Dynamic Forces and Stress Analysis in the Journal Bearing System

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Abstract

This paper is concerned with a stress analysis in a bearing under unbalanced forces of the journal. Some aspects of mathematical modeling of rotating structures were considered. "Finite Element Method" is formulated for modeling rotating structures. As an application, a test rotor mounted on two-lobe hydrodynamic bearings is presented. Unbalance response calculations for various unbalance magnitudes are carried out in the bearing location. The bearing coefficients were found at rotational speed of 4,000 rpm. An accurate identification of bearing force parameters, i.e. stiffness and damping coefficients is presented by a classical linearized model. The bearing support forces in flexible rotor-bearing systems are presented as a function of unbalance response of the journal. The calculation of the bearing stress due to rotor unbalance are carried out using ANSYS. The ANSYS program gives a good aids in understanding the stress analysis in the bearing under the action of journal rotation.

تحليل القوى الديناميكية و الاجهادات في منظومة كرسي التحميل

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الخلاصـــــــة

اهتم البحث بتحليل الاجهادات في كرسي التحميل تحت تأثير قوى عدم الاتزان للمحور. اخذ بنظر الاعتبار بعـــض أساسـيات النموذج الرياضي للهياكل الدوارة. تم صياغة طريقة العناصر المحددة لنمذجة الهياكل الدوارة. تطبيق هذه الطريقة وضعت لاختبار محور مثبت على كرسيين تحميل. أجريت حسابات لاستجابة عدم الاتزان لقيم عدم اتزان في موقع كرسي التحميل. تم أيجاد معاملات كرسي التحميل في سرعة دورا نية مقدارها **4000** دورة\الدقيقة. تم تحديد القيم الدقيقة لمتغيرات قوة كرسي التحميل، مثل المرونــه و معـــاملات التخميـــد كنموذج خطي كلاسيكي. وجد أن قوى كرسي التحميل في منظومة محور-كرسي تحميل تكتب على شكل دالة لاســتجابة عـــدم الاتـــزان للمحور. تم حساب الإجهاد المتولد في كرسي التحميل الناتج من عدم الاتزان باستعمال برنامج **ANSYS** . برنامج **ANSYS** ساعد الى حد ما في فهم تحليل الاجهاد في كرسي التحميل والناتج من تأثير دوران المحور.

Nomenclature

$C_{yy}C_{yx},C_{xy},C_{xx}$: elements of bearing damping

$[F_1],[F_2]$: unbalance force associated With $\cos\Omega t$ and $\sin\Omega t$

$[F^d]$: unbalance force of the disk

$K_{yy}K_{yx}K_{xy},K_{xx}$: elements of bearing stiffness

I_d , I_p : diametral and polar mass moment of inertia

m^d : mass of the disk

$\dot{V}_y,\dot{V}_x$: unbalance velocity response of the journal

$[M^e],[C^e],[K^e]$: mass, damping and stiffness for rotor element matrices

$[M^d],[C^d]$: mass and damping For disk element matrices

$[q]$: displacement vector

V_y ,V_x : unbalance displacements response of the journal

W_y,W_x :small angle rotations about Y and X axis, respectively

y^d,x^d :mass center eccentricities of the disk in Y and X axis directions measured at $t=0$

Ω : spin speed

Introduction

Rotor dynamic stability is a measure of the tendency of whirl vibration to grow or decay with time. The rotor instability vibrations in compressors and turbines have caused severe failures and costly downtime for several large projects [1].

To determine the unbalanced response of a general rotor with properties defined at several stations a numerical methods are used. Transfer matrices and finite element methods are popular, as it had been studied by Zorzi and Nelson [1], Polk [2], Nelson and McVaugh [3], Nevzat & Levent [4] had developed a computer program to calculate forward and backward whirl speeds, corresponding mode shapes and to evaluate rotor stability. The dynamic response of a multi-disk rotor system supported by fluid film bearing was presented by Sharan [5]. Earlier studies have suggested that a reliable estimation of the state of unbalance (both amplitude and phase) at multiple planes of a flexibly supported rotating machine from measured vibration data is possible using a single machine run-down. Sinha [6] proposed a method that can reliably estimate both the rotor unbalance and misalignment from a single machine run-down. Rotor-bearing system characteristics, such as natural frequencies, mode shapes, stiffness and damping coefficients, are essential to diagnose and to correct vibration problems during system operations. Among the above characteristics, accurate identification of bearing force parameters, i.e. stiffness and damping coefficients are some of the most difficult parameters to achieve. Field identification by unbalance response measurements is a simple and often reliable way to determine synchronous speed force coefficients [7]. The current study performs a dynamic analysis of a rotor supported by two fluid film journal bearings with non-linear suspension. The dynamics of the rotor centre and bearing centre have been studied by Chang-Jian and Chen [8], This analysis of the rotor–bearing system was investigated under the assumptions of a couple-stress lubricant and a short journal bearing approximation.

Mathematical Modeling

The mathematical modeling can be classified as three components:

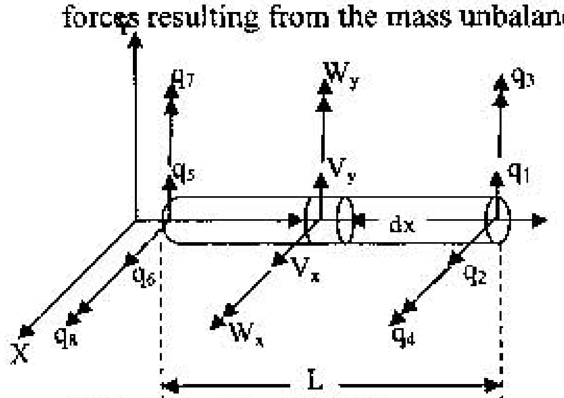
1-Modeling for rotor element.
2-Modeling for rigid disk element.
3-Modeling for bearing element.

1- Modeling for rotor element

A typical flexible rotor-bearing system consists of a rotor composed of discrete disks and rotor segments, and discrete elastic bearings. Figure 1 shows a typical rotor element of length L with the coordinates used to describe the end point displacements [q]. Each rotor element is modeled as an eight degree of freedom element with two rotations and two translations at each end. The displacements of a differential disk of thickness dx placed at a distance x from the element end are denoted by V_y , V_x , W_y and W_x. The equation of motion can be obtained for the finite rotor element as [9]

$$[M^e][\ddot{q}]+[C^e][\dot{q}]+[K^e][q]=[F] \quad .... (1)$$

Where [Me] is the symmetric mass matrix and $[C^e]$ and $[K^e]$ are the non symmetric damping and stiffness matrices for the rotor element respectively[4]. [F] is the generalized force vector which includes forces resulting from the mass unbalance.



2-Modeling for rigid disk

Fig.1 finite rotor element

The equation of motion of a rigid disk of mass md , diametral mass moment of inertia Id and polar mass moment of inertia Ip can be found according to method described in reference [4] as follows

$$[M^d][\ddot{q}]+[C^d][\dot{q}]=[F^d] \quad ....(2)$$

Where

$$[M^d]=\begin{bmatrix} m^d & 0 & 0 & 0 \\ 0 & m^d & 0 & 0 \\ 0 & 0 & I_d & 0 \\ 0 & 0 & 0 & I_d \end{bmatrix}$$

$$[C^d]=\begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & I_p\Omega \\ 0 & 0 & -I_p\Omega & 0 \end{bmatrix}$$

And the force due to journal unbalance can be written as[4]

$$[F^d]=\left[\begin{Bmatrix} y^d \\ x^d \\ 0 \\ 0 \end{Bmatrix}\cos\Omega t+\begin{Bmatrix} -x^d \\ y^d \\ 0 \\ 0 \end{Bmatrix}\sin\Omega t\right]M^d\Omega^2 \quad ....(3)$$

3-Modeling for bearing element

For the modeling of bearings, a classical linearized model with eight principal and cross-coupled stiffness and damping coefficients is used (Fig.2). Consider the tested bearing or (seal element) was considered as a point mass undergoing forced vibrations induced by external force functions. Equations for small amplitudes about an equilibrium position are described in linear form as given in reference [10]

$$\begin{bmatrix} C_{yy} & C_{yx} \\ C_{xy} & C_{xx} \end{bmatrix}[\dot{q}]+\begin{bmatrix} K_{yy} & K_{yx} \\ K_{xy} & K_{xx} \end{bmatrix}[q]=[F_b] \quad ....(4)$$

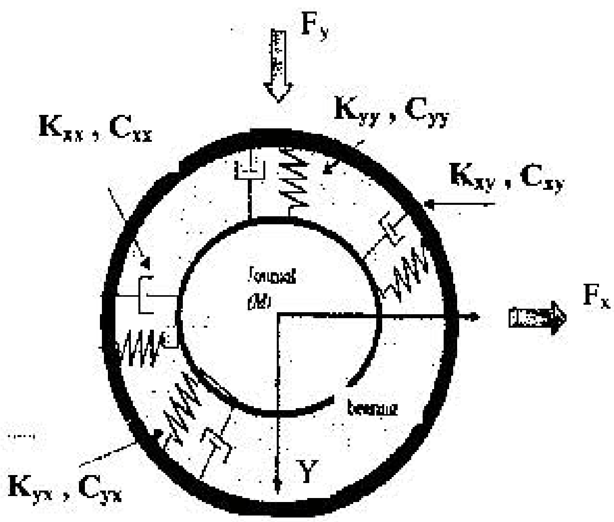


Fig.2 Representation of point mass and bearing force coefficients

Where $[F_b]$ can be represented as

$$F_x = -K_{xx}V_x - K_{xy}V_y - C_{xx}\dot{V}_x - C_{xy}\dot{V}_y$$

$$F_y = -K_{yx}V_x - K_{yy}V_y - C_{yx}\dot{V}_x - C_{yy}\dot{V}_y \text{....(5)}$$

Solution of System Equations

The equations of motion of the complete system can be obtained from the component equations as given in[4]

$$[M][\ddot{q}]+[C][\dot{q}]+[K][q]=[F]\text{.........(6)}$$

Where [M],[C], and [K] are the mass, damping, and stiffness matrices of the system obtained by assembling the element matrices. The forcing vector due to an unbalance force for rigid disk, eq. (3), can be also written as

$$[F^d]=[F_1]\cos(\Omega t)+[F_2]\sin(\Omega t)\text{.........(7)}$$

And the response (q), due to the unbalanced forces only, assuming a harmonic solution is

$$q = q_1 \cos\Omega t + q_2 \sin\Omega t$$

$$\dot{q} = -q_1\Omega \sin\Omega t + q_2\Omega\cos\Omega t \text{ ....(8)}$$

$$\ddot{q} = -q_1\Omega^2 \cos\Omega t - q_2\Omega^2 \sin\Omega t$$

Substituting eq.(8) in the equation of motion (6), then separating the sine and cosine terms gives

$$\begin{bmatrix} k - M\Omega^2 & c\Omega \\ -c\Omega & k - M\Omega^2 \end{bmatrix}\begin{bmatrix} q_1 \\ q_2 \end{bmatrix} = \begin{bmatrix} F_1 \\ F_2 \end{bmatrix}\text{......(9)}$$

then eq (9) can be solved in a single matrix equation.

Results And Discussion

Hydrodynamic forces have cross-coupled components that lead to stability problems for fixed geometry bearings. By referring to case study given in [5], it is possible to carry out the predicted unbalance forces of the grinding machine rotor. These unbalance forces can be shown in two cases as follow:

Case one, the cross–coupling reaches a value of approximately k_{yx}=33300x10^4N/m and k_{xy}= -14431x10^4N/m. It is evident from the data shown in Fig.3, with the plain bearings, instability is predicted to occur after a period of time.

Case two, where the cross–couplings are taken as k_{yx}= 33300x10^3, k_{xy}= 14413x10^3 N/m. This case was more stable in which the tilting pad bearings are predicated to yield stable operation(Fig.4). The data referred to above case one and case two were used in the forward calculations which is drawn in Fig.3 and Fig.4 which were found in good agreement with the work of Sharan and Rao[5]. The bearing has inside radius of the journal is 0.0508m, thickness 0.01m, and length 0.0508m. The material of the bearing is Aluminum alloy , elastic modulus is 73GPa.

For case one, the forces in the plain bearing for a grinding machine rotor increase with time which makes the rotor unstable, these forces are shown in Fig.3, while in Fig.4, with positive cross-coupling stiffness the forces are approximately in a sine wave equation with maximum forces in x and y-directions as 48.79 and 153.3 N respectively.

The minimum forces can be found from data of fig.4 in x and y-direction as -112.70N and -159.67N. The forces are shown in fig.4 should be avoided in the operation of rotating machinery as they cause fluctuating stresses, which in turn may rapidly induce fatigue failure. the maximum forces are compression on the bearing while the minimum forces are in tension.

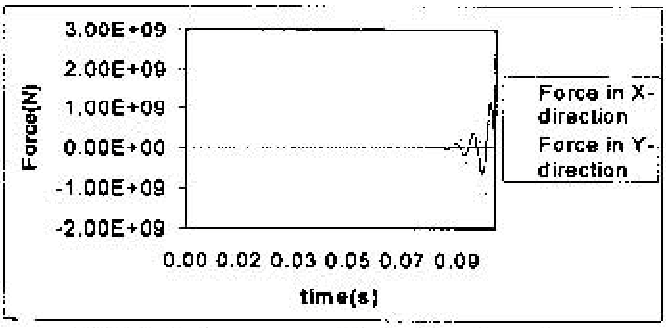


Fig.3 The force exerted in the bearing of a grinding machine rotor for case one

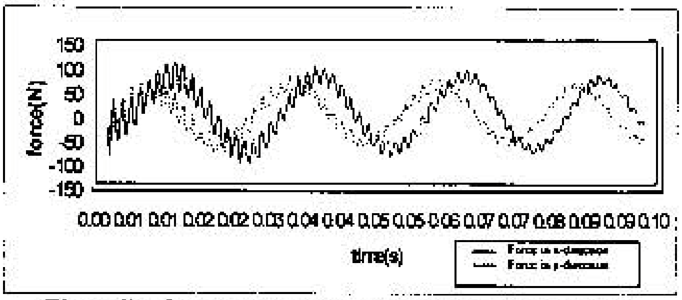
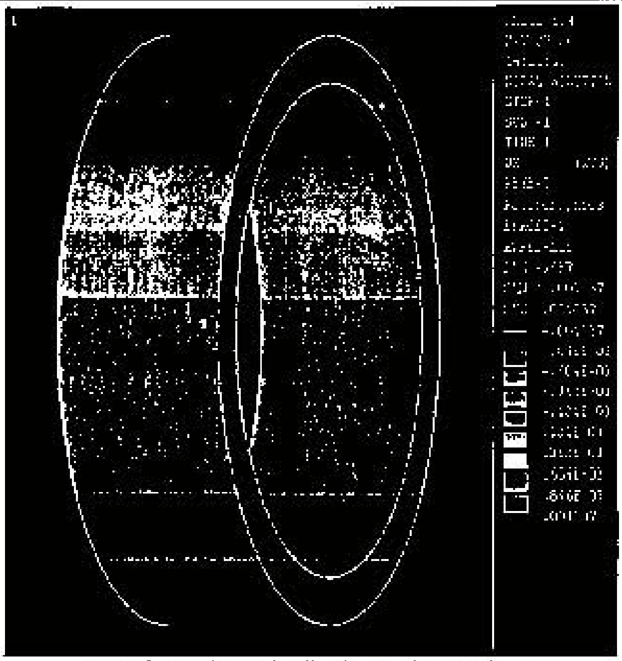


Fig.4 The force exerted in the bearing of a grinding machine rotor in the x and y-direction for case two

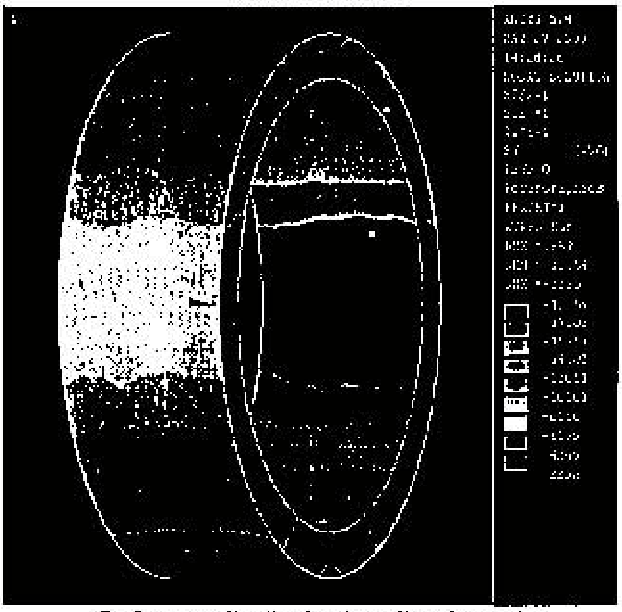
ANSYS data are illustrated in Fig.5 and Fig.6. Fig.5A shows the deformation of the bearing under action of maximum force. From Table 1 it can be found that the maximum deformation is (0.001087 m) occurred in the top surface for bearing, which mean increasing in the dimension. Also, it can be found that the maximum deformation (-0.001087 m) occurred in the bottom surface which mean decreasing in the dimensions. In contrast to Fig.6A, which the deformation was carried out under action of the minimum force, the top surface was deformed by (-0.001145 m) while the bottom surface was deformed by (0.001145 m).

Table1 Maximum and minimum deformations in the bearing

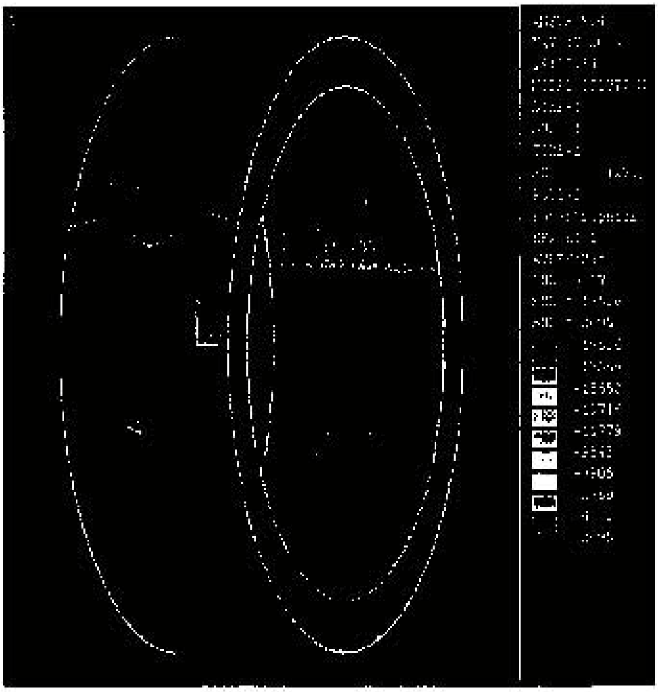
Case two	Deformation(m)
At maximum unbalance forces	0.001087
	-0.001087
At minimum unbalance forces	0.001145
	-0.001145



A- Deformations distribution under maximum unbalance forces

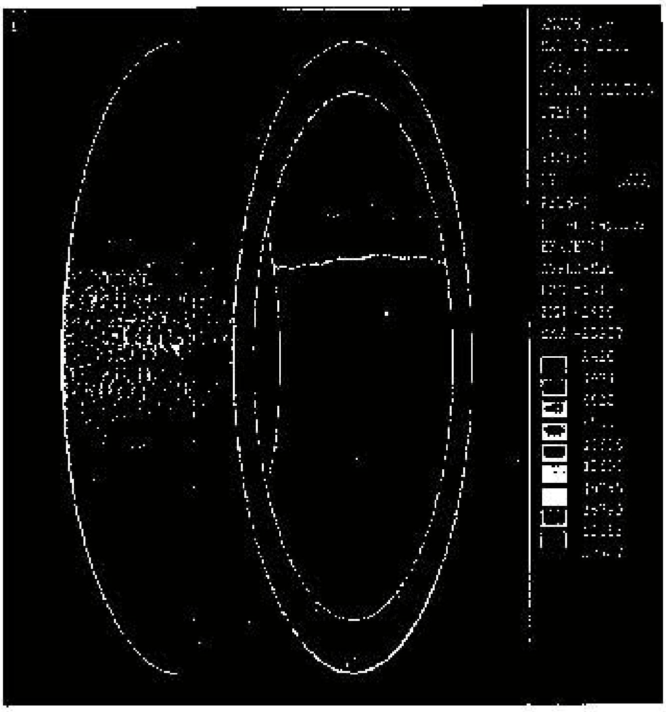


B- Stresses distribution in y-direction under maximum unbalance forces

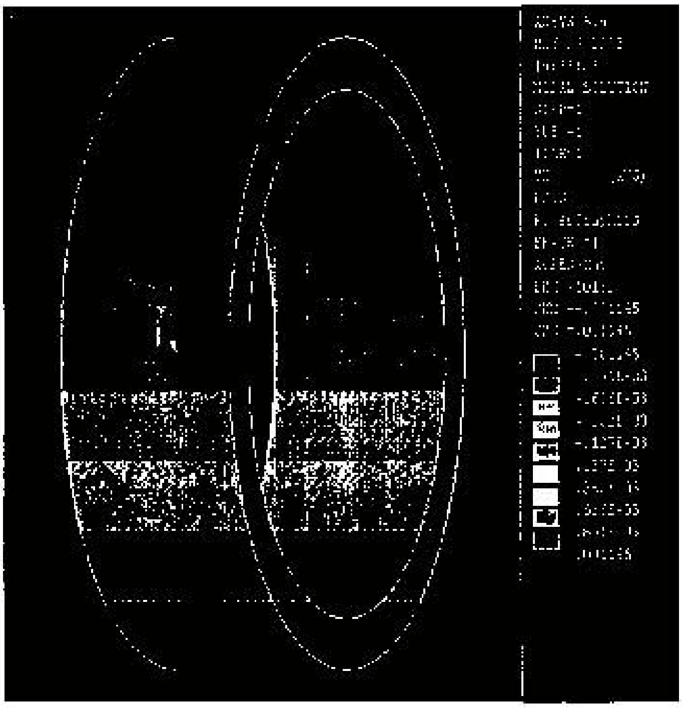


C- Stresses distribution in x-direction under maximum unbalance forces

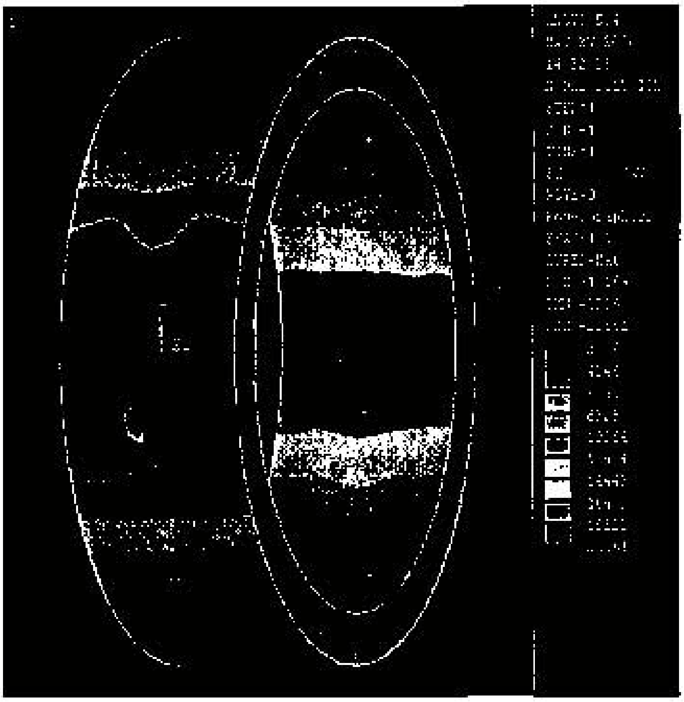
Fig.5 stresses and deformations distribution in the bearing under maximum unbalance forces (case two)



B- Stresses distribution in y-direction under minimum unbalance forces



A- Deformations distribution under minimum unbalance forces



C- Stresses distribution in x-direction under minimum unbalance forces

Fig.6 stresses and deformations distribution in the bearing under minimum unbalance forces (case two)

Fig.5B and Fig.6B show the stresses distribution in y-direction. From Table2 and Fig.5B, the maximum stress occur in besides and in the internal surface of magnitude (-2298 Pa) and the minimum stress is (-19854 Pa) in the external surface, top and bottom of the bearing. These stresses are compressive because of the compressive coefficient forces on the bearing surface. The maximum stress is found from Fig.6B is (20907 Pa) in the external surface, top and bottom, of the bearing. While the minimum stress is (2420 pa) occur in besides and in the internal surface, since the coefficient forces became tension on the bearing surface. It cause a tension stresses as in Fig.6B. Fig.5C and Fig.6C show the stresses distribution in x-direction. From Fig.5C, the maximum stress on the bearing is (-2095 Pa) in the top and bottom of the internal bearing surface and the minimum stress is (-19526 Pa) occur in besides and in the external surface. In Fig.6C the magnitude of the stress in besides and in the external surface is (20561 Pa) and the stress in the top and bottom of the of internal bearing surface is (2206 Pa).

Table 2 Maximum and minimum stresses in the bearing

Case two	Stress(pa) (y-direction)	Stress(pa) (x-direction)
At maximum unbalance forces	-2298	-2098
	-19854	-19526
At minimum unbalance forces	20907	20561
	2420	2206

Conclusions

1- ANSYS program gives good result in determining stress and deformation for journal bearing system.

2- The maximum deformation found in top and bottom of bearing when a minimum unbalance force is applied. The deformation in the top is negative(decrease the dimensions) while in the bottom is positive(increase the dimensions).

3- A fluctuating stresses are act on the bearing in x and y-direction. In y—direction, when the minimum forces are act, a maximum tensile stress occur and located at the external surface in the top and bottom of the bearing. When the maximum forces are act, a maximum compressive stress occur in the internal surface beside the bearing.

4- In x-direction, when the minimum forces are act on the bearing, a maximum tensile stress is occur and located at external surface beside the bearing. But when the maximum forces are act on the bearing, a maximum compressive stress is occur and located at the internal surface in the top and bottom of the bearing.

4- Maximum fluctuating stress is occur in y-direction which cause a fatigue failure.

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