

On Chaotic Properties of Convergent Sequences Maps on G -space

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Abstract

We discuss the convergence for some chaotic properties on G -space. We give a new definition of sensitive dependence on initial conditions and expansive on a topological G -space, also we prove some results about them and we study mixing , expansive , blending and minimal by a G -orbital convergence. Also, we study and prove some of these properties of a sequence of maps by a conjugation and a product on G -space .

Key words: G -mixing map, G -space, G -sensitive dependence on initial conditions map, G -strongly blending, G -orbital convergence.

الخلاصة

ناقشنا التقارب لبعض الخواص الفوضوية في فضاء G حيث عرفنا الحساسية المعتمدة على الشروط الابتدائية في فضاء G -التبولوجي ودرسنا بعض خواصها . كذلك درسنا الخلط ، المزج والدالة الأصغرية بواسطة التقارب المساري- G ، أخيرا درسنا وبرهنا بعض هذه الخواص لمتتابعة من الدوال بواسطة الترافق والضرب الجدائي في فضاء G .
الكلمات المفتاحية: دالة الخلط- G ، فضاء G ، دالة الحساسية المعتمدة على الشروط الابتدائية- G ، دالة ذات المزج العالي- G ، التقارب المساري- G .

1.Introduction

The chaos theory is one of the leading areas of researches in Mathematics. In the literatures, there are many researches on chaotic behavior of maps in topological dynamical systems. Also, there are several chaotic properties (for example; transitive, mixing, sensitivity etc.) which are studied deeply. (Das and Das, 2012), have introduced several theorems of convergence on G -space, and they have studied some results of topological transitive on metric G -space, also they showed that there are results giving sufficient conditions for the topological G -transitivity of the limit maps. (Mangang, 2014), has studied the minimality and equicontinuity of a sequence of maps in iterative way, he has proved that if P_n is a sequence of equicontinuous maps converges orbitally to a map P then P is also equicontinuous. (Thakkar and Das, 2014), have studied the expansiveness of a sequence of maps and they proved that it is preserved by a product. In our work, we generalize some results to above researchers on G -space. Also, we prove many results by a product and topological conjugation on G -space .

2.Preliminaries

Let \mathbb{Z} be the set of integers and \mathbb{N} be the set of natural numbers. We call a triple (Y, G, θ) be a transformation group, if it satisfy the following: Y be a Hausdorff space, G is a topological group and $\theta: G \times Y \rightarrow Y$ is a continuous action of G on Y . For $\theta(g, y)$ will be denoted by $g.y$. By trivial action of G on Y , we mean $g.y = y$ for all $g \in G, y \in Y$, (Das R. 2013). If $P: Y \rightarrow Y$ be a map on G -space, we say P is a G -map.

In this section, we introduce some necessary definitions which we need in our results.

Definition:(2.1) (Das and Das, 2012)

Let (Y, d) be a metric G -space, $\{P_n\}_{n=0}^{\infty}$ be a sequence of continuous maps from a metric G -space Y to itself. $\{P_n\}_{n=0}^{\infty}$ is **G -orbitally convergent** to a G -map

$P: Y \rightarrow Y$ if there exists an $n_0 \in \mathbb{N}$ for every $\epsilon > 0$ such that $\forall g, h \in G$, we have $d(g.P_n^m(y), h.P_n^m(y)) < \epsilon$, for all $y \in Y$, for all $m \in \mathbb{N}$, and for all $n \geq n_0$.

Definition:(2.2)

Let $P: Y \rightarrow Y$ be a G -map. P is called **topologically G -transitive** if for all A, B nonempty open sets of Y there exist $n \in \mathbb{N}$ and $g \in G$ such that $g.P^n(A) \cap B \neq \emptyset$.

Definition:(2.3)

Let $P: Y \rightarrow Y$ be a G -map. P is called **topologically G -mixing** if for all A, B nonempty open sets of Y there exist $n_0 \in \mathbb{N}$ and $g \in G$ such that $g.P^n(A) \cap B \neq \emptyset$, for all $n \geq n_0$.

Remark:(2.4)

We notice that from Definitions (2.2) and (2.3) a topological G -mixing implies a topological G -transitive. But the converse is not true.

It is know that there is a new definition of a chaos is " blending ". Although there is similarity between a blending and transitivity but there is no relation between them.

Definition:(2.5)

Let $P: Y \rightarrow Y$ be a G -map. P is called **G -weakly blending**, if for any nonempty open sets A and B of Y there exist $n > 0$ and $g, h \in G$ such that $g.P^n(A) \cap h.P^n(B) \neq \emptyset$.

Definition:(2.6)

Let $P: Y \rightarrow Y$ be a G -map. P is called **G -strongly blending** if for any nonempty open sets A and B of Y there exist $n > 0$ and $g, h \in G$ such that $g.P^n(A) \cap h.P^n(B) = W$; where W is open subsets of Y .

Remark:(2.7)

From Definitions (2.5) and (2.6), we can say that G -strongly blending implies G -weakly blending. But the converse is not true.

It is well known that the definition of sensitive dependence on initial conditions is metric property thus, we give a new definitions of a sensitive dependence on initial conditions and expansiveness on a topological G -space different from definitions on metric spaces as follows.

Definition:(2.8)

Let $P: Y \rightarrow Y$ be a G -map. P is called **G -sensitive dependence on initial conditions** at $y \in Y$ if for any open set A , containing y there exist $g, h \in G$ and $z \in A$ with $z \notin G(y)$, then there are $n \in \mathbb{N}$ and B open set of Y such that $g.P^n(y) \in B$ and $h.P^n(z) \notin \bar{B}$. Equivalently, if $\forall z \in G(y), \forall n \in \mathbb{N}$ then $g.P^n(y) \in V$ and $h.P^n(z) \in \bar{V}$.

Definition:(2.9)

Let $P: Y \rightarrow Y$ be a G -map. P is called **G -expansive** at $y \in Y$ if for any open set A , containing y there exist $g, h \in G$ and for all $z \in A$ with $z \notin G(y)$, then there are $n_0 \in \mathbb{Z}$ and open set B of Y such that $g.P^n(y) \in B$ and $h.P^n(z) \notin \bar{B}$, $\forall n \geq n_0$, if n is positive integer then P is positive G -expansive. Equivalently, if there exist $y, z \in Y$, $z \in G(y)$, $\forall n \in \mathbb{N}$ then $g.P^n(y) \in B$ $h.P^n(z) \in \bar{B}$.

Remark:(2.10)

We notice that from Definitions (2.8) and (2.9), positive G -expansive implies G -expansive and G -expansive implies G -sensitive dependence on initial condition. But the converse is not true.

Definition:(2.11)

Let Y and Z be G_1 and G_2 spaces respectively. The maps $P: Y \rightarrow Y$ and $Q: Z \rightarrow Z$ are said to be G -topologically conjugate if:

(1) there exists a homeomorphism $\varphi: Y \rightarrow Z$ such that $\varphi P = Q\varphi$.

(2) $\varphi(g_1 \cdot y) = g_2 \cdot \varphi(y)$; for each $g_1 \in G_1, g_2 \in G_2$ and y in Y .

In this case, we say that P and Q are G -topologically conjugate.

Definition:(2.12)

Let $P: Y \rightarrow Y$ be a G -map. P is called **G -minimal** if each G -orbit of $y \in Y$ is dense in Y .

3. Chaotic Properties via Convergence Orbital

In this section, we define some definitions and we prove results for some chaotic properties by convergence orbital on G -space.

Definition:(3.1)

Let $P: Y \rightarrow Y$ be a G -map. P is called **equicontinuous** on a metric G -space if for any $\epsilon > 0$ there exists $\delta > 0$ and there are $g, h \in G$ such that when $d(y, z) < \delta$ then $d(g \cdot P^m(y), h \cdot P^m(z)) < \epsilon$, for all $y, z \in Y, \forall m \geq 1$.

Theorem:(3.2)

Let (Y, d) be compact metric G -space, $\{P_n\}_{n=0}^\infty$ be a sequence of continuous maps G -orbitally convergent to a map $P: Y \rightarrow Y$. If P_n is G -equicontinuous then so is P .

Proof:

Since P_n is a G -orbitally convergent to P , then there is $k \in \mathbb{N}$ for all $y \in Y$ and for all $\epsilon > 0$ such that $d(g_1 \cdot P_n^m(y), h_1 \cdot P^m(y)) < \frac{\epsilon}{4}, \forall g_1, h_1 \in G, \forall m \in \mathbb{N}$ and $\forall n \geq k$, also $\forall g_2, h_2 \in G, \forall z \in Y$, we have $d(g_2 \cdot P_n^m(z), h_2 \cdot P^m(z)) < \frac{\epsilon}{4}$. Since P_n is G -equicontinuous, then there exists $\delta > 0$, for all $\epsilon > 0$, such that $d(g_1 \cdot P_n^m(y), g_2 \cdot P_n^m(z)) < \frac{\epsilon}{4}$. Thus, we have $d(h_1 \cdot P^m(y), h_2 \cdot P^m(z)) \leq d(g_1 \cdot P_n^m(y), h_1 \cdot P^m(y)) + d(g_1 \cdot P_n^m(y), h_2 \cdot P^m(z)) \leq d(g_1 \cdot P_n^m(y), h_1 \cdot P^m(y)) + d(g_1 \cdot P_n^m(y), g_2 \cdot P_n^m(z)) + d(g_2 \cdot P_n^m(y), h_2 \cdot P^m(z)) < \frac{\epsilon}{4} + \frac{\epsilon}{4} + \frac{\epsilon}{4} = \frac{3}{4}\epsilon$. And consequently, we get that P is G -equicontinuous.

We need to generalize the definitions: G -Lyapunov stable point and G -sensitive dependence on initial conditions point on metric G -space:

Definition:(3.3)

Let $P: Y \rightarrow Y$ be a G -map. A point $y \in Y$ is called **G -Lyapunov stable** of P on a metric G -space if there exist $\delta > 0$ and $g, h \in G$ for all $\epsilon > 0$ such that for all $z \in Y$, we have $d(g \cdot P^n(y), h \cdot P^n(z)) < \epsilon, \forall n > 0$ with $d(y, z) < \delta$.

Theorem:(3.4)

Let (Y, d) be a compact metric G -space, $\{P_n\}_{n=0}^\infty$ be a sequence of continuous maps G -orbitally convergent to a G -map $P: Y \rightarrow Y$. If P_n has G -Lyapunov stable point then P also has G -Lyapunov stable point.

Proof:

Since P_n G -orbitally convergent to P , then there exists $k \in \mathbb{N}$ for all $\epsilon > 0$ such that $d(g \cdot P_n^m(y), h \cdot P^m(y)) < \frac{\epsilon}{3}, \forall n \geq k, \forall y \in Y, \forall m \in \mathbb{N}$ and $\forall g, h \in G$. Suppose that $\delta > 0$ and let $y \in Y$, since P_n has G -Lyapunov stable point then for all $\epsilon > 0$ and for all $z \in Y$ there exist $g_1, h_1 \in G$ such that $d(g_1 \cdot P_n^m(y), h_1 \cdot P_n^m(z)) < \frac{\epsilon}{3}, \forall m, n \in \mathbb{N}$. Now, we have $d(h_1 \cdot P^m(y), h_2 \cdot P^m(z)) \leq d(g_1 \cdot P_n^m(y), h_1 \cdot P^m(y)) + d(g_1 \cdot P_n^m(y), h_2 \cdot P^m(z)) \leq d(g_1 \cdot P_n^m(y), h_1 \cdot P^m(y)) + d(g_1 \cdot P_n^m(y), h_1 \cdot P_n^m(z)) + d(h_1 \cdot P_n^m(z), h_2 \cdot P^m(z)) \leq \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} < \epsilon$. This mean $d(h_1 \cdot P^m(y), h_2 \cdot P^m(z)) < \epsilon$ for some $h_1, h_2 \in G$. Thus, P has G -Lyapunov stable point.

Definition:(3.5)

Let $P: Y \rightarrow Y$ be a G -map. A point $y \in Y$ is called **G -sensitive dependence on initial conditions** on a metric G -space if for all $\delta > 0$ there exists $\epsilon > 0$ such that

there are $z, w \in B_\delta(y)$ and $n \in \mathbb{N}$ implies that $d(g.P^n(z), h.P^n(w)) \geq \epsilon$ for some $g, h \in G$.

Remark:(3.6)

We can say $\{P_n\}_{n=0}^\infty$ is not converges G -orbitally to a G -map P if there exist $\epsilon > 0$ and $k \in \mathbb{N}$ such that for all $y \in Y$, we have $d(g.P_n^m(y), h.P^m(y)) \geq \epsilon$, $\forall n \geq k, \forall g, h \in G$.

Theorem:(3.7)

Let (Y, d) be a compact metric G -space, $\{P_n\}_{n=0}^\infty$ be a sequence of continuous maps is not G -orbitally convergent to a G -map $P: Y \rightarrow Y$. Then P has G -sensitive dependence on initial conditions point if P_n has G -sensitive dependence on initial conditions point.

Proof:

Let $\delta > 0$ and let $y \in Y$. Since P_n is not G -orbitally convergent to P , then by Remark (3.6) there exist $\epsilon > 0$ and $k \in \mathbb{N}$ such that $d(g.P_n^m(y), h.P^m(y)) \geq \frac{\epsilon}{3}$, for all $y \in Y, \forall n \geq k, \forall g, h \in G$. Since P_n has G -sensitive dependence on initial conditions point then there exist $z, w \in B_\delta(y)$ and $m \in \mathbb{N}$ such that $d(g_1.P_n^m(z), h_1.P_n^m(w)) \geq \frac{\epsilon}{3}$ for some $g_1, h_1 \in G$. Now we have $d(h_1.P^m(z), h_2.P^m(w)) \geq d(g_1.P_n^m(z), h_1.P^m(z)) + d(g_1.P_n^m(z), h_1.P_n^m(w)) + d(h_1.P_n^m(w), h_2.P^m(w)) \geq \frac{\epsilon}{3} + \frac{\epsilon}{3} + \frac{\epsilon}{3} \geq \epsilon$ so that $d(h_1.P^m(z), h_2.P^m(w)) \geq \epsilon$ and thus, P has G -sensitive dependence on initial conditions point.

Now, we give a new definition of G -orbitally convergence on topological G -space as follows:

Definition:(3.8)

Let $\{P_n\}_{n=0}^\infty$ be a sequence of continuous maps from topological G -space Y to itself. $\{P_n\}_{n=0}^\infty$ is called G -orbitally convergent to P if for all nonempty open set B of Y there exists $k \in \mathbb{N}$ such that $\forall g, h \in G$, we have $g.P_n^m(y), h.P^m(y) \in B, \forall n \geq k, \forall y \in Y$ and $\forall m \in \mathbb{N}$.

Theorem:(3.9)

Let $\{P_n\}_{n=0}^\infty$ be a sequence of continuous maps from a compact G -space Y to itself. If $\{P_n\}_{n=0}^\infty$ converges G -orbitally to a G -map P with $\{P_n^m(y)\}_{n=0}^\infty$ is dense set in $Y, \forall y \in Y, \forall m \in \mathbb{N}$ then P is G -minimal.

Proof:

Let B be any open set of Y , since P_n is G -orbitally convergent to P , then there exists $k \in \mathbb{N}$ such that $g.P_n^m(y), h.P^m(y) \in B, \forall n \geq k, \forall y \in Y, \forall m \in \mathbb{N}$ and $\forall g, h \in G$. Since $\{P_n^m\}_{n=0}^\infty$ is dense set in $Y, \forall y \in Y$ then $\overline{\{P_n^m(y)\}_{n=0}^\infty} = Y$, since $P_n(y) \rightarrow P(y), \forall y \in Y$ then $P_n^m(y) \rightarrow P^m(y), \forall y \in Y, \forall m \in \mathbb{N}$ as $n \rightarrow \infty$. $\{\{P_n^m(y)\}_{m=0}^\infty\}_{n=0}^\infty$ is a sequence of dense sets converges to a dense set, say A . By above $P_n^m(y) \rightarrow P^m(y), \forall y \in Y, \forall m \in \mathbb{N}$ as $n \rightarrow \infty$ so that $A = \{P^m(y)\}_{m=0}^\infty$ also dense set in $Y, \forall y \in Y$, that is, $\overline{\{P^m(y)\}_{m=0}^\infty} = Y$, and consequently P is G -minimal. اكتب المعادلة هنا.

4. G-topological Conjugation

In this section, we show that some chaotic properties are preserved by a topological conjugation on G -space.

Theorem:(4.1)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively, such that $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be

G -topologically conjugate by a homeomorphism $\varphi: Y \rightarrow Z$. If $\{P_n\}_{n=0}^\infty$ is a G_1 -mixing on Y then $\{Q_n\}_{n=0}^\infty$ is also G_2 -mixing on Z .

Proof:

Let $P_n: Y \rightarrow Y$ and $Q_n: Z \rightarrow Z$ be a topologically conjugated by $\varphi: Y \rightarrow Z$ such that $\varphi \circ P_n = Q_n \circ \varphi$. Let A_2, B_2 be a nonempty open sets in Z . A homeomorphism of φ implies that there exist A_1, B_1 in Y such that $\varphi^{-1}(A_2) = A_1$, $\varphi^{-1}(B_2) = B_1$, where $\varphi^{-1}(A_2), \varphi^{-1}(B_2)$ are nonempty open sets in Y . We have $\mu(g_1) = g_2$, $g_1 \in G_1, g_2 \in G_2$. Since P_n is G_1 -mixing then there exist $k_0 \in \mathbb{N}$ and $g_1 \in G_1$ such that $g_1 \cdot P_n^k(A_1) \cap B_1 \neq \emptyset, \forall k \geq k_0, \forall n \geq 0$, this mean there exists $y \in A_1$ such that $g_1 \cdot P_n^k(y) \in B_1$ which implies $g_1 \cdot P_n^k(A_1) \subseteq B_1$. Now, we have $\varphi(g_1 \cdot P_n^k(A_1)) \subseteq \varphi(B_1)$ implies that $g_2 \cdot \varphi(P_n^k(A_1)) \subseteq B_2$. Since P_n conjugates to Q_n by $\varphi, \forall n \geq 0$ then (by induction), we get P_n^k conjugates to Q_n^k by $\varphi, \forall n \geq 0$. So $g_2 \cdot Q_n^k(\varphi(A_1)) \subseteq B_2$ implies $g_2 \cdot Q_n^k(A_2) \subseteq B_2$, this means there is $z \in A_2$ such that $g_2 \cdot Q_n^k(z) \in B_2$ and consequently $g_2 \cdot Q_n^k(A_2) \cap B_2 \neq \emptyset$, for some $g_2 \in G_2, \forall n \geq 0$. Thus, we get that $\{Q_n\}_{n=0}^\infty$ is also G_2 -mixing on Z .

Since G -mixing implies to G -transitive, we notice that Theorem (4.1) is also satisfying of G -transitive as the following corollary:

Corollary:(4.2)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively, such that $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be G -topologically conjugate by a homeomorphism $\varphi: Y \rightarrow Z$. If $\{P_n\}_{n=0}^\infty$ is a G_1 -transitive on Y then $\{Q_n\}_{n=0}^\infty$ is also G_2 -transitive on Z .

Theorem:(4.3)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively, such that $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be G -topologically conjugate by a homeomorphism $\varphi: Y \rightarrow Z$. If $\{P_n\}_{n=0}^\infty$ is a G_1 -strongly blending on Y then $\{Q_n\}_{n=0}^\infty$ is also G_2 -strongly blending on Z .

Proof:

Let $P_n: Y \rightarrow Y$ and $Q_n: Z \rightarrow Z$ be a G -topologically conjugated by $\varphi: Y \rightarrow Z$ such that $\varphi \circ P_n = Q_n \circ \varphi$. Let A_2, B_2 be any nonempty open subsets of Z . Since φ is a homeomorphism then $\varphi^{-1}(A_2) = A_1, \varphi^{-1}(B_2) = B_1$ and $\varphi^{-1}(V) = U$ are nonempty open sets in Y , where V is open subset of Z . We have $\mu(g_1) = g_2, \mu(h_1) = h_2, g_1, h_1 \in G_1, g_2, h_2 \in G_2$. Since P_n is G_1 -strongly blending then there exist $k > 0$ and $g_1, h_1 \in G_1$ such that $g_1 \cdot P_n^k(A_1) \cap h_1 \cdot P_n^k(B_1) = U, \forall n \in \mathbb{N}$. Now, we have $\varphi(g_1 \cdot P_n^k(A_1) \cap h_1 \cdot P_n^k(B_1)) = \varphi(U)$ implies that $\varphi(g_1 \cdot P_n^k(A_1)) \cap \varphi(h_1 \cdot P_n^k(B_1)) = \varphi(U)$ so that $g_2 \cdot \varphi(P_n^k(A_1)) \cap h_2 \cdot \varphi(P_n^k(B_1)) = V$. Since P_n conjugates to Q_n by $\varphi, \forall n \geq 0$ then P_n^k conjugates to Q_n^k by $\varphi, \forall n \geq 0$. So $g_2 \cdot Q_n^k(\varphi(A_1)) \cap h_2 \cdot Q_n^k(\varphi(B_1)) = V$ implies $g_2 \cdot Q_n^k(A_2) \cap h_2 \cdot Q_n^k(B_2) = V$, for some $g_2, h_2 \in G_2, \forall n \in \mathbb{N}$. Thus, we get that $\{Q_n\}_{n=0}^\infty$ is G_2 -strongly blending on Z .

Since G -strongly blending is G -weakly blending then Theorem (4.3) is also satisfying of G -weakly blending as the following corollary:

Corollary:(4.4)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively, such that $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be G -topologically conjugate by a homeomorphism $\varphi: Y \rightarrow Z$. If $\{P_n\}_{n=0}^\infty$ is a G_1 -weakly blending on Y then $\{Q_n\}_{n=0}^\infty$ is also G_2 -weakly blending on Z .

Theorem:(4.5)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively, such that $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be G -topologically conjugate by a homeomorphism $\varphi: Y \rightarrow Z$. If $\{P_n\}_{n=0}^\infty$ is a G_1 -minimal on Y then $\{Q_n\}_{n=0}^\infty$ is G_2 -minimal on Z .

Proof:

Let $P_n: Y \rightarrow Y$ and $Q_n: Z \rightarrow Z$ be a G -topologically conjugated by $\varphi: Y \rightarrow Z$ such that $\varphi \circ P_n = Q_n \circ \varphi$. Let A, B be any nonempty open subsets of Y and Z respectively. By hypothesis, for all point $y \in Y$ there exist $k \in \mathbb{N}$ and $g_1 \in G_1$ such that $g_1 \cdot P_n^k(y) \in A$. Now, we have $\mu(g_1) = g_2, g_1 \in G_1, g_2 \in G_2$. Since φ is a homeomorphism then φ^{-1} is a continuous such that $\varphi^{-1}(B)$ is a nonempty open set in Y , where $g_1 \cdot P_n^k(y) \in \varphi^{-1}(B)$. So, we have $\varphi(g_1 \cdot P_n^k(y)) \in B$ implies $g_2 \cdot \varphi(P_n^k(y)) \in B$. Since P_n conjugates to Q_n by $\varphi, \forall n \geq 0$ then (by induction) P_n^k conjugates to Q_n^k by $\varphi, \forall n \geq 0$ implies that $g_2 \cdot Q_n^k(\varphi(y)) \in B$. Since φ is onto then for all $z \in Z, \varphi(y) = z$. So, we get $g_2 \cdot Q_n^k(z) \in B$, for all $z \in Z$ and consequently $\{Q_n\}_{n=0}^\infty$ is G_2 -minimal on Z .

5. Product map

In this section, we show that some chaotic properties are preserved by product map on G -space.

We recall the next theorem which is proved by (Das R. and Das T. 2012), on metric G -space, also, we can generalize this on topological G -space.

Theorem:(5.1) (Das R. and Das T. 2012)

Let (Y, d) be a metric G -space, $\{P_n\}_{n=0}^\infty$ be a sequence of self-maps converging G -orbitally to a G -map $P: Y \rightarrow Y$. If $\{P_n\}_{n=0}^\infty$ is G -transitive then so is P .

We can satisfying Theorem (5.1) on product map for two sequences when Y, Z are G -spaces as the following:

Theorem:(5.2)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively. If $\{P_n \times Q_n\}_{n=0}^\infty$ be a sequence of $G_1 \times G_2$ -mixing maps on $Y \times Z$ converging G -orbitally to a map $P \times Q: Y \times Z \rightarrow Y \times Z$ then $P \times Q$ is also $G_1 \times G_2$ -mixing.

Proof:

Let A_1, A_2 be a subsets of Y and B_1, B_2 be a subsets of Z so $(A_1 \times B_1)$ and $(A_2 \times B_2)$ be a nonempty open subset of $Y \times Z$ and let $U \times V = (U_1 \times V_1) \cup (U_2 \times V_2)$. Since $P_n \times Q_n$ converges G -orbitally to $P \times Q$, then for all $(y, z) \in Y \times Z$ and for every $U \times V$ open subset of $Y \times Z$ then there exists $k \in \mathbb{N}$ such that $(g_1 \cdot P_n^{m_1}(y), h_1 \cdot Q_n^{m_2}(z))$ and $(g_2 \cdot P_n^{m_1}(y), h_2 \cdot Q_n^{m_2}(z)) \in U_1 \times V_1, \forall m_1, m_2 \in \mathbb{N}, \forall g_1, g_2 \in G_1, h_1, h_2 \in G_2, \forall n \geq k$. Since $(A_2 \times B_2)$ is open, there exist $U \times V$ open set of $Y \times Z$ and $(w_1, w_2) \in Y \times Z$ such that $(w_1, w_2) \in U \times V \subset (A_2 \times B_2)$. Since $P_n \times Q_n$ is $G_1 \times G_2$ -mixing, then there exist $k_1, k_2 \in \mathbb{N}$ and $g_1 \in G_1, h_1 \in G_2$ such that $(g_1 \cdot P_n^{m_1}(A_1) \cap A_2) \times (h_1 \cdot Q_n^{m_2}(B_1) \cap B_2) \neq \emptyset, \forall m_1 \geq k_1, \forall m_2 \geq k_2$, hence there exists $(a_1, b_1) \in (A_1 \times B_1)$ satisfying $(g_1 \cdot P_n^{m_1}(a_1) \times h_1 \cdot Q_n^{m_2}(b_1)), (w_1, w_2) \in U_2 \times V_2$ thus, we have $g_1 \cdot P_n^{m_1}(a_1) \times h_1 \cdot Q_n^{m_2}(b_1), g_2 \cdot P_n^{m_1}(a_1) \times h_2 \cdot Q_n^{m_2}(b_1) \in U_1 \times V_1$ and $g_1 \cdot P_n^{m_1}(a_1) \times h_1 \cdot Q_n^{m_2}(b_1), (w_1, w_2) \in U_2 \times V_2$. So we get $g_1 \cdot P_n^{m_1}(a_1) \times h_1 \cdot Q_n^{m_2}(b_1), (w_1, w_2) \in U_1 \times V_1 \cup U_2 \times V_2 = U \times V$ which implies $g_1 \cdot P_n^{m_1}(a_1) \times h_1 \cdot Q_n^{m_2}(b_1) \in U \times V \subset (A_2 \times B_2)$

so that $(g_1.P^{m_1}(A_1) \times h_1.Q^{m_2}(B_1)) \cap (A_2 \times B_2) = (g_1.P^{m_1}(A_1) \cap A_2) \times (h_1.Q^{m_2}(B_1) \cap B_2) \neq \emptyset$, and consequently $P \times Q$ is $G_1 \times G_2$ -mixing.

Since G -mixing implies G -transitive then Theorem (5.2) is also satisfy of G -transitive as:

Corollary:(5.3)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively . If $\{P_n \times Q_n\}_{n=0}^\infty$ be a sequence of $G_1 \times G_2$ -transitive maps on $Y \times Z$ converging G -orbitally to a map $P \times Q: Y \times Z \rightarrow Y \times Z$ then $P \times Q$ is also $G_1 \times G_2$ -transitive.

Theorem:(5.4)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively . If $\{P_n\}_{n=0}^\infty$ be G_1 -expansive on Y and $\{Q_n\}_{n=0}^\infty$ be G_2 -expansive on Z then $\{P_n \times Q_n\}_{n=0}^\infty$ is $G_1 \times G_2$ -expansive on $Y \times Z$.

Proof:

Let $(y, z) \in Y \times Z$ and there are $g \in G_1, h \in G_2$ such that for any $n, k \geq 0$, $(g, h). (P \times Q)_n^k(y, z) = g.P_n^k(y), h.Q_n^k(z)$. Let B_1 and B_2 be any two nonempty open subsets of Y and Z respectively . Now, let $B = B_1 \cap B_2$ such that $\bar{B} = \bar{B}_1 \cap \bar{B}_2$ and $(y_1, z_1), (y_2, z_2) \in Y \times Z$. If for all $n, k \geq 0$, $(g_1, h_1). (P_n^k \times Q_n^k)(y_1, z_1) \in B$, $(g_2, h_2). (P_n^k \times Q_n^k)(y_2, z_2) \in \bar{B}$, $g_1, g_2 \in G_1, h_1, h_2 \in G_2$ then $g_1.P_n^k(y_1), h_1.Q_n^k(z_1) \in B \subseteq B_1$, $g_2.P_n^k(y_2), h_2.Q_n^k(z_2) \in \bar{B} \subseteq \bar{B}_1$ and $h_1.Q_n^k(z_1) \in B \subseteq B_2, h_2.Q_n^k(z_2) \in \bar{B} \subseteq \bar{B}_2$ for all $n, k \geq 0$. Notes that by G -expansiveness of P_n and Q_n , we get $y_1 = y_2$ and $z_1 = z_2$, that is, $(y_1, z_1) = (y_2, z_2)$ so that $\{P_n \times Q_n\}_{n=0}^\infty$ is $G_1 \times G_2$ -expansive on $Y \times Z$.

Since G -expansive implies to G -sensitive then Theorem (5.4) is also satisfy of G -sensitive as:

Corollary:(5.5)

Let Y and Z be G_1 and G_2 spaces respectively, $\{P_n\}_{n=0}^\infty$ and $\{Q_n\}_{n=0}^\infty$ be two sequences of self-maps on Y and Z respectively . If $\{P_n\}_{n=0}^\infty$ be G_1 -sensitive on Y and $\{Q_n\}_{n=0}^\infty$ be G_2 -sensitive on Z then $\{P_n \times Q_n\}_{n=0}^\infty$ is $G_1 \times G_2$ -expansive on $Y \times Z$.

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