Radial Crack in Rotating Hollow Thick Disc.

This case deals with disc made from bi-directional glass fiber reinforced epoxy case of Quasi-Isotropic containing double edge radial crack, because of symmetry only one quarter of the disc is used to analysis the problem. The geometry of the disc specimen, dimension and the finite element mesh are illustrated in Fig.1A and Fig.1B. A total of 80 quadrilaterals 9 nodes standard element were used.



A-Disc specimen



B- Finite element mesh.

Fig.1 Disc specimen and example of finite element mesh for case 1.

Case Study (2) Single Edge Crack in Rotating Blade

This case deals with single edge crack in blade (8*34 cm) made from boron fiber reinforced aluminum matrix [8, 9]. The materials constant are given in Table 2. The finite element meshes are illustrated in the Fig.2 by using 80 quadrilateral 9-nodes element.

Appendix contains the mechanical properties of glass fiber reinforced epoxy Table1 and boron fiber reinforced aluminum matrix Table2.

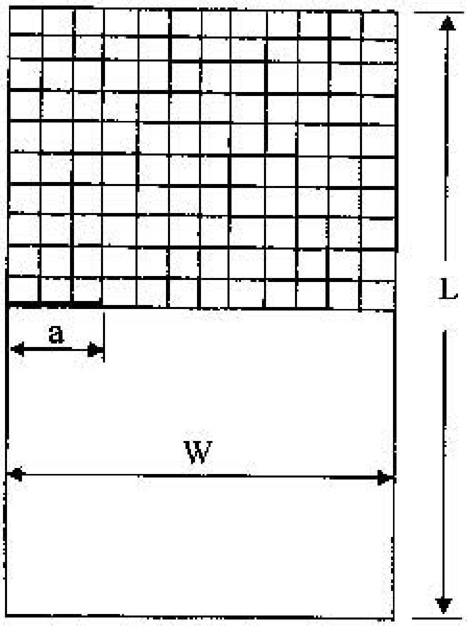


Fig.2 Blades finite element mesh for crack length (8 mm) for case 2.

6-Results and Discussion

Fig.3 shows the stress intensity factor at different crack ratios calculated from J-Integral technique and extrapolation method (stress and displacement extrapolation methods). It can clear from this figure that there a good agreement between these two methods. It can be seen that as crack length increased the stress intensity factor increased resulting from increasing the stresses at the crack tip as crack extension in the material.

Fig.4 and Fig.5 shows the stress intensity factor and J-values vs. crack length to width ratios that obtained from J-Integral and extrapolation methods. A little deference observed between J-Integral and extrapolation method results.

It can be seen that the stress intensity factor for fiber reinforced metal matrix composite is larger than that obtained from fiber-reinforced materials composite under the same conditions and for the same crack length to width ratios. This attributed that the modulus of elasticity of fiber-reinforced material is less than that for metal reinforced metal matrix composite.

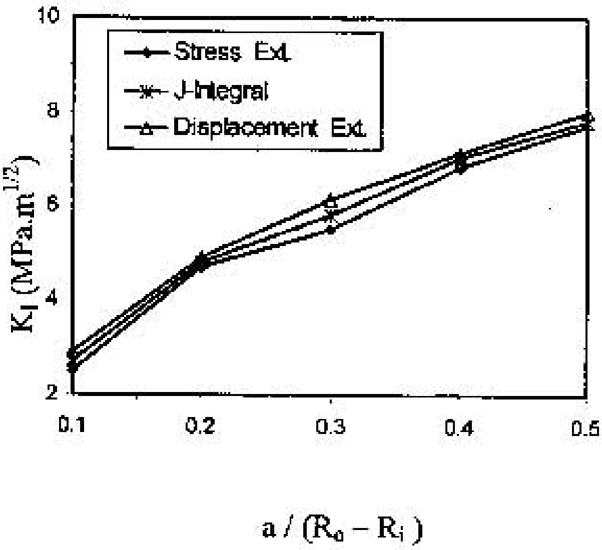


Fig.3 Stress intensity factor at different crack length to width ratios for case 1

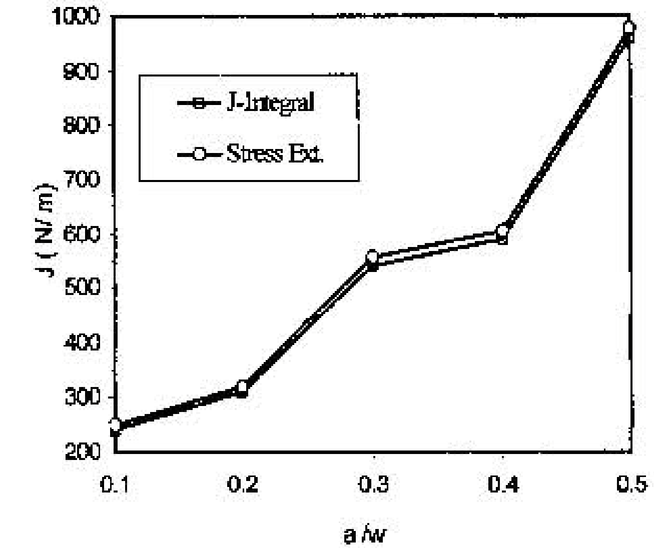


Fig.4 J-values at different crack length to width ratios for case 2.

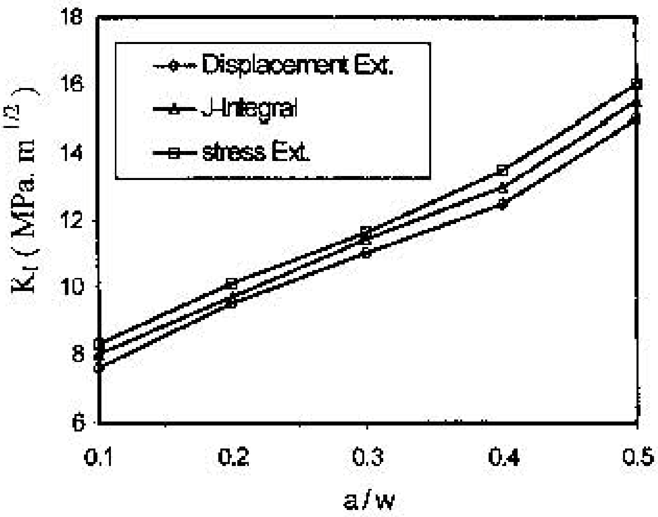


Fig.5 K_I – values at different crack length to width ratios for case 2.

7-Conclusios

From the pervious section, we can conclusions the following remarks:

1- The finite element method and J-Integral methods is a good approach for the analysis of composite materials.

2- The stress intensity factor increase as crack length increase resulting from increasing stress at the crack tip.

2- The fracture toughness for fiber reinforced composite materials is less than that for metal matrix composite under the same conditions.

Acknowledgement

The author expresses thanks to all staff of mechanical engineering department for there help and support during this work.

8- References

[1] Dharan C. K. H. "Fracture mechanics of composite material" Journal of engineering Material and Technology, vol.100, pp.233-246, 1978.

[2] Shiratori E., Ikegami K. and Shingo T. "Rotating strength of laminated composite discs", Bulletin of the JSME, vol.23, no.180, pp.822-830, 1980

[3] Ikegami, K. and Shiratori, E., "Study on the high speed rotating disc reinforced by laminating and hoop winding method", Bulletin of the JSME, vol.24, no.189, pp.501-506, 1983.

[4] O-Il Byon and Masuji Uemura, "Optimum lamination in fiber reinforced flywheel taking account of

the curing stresses" Report of the Research Institute of Industrial Technology, Nihon University, Number 9, 1979.

[5] Genta, G., Gola, M. and Guglitta, A., "Axisymmetric computation of the stress distribution in orthotropic rotating disc" Int. J. Mech. Sci., vol. 24, no.1,pp.21-26,1983.

[6] Onur S. "Stress analysis of a thermoplastic composite disc under uniform temperature distribution" J. of thermoplastic composite, vol.19, pp.61-77, 2006.

[7] Shramh, K. M. "Domain loading fracture mechanics analysis of welding components using finite element method", M.Sc. thesis, Basrah University, Iraq, 1999.

[8] Martin Grayson "The encyclopedia of composite material and components" John Wily and Sons Inc., 1983

[9] Brain N. Cox and David B. Marshall "Crack initiation in fiber reinforced brittle laminates" J. AM. Ceram. Soc. Vol. 79, 1996.

APPENDIX TABLES

Table 1. Material constant of E-Glass and epoxy resin [Ref.3]

	Constants				
materials	E(GPa)	σ_u (MPa.)	γ	ρ (kg/m)	V_f %
E-Glass fiber	74	1500	0.22	2540	45
Epoxy-Resin	3.21	4100	0.38	1180	50

Table2. Material constant of boron-fiber and aluminum matrix [Ref.7]

	Constants				
materials	E(GPa)	σ_u (MPa.)	γ	ρ (kg/m)	V_f %
Boron fiber	385	2800	0.25	2630	50
Aluminum	70	310	0.22	2800	50