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On -MΘ - C-Multifunctions

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ABSTRACT

In this work we introduce anew concept namely MΘ - C -Multifunction.These are the Multifunction $F:X\rightarrow Y$ such that the inverse of every compact subset in Y is Θ - closed in X .We proved several Theorems about MΘ-C- Multifunction and we study the relations of MΘ-C- Multifunction with other types of functions.

Introduction

Let $f: X\rightarrow Y$ be a Θ - continuous function from a Topological space X in to a Θ-T_2 space Y, if $K\subset Y$ is compact , then K is Θ-closed in Y.

But f is Θ-continuous, so $f^{-1}(K)$ is Θ-closed.

This means that the inverse of each compact set in Y is Θ-closed in X.

This motivates the definition of MΘ-C- function.

In This work , we generalized These ideas to Multifunctions .

If K is a subset of a Topological space , Then Cl(K) denoted the closure of K and K° denoted the interior of K .

2-Preliminaries

In this section we recall the Basic definitions needed this work.

(2-1) Definition [5]

Let X & Y be a Topological spaces , A multifunction $F: X\rightarrow Y$ is a correspondence from X to Y with F(x) a nonempty subset of Y for each $x\in X$.

(2-2) Definition [3]

If $F: X\rightarrow Y$ is a Multifunction , Then the Graph of F (G(F)) is the subset $\{(x , y) : x\in X , y\in F(x) \}$ of X x Y .

(2-3) Definitions

if X and Y are Topological spaces and $F: X\rightarrow Y$ be Multifunction we will say that F has closed graph if G(F) is closed subset of the product X x Y [2] .

A function $F: X\rightarrow Y$ has Θ-closed graphs iff for each $(x , y)\in X \times Y - G(F)$ these are sets $V\in\sum(x)$ in X (where $\sum(x)$ denoted the family of open subsets which contain x) and $W\in\sum(y)$ in Y (where $\sum(y)$ denoted the family of open subsets which contain y) with (cl (v) x cl (w)) $\cap$ G(F) = φ [1] .

(2-4)Definition [6]

Let D be a non empty set and let ≥ be a binary relation on D we say that the relation ≥ directs D iff the following three conditions hold :

1- for every $a\in D$, $a\geq a$.
2- if $a\geq b$ and $b\geq c$ then $a\geq c$.
3- for each $a,b\in D$ $\exists c\in D \ni c\geq a$ and $c\geq b$.

The pair (D, ≥) is called directed set.

A net in the space X is a function $f: D\rightarrow X$ where (D, ≥) is a directed set .

(2-6) Definition [1]

we will say that Multifunction $F: X\rightarrow Y$ has a Θ - sub closed graph if for each $x\in X$ and net $\{x_\alpha\}$ in X - {x} with $x_\alpha \xrightarrow[\theta]{} x$ and net $\{y_\alpha\}$ in Y with $y\in F(x_\alpha)$ for each α , and $y_\alpha \xrightarrow[\theta]{} y$ in Y , we have $y\in F(x)$.

(2-7) Definition [2]

we will say that a Multifunction $F: X\rightarrow Y$ has (closed , Θ-closed , compact) point image if F(x) is (closed , Θ-closed , compact) in Y for each $x\in X$.

(2-8) Fact [1]

Let $F: X\rightarrow Y$ be Multifunction from a space X into a space Y , Then F has Θ - closed graph iff F has a Θ - sub closed graph and Θ-closed point image .

(2-9) Definitions

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1- A Topological space (X , T) is said to satisfy the Θ - T_2 - space if given a pair of distinct Pointes x , y $\in$ X $\exists$ two Θ - open sets U , V s.t $x \in U$ and $y \in V$
and $U \cap V = \varphi$.

2- A Pointe x is said to be a Θ - limit Pointe of a set K if for every open set U with $x \in U$, $(cl(U) - \{x\}) \cap K \neq \varphi$.

1- Let (X , T) be a Topological space , $K \subseteq X$ is called a Θ - neighborhood of a Pointe $x \in X$ if $\exists$ Θ - open set $U \in T$ with $x \in U$ such that $U \subseteq K$.

(2-10) Definition [5]

Let F: X→Y be Multifunction from a space X into a space Y, if $K \subseteq Y$ then

1- $F^{+}(K) = \{ x \in X : F(x) \subseteq K\}$ this called upper inverse of K .

2- $F^{-1}(K) = \{ x \in X : F(x) \cap K \neq \varphi\}$ this called lower inverse of K .

(2-11) Example

Let X={a , b , c} and T={X , φ , {a}, {b}, {a , b}} be a Topology on a set X and Let Y={1 , 2 , 3} and T={Y , φ , {1 , 2}} be a Topology on a set Y , and let F:X→Y be Multifunction defined as following :-

F(a) = {1 , 2} , F(b)= F(c)={2 , 3}and let $K \subseteq Y$ $\ni$ K={1 , 2} then

$F^{+}(K) = \{ x \in X : F(x) \subseteq K\} = a$.

$F^{-1}(K) = x \in X : F(x) \cap K \neq \varphi\} = \{a , b , c\} = X$.

(2-12) Definitions [4]

1- A Multifunction F: X→Y on a Topological space X into a Topological space Y is a Θ - upper - semi - continuous (Θ - u . s . c) iff for each Θ-closed set $K \subseteq Y$ Then $F^{-1}(K)$ is Θ-closed in X further , F is a Θ - lower - semi - continuous (Θ - l . s . c) iff $F^{-1}(K)$ is Θ - open for each Θ - open set $K \subseteq Y$ then F is Θ - continuous in case it is both (Θ - u . s . c) and (Θ - l . s . c) .

2- A Multifunction F: X→Y is almost - Θ - upper - semi - continuous (a . Θ - u . s . c) iff for each $x \in X$ and each open set $V \subseteq Y$ with $F(x) \subseteq V$, $cl(F^{+}(V))$ is a neighborhood of x . Further , F is almost - Θ - lower - semi - continuous (a . Θ - l . s . c) iff for each open set $V \subseteq Y$ the set $cl(F^{-1}(V))$ is a neighborhood of each $x \in F^{-1}(V))$. Then F is almost Θ - continuous (a . Θ - c) iff it is both (a . Θ - u . s . c) and (a . Θ - l . s . c) .

(2-13) Definition [1]

1- A point x is in the Θ - closure of a subset K of a space X ($x \in cl_{\Theta}(K)$) if each $V \in \sum(x)$ satisfies $K \cap cl(V) \neq \varphi$.

2- A subset K is Θ-closed iff $cl_{\Theta}(K) = K$.

3- MΘ - C-Multifunction

In this section we introduce a new concept , namely MΘ - C -Multifunction defined as follows :

(3-1) Definitions

1- Let F: X→Y be Multifunction we say that F is Θ- lower- MC-Multifunction iff for each compact $K \subseteq Y$, Then $F^{-1}(K)$ is Θ-closed in X .

((we will use $M^{-}\Theta$ - C- Multifunction to denoted Θ- lower- MC- Multifunction)) .

2- Let F: X→Y be Multifunction we say that F is Θ- upper- MC-Multifunction iff for each compact $K \subseteq Y$, Then $F^{+}(K)$ is Θ-closed in X .

((we will use $M^{+}\Theta$-C- Multifunction to denoted Θ- upper- MC- Multifunction)).

3- If F:X→Y is Θ- upper- MC-Multifunction and Θ- lower - MC-Multifunction Then we say that F is MΘ -C-Multifunction .

(3-2) Theorem

If F: X→Y is Multifunction with Θ- sub closed graph then F is an $M^{-}\Theta$ - C - Multifunction .

Proof :

Let K be compact subset in Y . we must prove that $F^{-1}(K)$ is Θ-closed in X .

Let p be a Θ - limit Pointe of $F^{-1}(K)$, There are a nets $\{x_\alpha\}$ in $X - \{p\}$ and $\{y_\alpha\}$ in K , with $x_\alpha \xrightarrow[\theta]{} p$ and $y_\alpha \in F(x_\alpha)$ for each α .

Some subnet $\{y_{\alpha n}\}$ of $\{y_\alpha\}$ Θ - converges to some $y \in K$ this gives $y \in F(p)$, so $p \in F^{-1}(K)$, Then $F^{-1}(K)$ is Θ-closed in X .

(3-3) Corollary

If F: X→Y is Multifunction with Θ- closed graph then F is an $M^{-}\Theta$ - C - Multifunction .

(3-4) Remark

If F: X→Y is Θ- lower - MC-Multifunction then G(F) is not necessarily Θ-closed .

The following Theorem show if F: X→Y is Θ- lower - MC-Multifunction from a space X into a locally compact Θ-T_2- space Y,Then G(F) will be Θ-closed

(3-5) Theorem

Let F: $X\rightarrow Y$ be $M^{-}\Theta$ - C - Multifunction with Θ-closed point images from a space X into a locally compact Θ-T_2- space Y then G(F) is Θ-closed .

Proof :

suppose (x , y) $\notin$ G(F) , then y $\notin$ F(x) and so there exists disjoint sets U_1 , U_2 with $y\in U_1$ and $F(x)\subseteq U_2$, Further , There is a compact neighborhood W of y such that $W \subseteq U_1$, Then $F^{-1}(W)$ is Θ-closed in X and x $\notin F^{-1}(W)$ thus there is an open set V such that $x\in V$ and $V \cap F^{-1}(W) = \varphi$, Hence V x W is a neighborhood of (x , y) which misses G(F) and so G(F) is Θ-closed .

(3-6) Theorem

If F: $X\rightarrow Y$ is (Θ - u . s . c) Multifunction from a space X into Θ-T_2- space Y Then F is $M^{-}\Theta$ - C - Multifunction .

Proof :

Let K be a compact subset in Y , since Y is Θ-T_2-space , Then K is Θ-closed in Y {A compact subset of Θ- Hausdorff space is Θ-closed [6]} since F is (Θ - u . s . c) Then $F^{-1}(K)$ is Θ-closed in X , Then F is $M^{-}\Theta$ - C - Multifunction .

(3-7) Corollary

Let F: $X\rightarrow Y$ be Θ - continuous Multifunction from a space X into Θ-T_2- space Y, Then F is $M^{-}\Theta$ - C - Multifunction .

Before we state our next Theorem , we recall the following definitions :-

(3-8) Definitions

1- Let F: $X\rightarrow Y$ be Multifunction , we say that F is a lower compact Multifunction iff for each compact set $K \subseteq Y$, Then $F^{-1}(K)$ is compact in X .

2- Let F: $X\rightarrow Y$ be Multifunction , we say that F is a upper compact Multifunction iff for each compact set $K \subseteq Y$, Then $F^{+}(K)$ is compact in X . Then if F lower compact Multifunction and upper compact Multifunction then_we say that F is compact Multifunction .

(3-9) Theorem

Let F: $X\rightarrow Y$ be a lower (upper) compact Multifunction from a Θ - T_2- space X in to a space Y then F is $M^{-}\Theta$ - C($M^{+}\Theta$-C) – Multifunction – respectively .

Proof :

Let K be a compact subset in Y, since F is lower (upper) compact Multifunction Then $F^{-1}(K)$ & $F^{+}(K)$ is compact in X . since X is a Θ - T_2-space ,
Then $F^{-1}(K)$ & $F^{+}(K)$ is Θ-closed in X, then F is $M^{-}\Theta$- C($M^{+}\Theta$-C)-Multifunction .

(3-10) Corollary

Let F: $X\rightarrow Y$ be compact Multifunction from a Θ - T_2- space X in to a space Y Then F is MΘ - C - Multifunction.

(3-11) Theorem

Let F: $X\rightarrow Y$ be (a . Θ - u . s . c) with compact point image, and let F is a Θ - upper - MC - Multifunction , and suppose that each $y\in Y$ has a base for as neighborhoods consisting of compact sets , Then F is (Θ - u . s . c) .

Proof :

Let F is a (a . Θ - u . s . c) with compact point image and let F is $M^{+}\Theta$-C- Multifunction . we must prove that F is (Θ - u . s . c) .

Let $F(x) \subseteq U$ be an open set , then from the hypothesis there exists a compact set W such that $F(x) \subseteq W^{\circ} \subseteq W \subseteq U$, then $F^{+}(W)$ is Θ-closed set containing $F^{+}(W^{\circ})$, thus $F^{+}(cl(W^{\circ})) \subseteq F(W)$ and so there is an open set V with $x\in V\subseteq F^{+}(cl(W^{\circ}))$, But $F(V) \subseteq U$, Then F is (Θ - u . s . c) .

(3-12) Theorem

Let F: $X\rightarrow Y$ be (a . Θ - l . s . c) with compact point image, and let F is Θ - lower - MC - Multifunction , and suppose that each $y\in Y$ has a base for as neighborhoods consisting of compact sets , Then F is (Θ - l . s . c) .

Proof :

Similar to the proof of Theorem [3-11]

(3-13) Theorem

Let F: $X\rightarrow Y$ be $M^{-}\Theta$ - C- Multifunction from a space X into a compact space Y and let H : $Y\rightarrow Z$ be a (Θ - u . s . c) Multifunction from a space Y into a Θ - T_2- space Z , Then HoF: $X\rightarrow Z$ is $M^{-}\Theta$ - C- Multifunction .

Proof :

Let K be a compact subset in Z , since Z is Θ - T_2-space , Then K Θ-closed in Z since H is a (Θ - u . s . c)

, Then $H^{-1}(K)$ is Θ-closed in Y , since Y is compact , Then $H^{-1}(K)$ is compact in Y , since F is $M^{*}Θ$ - C- Multifunction then $F^{-1}(H^{-1}(K) = (HoF)^{-1}(K)$ is Θ-closed in X then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

(3-14) Corollary

Let F: X→Y be MΘ-C-Multifunction from a space X into a compact space Y and let H : Y→Z be Θ - continuous Multifunction from a space Y into a Θ - T_2- space Z , Then HoF: X→Z is MΘ-C-Multifunction .

(3-15) Theorem

Let F: X→Y be (Θ - u . s . c) Multifunction from a space X into a space Y and let H : Y→Z be $M^{*}Θ$ - C- Multifunction , Then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

(3-16) Corollary

Let F: X→Y be Θ - continuous Multifunction from a space X into a space Y and let H :Y→Z be MΘ-C-Multifunction ,Then HoF: X→Z is MΘ-C-Multifunction .

(3-17) Theorem

LeF: X→Y be $M^{*}Θ$ - C- Multifunction from a space X into a compact space Y and let H : Y→Z be $M^{*}Θ$ - C- Multifunction , Then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

Proof :

Let K be a compact subset in Z , since H is $M^{*}Θ$ - C- Multifunction then $H^{-1}(K)$ is Θ-closed in Y , since Y is compact then $H^{-1}(K)$ is compact , since F is $M^{*}Θ$ - C- Multifunction then $F^{-1}(H^{-1}(K) = (HoF)^{-1}(K)$ is Θ-closed in X then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

(3-18) Corollary

Let F: X→Y be MΘ-C-Multifunction from a space X into a compact space Y and let H :Y→Z be MΘ-C-Multifunction ,Then HoF: X→Z is an MΘ-C-Multifunction

(3-19) Theorem

Let F: X→Y be $M^{*}Θ$ - C- Multifunction and let W⊂ X then F|W : W→Y is $M^{*}Θ$ - C- Multifunction .

Proof :

Let g = F|W : W→Y and let K be a compact subset in Y , since F is an $M^{*}Θ$ - C- Multifunction , Then $F^{-1}(K)$ is Θ-closed in X . Now $g^{-1}(K) = F^{-1}(K) \cap W$ then $g^{-1}(K)$ is Θ-closed in W .

Then g = F|W : W→Y is $M^{*}Θ$ - C- Multifunction .

(3-20) Corollary

Let F: X→Y be MΘ - C -Multifunction and let W⊂ X , Then F|W : W→Y is MΘ-C-Multifunction .

(3-21) Theorem

Let F: X→Y be $M^{*}Θ$ - C- Multifunction and let W⊂ X , Then $(F|W)_p$: W→F(W) is $M^{*}Θ$ - C- Multifunction .

Proof :

Similar to the proof of Theorem [3-19] .

(3-22) Corollary

Let F: X→Y be MΘ -C-Multifunction and let W⊂ X , Then $(F|W)_p$: W→F(W) is MΘ - C -Multifunction .

(3-23) Theorem

Let F: X→Y be (Θ - u . s . c) Multifunction from a space X into Θ - T_2- space Y, and let H: Y→Z be a lower compact Multifunction ,Then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

Proof :

Let K be a compact subset in Z , since H is lower compact Multifunction, Then $H^{-1}(K)$ is compact in Y , since Y is Θ - T_2- space , Then $H^{-1}(K)$ is Θ-closed in Y, since F is (Θ - u . s . c) , Then $F^{-1}(H^{-1}(K) = (HoF)^{-1}(K)$ is Θ-closed in X then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

(3-24) Corollary

Let F: X→Y be Θ - continuous Multifunction from a space X into Θ - T_2- space Y, and let H: Y→Z be a compact Multifunction , Then HoF: X→Z is MΘ - C -Multifunction .

(3-25) Theorem

Let F: X→Y be a lower compact Multifunction from Θ - T_2- space X into Y, and let H : Y→Z be a lower compact Multifunction , Then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

Proof :

Let K be a compact subset in Z , since H is a lower compact Multifunction then $H^{-1}(K)$ is compact subset in Y , since F is a lower compact Multifunction then $F^{-1}(H^{-1}(K) = (HoF)^{-1}(K)$ is compact in X , since X is Θ - T_2- space then $(HoF)^{-1}(K)$ is Θ-closed in X , Then HoF: X→Z is $M^{*}Θ$ - C- Multifunction .

(3-26) Theorem

Let FX→Y be (Θ - u . s . c) Multifunction from a space X into Y , and let H : Y→Z be (Θ - u . s . c)

Multifunction from a space Y into Θ - T_2- space Z , Then HoF: $X\to Z$ is $M^{-}\Theta$ - C- Multifunction .

Proof :

Let K be a compact subset in Z , since Z is Θ - T_2-space then K is Θ-closed in Z , since H is (Θ - u . s . c) then $H^{-1}(K)$ is Θ-closed in Y , since F is (Θ - u . s . c) ,Then

$F^{-1}(H^{-1}(K) = (HoF)^{-1}(K)$ is Θ-closed in X,Then HoF: $X\to Z$ is $M^{-}\Theta$-C- multifunction .

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الخلاصة:

في هذا البحث قدمنا مفهوماً جديداً يدعى بالدوال متعددة القيم MΘ-C وهذه الدوال $F:X\to Y$ بحيث أن كل مجموعة متراصة مثل K في Y المعكوس لها (K) f -1 هو Θ – مغلقة في X .ثم برهنا عدة مبرهنات عن الدوال متعددة القيم MΘ-C وقمنا بدراسة العلاقة بين الدالة متعددة القيم MΘ-C وبعض أنواع الدوال الأخرى .