A New Design for Linear Phase FIR Digital Filter with Efficient Realization

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Abstract:

In this paper, the design of linear phase FIR digital filter using Frequency Sampling method is presented. Such design is achieved with a reduction in the maximum stop-band ripples utilizing optimal transition-band sample value throughout the use of Golden Section search method for single transition samples, and with aid of Steepest Descent method for double transition samples. The realization requirements of such filters are reduced by the use of a new analytic design. The reduction can be increased to 50% of the whole filter structure. Therefore, the designed FIR filter offers global properties, minimum stop-band, minimum pass-band, average deviation, and reduced structure complexity.

Key words: *FIR digital filter, Frequency sampling method, Optimal transition-band samples, Reduced structure complexity.*

طريقة جديدة لتصميم مرشح رقمي ذو الدفقة المحدودة و الطور الخطي مع تحقيق كفوء

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الخلاصة:

في هذا البحث تم تصميم مرشحات رقمية نوع إستجابة الدفقة المحدودة ذات الطور الخطي (FIR)بإستخدام طريقة العينات الترددية. تمت معالجة المشاكل التي تعاني منها المرشحات من هـذا النوع وهي التعقيدات الناتجة عن التصميم بمراتب عالية للحصول على حزمة إنتقال ضيقة للإسـتجابة الترددية مع مراعاة كون الإستجابة الترددية هي الأمثل في حزمتي المنع والمرور وأظهرت الطريقـة التحليلية للتصميم نتائج مفيدة من جهة تحقيق المرشح حيث نتج عدد من المعاملات ذات قيمة صفرية مما أدى إلى إنخفاض بعدد المضارب المطلوبة للبناءإلى ما يقارب نصف مرتبة المرشح.

إن جمع تلكما الخاصيتين في نوع المرشحات المستخدم (FIR) أدى إلى وجود طريقة جديدة للتصميم تضمنت الإستجابة الترددية المثلى وتحقيق بناء أقل تعقيداً من الطرق السابقة.

1- Introduction

FIR digital filters are straightforward to use in applications, whenever linear phase is require, or it is important not to generate noise within the filter [1]-[3]. Among all design techniques, there are essentially two well-known classes of design techniques for linear phase FIR filters; namely- the window method and Remez Exchange method. All of these design methods have weakness and strength points that must be known before use, to obtain the best performance [4]-[9].

Recently, many other non-classical methods are reported, such as Genetic Algorithm [10], Parks-McClellan, and Frequency sampling method [7],[8],[10]. It is well known that exact linear phase can only be achieved in FIR digital filter in which the impulse response is symmetric [7]-[9]. But to get the desired performance (*sharp transition region or large stop-band attenuation*) the FIR filter may need too large number of coefficients, which leads to complexity in design, realization, and computation, especially for high order filters [12],[13].

The realization of FIR digital filter is, basically made by direct method [4]-[8]. Later, many attempts are carried out to reduce the hardware implementations, such as, Chebyshev structure [14],[15], Minimum adder Multiplier Blocks [16], Dumpster MacLoad algorithm [1], and Sum of Powers of two [17]. All the above filter realization methods reduce the number of required multiplications, and /or reduce additions, or replacing multipliers with shifters and adders.

In this paper, a modification is achieved in both design and realization of FIR filters. From the design side of view, optical methods (Golden Section search, Steepest Descent algorithm) are used to improve the stop-band and pass-band intervals response of the FIR filter that is previously designed by the Frequency Sampling method. The design enhancement covers, in this paper, both single (a_{t1}) and double (a_{t1}, a_{t2}) transition- band samples. Such improvement is dealt with in Section 2. In Section 3, and throughout the use of Cholesky decomposition method the Frequency Sampling design gives rise to so many zero coefficients, thus, reducing the realization complexity of the above filter.

The FIR filter, that is optimally designed in section 2, is efficiently realized via the use of modified Frequency Sampling method of section 3, resulting in a novel FIR filter design and realization. This is well described in Section 4. Illustrative examples are shown in Section 5. In Section 6 some conclusions are given.

2- A Modified Design Method

Frequency Sampling method is one of the methods used in FIR digital filter design. This method has no direct counterpart in the theory of analog filters. It gives great flexibility over the choice of filter magnitude characteristic. It also produces FIR filter, which offers the advantages of linear phase response [10]. The desired frequency response in such method is specified in discrete frequency domain, and then by the use of the Inverse Discrete Fourier Transform (IDFT), the corresponding discrete time impulse response is obtained [8].

Frequency Sampling filters are based on specification of set of samples of the desired frequency response at N uniformly spaced points around a unit circle as shown in Fig. 1. The relation determines the set of frequencies that have been used until the point

$$f_i = \frac{i}{N}, \quad i = 0,1,\ldots, N\text{-}1 \qquad (1)$$

corresponding to the N frequencies at which an N point DFT is evaluated. There is a second set of uniformly spaced frequencies for which a Frequency Sampling structure can conveniently be obtained. The relation determines this set of frequencies is

$$f_i = \frac{(i+1/2)}{N}, \quad i=0,1,\ldots,N\text{-}1 \quad (2)$$

Eqn. (1) is called Type 1, while eqn. (2) is called Type 2.

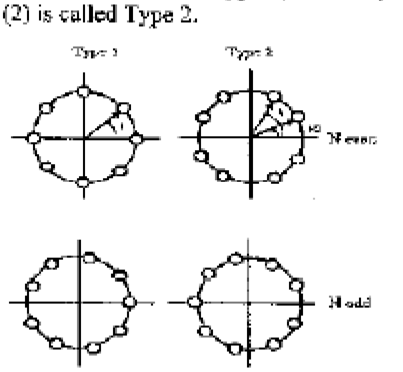

Fig. 1Two types of frequency sampling grid points for N-even and N-odd.

The angular spacing between frequency sampling points is equal to $\Theta = 1/N$. Type 1 design has the initial point at $f = 0$, whereas Type 2 design has the initial point at $f = \Theta/2$ [4],[18].

For symmetric odd length filters, the formula for obtaining FIR coefficients is [8]:

$$h[n] = \frac{1}{N}\left[A[0] + \sum_{j=0}^{M} 2A[j]\cos\left[\frac{2\pi(n-M)j}{N}\right]\right] \qquad (3)$$

where

N is number of FIR samples,

$A[.]$ is discrete desired frequency samples, and

$$M = \left(\frac{N-1}{2}\right)$$

The technique of Frequency Sampling can be used to design a wide variety of filters (Low-pass filters (LPF), Band-pass filters (BPF), and wide-band filters (WBF)). Frequency response usually contains ripples in the stop and pass-band intervals. The samples in these bands are initially specified (*fixed values given in the design*), but the frequency samples which occur in transition-band are made to be unspecified variables whose values are chosen by an optimization algorithm which minimizes some function of the approximation error of the filter, like reduction of pass-band ripples level or, reduction of stop-band ripples level or even both. Arbitrary choice of sample's value in transition-band interval (e.g., 0.5 of the maximum value) may lead to some reduction in ripples level. As example, if the total number of samples (N)=19 and pass-band samples (k)=9, the frequency response will be as shown in Fig. 2. When transition-band sample (a_{tt}) with value 0.5 is chosen, the frequency response is also shown in Fig. 2. It can be seen from comparing the last two responses that the peek stop-band ripple has been reduced from -15.5dB to -29dB, and average deviation in pass-band has been these reduced from 5% to 3.5%. More reduction in these ripples can be obtained by optimizing the single transition-band sample's value (a_{tt}) [4],[9]. Such optimization is achieved by using Golden Section search. The name comes from the fact that the most efficient search results when the middle point of the bracket is a function distance 0.61803 from one end point and 0.38197 from the other [9]. In addition to, limitation in transition-band sample values in search (0.61803-0.38197), the starting point in search from minimum peak is the frequency at first sample after the optimized one. Normally, the optimization process is terminated when the peak ripple changes by a value that is less than a given value called stopping tolerance. For our work, the stopping tolerance is chosen (0.001). When the Golden Section method is applied to the filter with N=19, and K=9, the optimized single transition-band sample (a_{t1}) value is 0.404639 (see Fig. 3.a) that reduces peak ripple to -41.4dB as shown in Fig. 2.

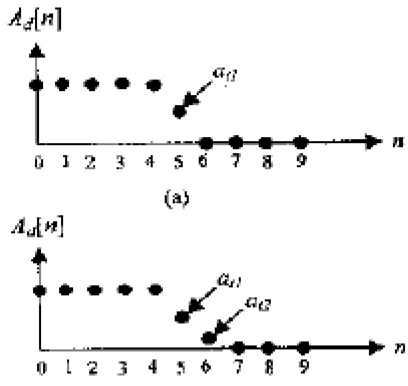

Fig. 3 Discrete frequency magnitude response for a low pass FIR filter with
(a) Single transition-band sample,
(b) Double transition-band samples.

Using double transition-band samples produce more and more reduction in stop-band ripples in the FIR digital filters response [9]. Golden Section search and steepest Descent methods are both used in optimizing double transition-band samples (see Fig. 3.b). For the same filter (N=19, k=9) with optimized double transition-band samples a_{t1}=0.5668437, a_{t2}=0.0904549. Fig. 2 illustrates also the FIR filter response with peak ripple of -73.3dB, and average deviation 1.3% from the ideal filter. It can be seen that the average deviation in pass-band decreases for each of the above cases, as well, Table 1 shows the reduction in the stop-band peak ripple and pass-band average deviation (Δ%) from the ideal filter for each of the discussed cases.

3- New Design leads to Reduced Realization

A non-classical design method is to be considered here. Such a method depends on the design of a mirror image polynomial $P(z)$ for the sampled version of the desired frequency response. This mirror image polynomial can be expressed by

$$P(z) = \sum_{n=-N}^{N} h(n) z^{n} \qquad (4)$$

and

$$P(e^{j\omega}) = h(0) + 2\sum_{n=1}^{N} h(n)\cos(n\omega) \qquad (5)$$

$P(e^{j\omega})$ is required to meet the corresponding amplitude response $A(\omega)$ of the digital filter. To achieve such sampled version of the amplitude response $A(\omega_m)$ for $m = 0,1,\ldots,M$, when ω_0=0 and $\omega_M = \pi$, the error is defined as

$$E(\omega_m) = \left| P(e^{j\omega_m}) - A(\omega_m) \right| \qquad (6)$$

is forced to be zero at $m = 0,1,\ldots,M$, i.e.,

$$P(e^{j\omega_m}) = A(\omega_m) \quad (7)$$

or

$$h(0)+2\sum_{n=1}^{N} h(n)\cos(n\omega_m) = A(\omega_m)$$
$$\text{for } m = 0,1,\ldots,M \quad (8)$$

Eqn. (8) can be written in matrix form as

$$\boldsymbol{CH} = \boldsymbol{A} \quad (9)$$

where

$$C=\begin{bmatrix} 1 & \cos\omega_0 & \cos 2\omega_0 & \ldots & \cos N\omega_0 \\ 1 & \cos\omega_1 & \cos 2\omega_1 & \ldots & \cos N\omega_1 \\ 1 & \cos\omega_2 & \cos 2\omega_2 & \ldots & \cos N\omega_2 \\ \vdots & \vdots & \vdots & & \\ 1 & \cos\omega_N & \cos 2\omega_N & \ldots & \cos N\omega_N \end{bmatrix}$$

(10.a)

$$H' = [h(0) \quad 2h(1) \quad 2h(2) \quad \ldots \quad 2h(n)]$$

(10.b)

and

$$A' = [A(\omega_0) \quad A(\omega_1) \quad A(\omega_2) \quad \ldots \quad A(\omega_M)]$$

(10.c)

The system of Eqn. (9) can be solved in an efficient manner by choosing $N = M$, $\omega_0 = 0$, $\omega_1 = \pi/N$ and

$$\omega_{n+1} - \omega_n = \omega_n - \omega_{n-1} = \pi/N \quad (11)$$

The $\boldsymbol{C}$ matrix will become symmetric, while $\boldsymbol{H}$ and $\boldsymbol{A}$ are column vectors of order $(N+1)$. Cholesky decomposition [19] is used for solving the matrix Eqn. (9). Matrix $\boldsymbol{C}$ is expressed in the form

$$\boldsymbol{C} = \boldsymbol{SRS}^t \quad (12)$$

where $\boldsymbol{S}$ is the lower triangular matrix (whose main diagonal elements are all 1's) and $\boldsymbol{R}$ is a diagonal matrix. The elements of the matrices $\boldsymbol{S}$ and $\boldsymbol{R}$ are readily determined by solving for the $(i,j)^{th}$ element of both sides of Eqn. (12), giving

$$c_{ij} = \sum_{k=0}^{j} s_{ik} r_k s_{jk} \text{ for } 0 \le j \le i-1 \quad (13)$$

or

$$s_{ij} r_{ii} = c_{ij} - \sum_{k=0}^{j-1} s_{ik} r_k s_{jk}$$
$$\text{for } 0 \le j \le i-1 \quad (14)$$

and for the diagonal elements

$$c_{ii} = \sum_{k=0}^{i} s_{ik} r_k s_{ik} \quad (15)$$

or

$$r_{ii} = c_{ii} - \sum_{k=0}^{i-1} s_{ik}^{\,2} r_k \quad (16)$$

with

$$r_0 = c_{00} = 1 \quad (17)$$

The recursive Eqns. (13) to (17) can be used to solve $\boldsymbol{S}$ and $\boldsymbol{R}$ matrices. Once these matrices have been determined, it is relatively simple to solve for the column vector $\boldsymbol{H}$ in a two step procedure. From Eqns. (9) and (12), we get

$$\boldsymbol{SRS}^t\boldsymbol{H} = \boldsymbol{A} \quad (18)$$

which can be rewritten as

$$\boldsymbol{SB} = \boldsymbol{A} \quad (19.a)$$

and

$$\boldsymbol{RS}^t\,\boldsymbol{H} = \boldsymbol{A} \quad (19.b)$$

or

$$\boldsymbol{S}^t\,\boldsymbol{H} = \boldsymbol{R}^{-1}\,\boldsymbol{B} \quad (19.c)$$

Thus, using the matrix $\boldsymbol{S}$, Eqn. (19.a) can be solved for the column vector $\boldsymbol{B}$ using a recursion of the form

$$b_i = a(\omega_i) - \sum_{j=0}^{i-1} s_{ij} b_j$$
$$\text{for } 1 \le i \le N \quad (20)$$

with initial condition

$$b_0 = a(\omega_0) \tag{21}$$

Having obtaining $\boldsymbol{B}$, Eqn. (19,e) can be solved recursively for $\boldsymbol{H}$ using the relation

$$h[i] = \frac{b_i}{2r_i} - \sum_{j=i+1}^{N} s_{ji} h[j] \quad \text{for } 1 \le i \le N-1 \tag{22}$$

and

$$h[0] = \frac{b_0}{r_0} - 2\sum_{j=1}^{N} s_{j0} h[j] \tag{23}$$

with initial condition

$$h[N] = \frac{q_N}{2r_N} \tag{24}$$

It should be noted that, the index i in Eqn. (22) proceeds backwards from $i = N-1$ to $i = 1$. Knowing the coefficient matrix $\boldsymbol{H}$, the mirror image polynomial is then formed as in Eqn. (4), *i.e.*,

$$P(z) = \sum_{n=-N}^{N} h(n) z^{n}$$

As an example, if N=7 and K=3, the resulted coefficient will be as:

$h[0] = 0.357143$, $h[\pm 1] = 0.2820812$,

$h[\pm 2] = 0.12871$, $h[\pm 3] = -0.0981305$,

$h[\pm 4] = -0.08907$, $h[\pm 5] = -0.0494301$,

$h[\pm 6] = 0.0317887$, $h[\pm 7] = -0.0357143$.

While solving for these coefficients $h[n]$ of digital FIR using the above Frequency Sampling method with the samples being arranged in generally acceptable forms, the method leads to some zero coefficients. For single transition-band sample, the sampled version vector A can be put in a form to match the desired frequency response:

$$A^t = [\underbrace{1 \;\; \dots \;\; \dots}_{\text{Pass-band } K \text{ samples}} \;\; \underbrace{0.5}_{\text{Single transition-band sample}} \;\; \underbrace{0 \;\; \dots \;\; \dots \;\; 0}_{\text{Stop-band } (N\text{-}K) \text{ samples}}] \tag{25}$$

with $\omega_c = \left(\frac{(K-1)}{N}\pi\right)$ rad., and a transition-band of $\frac{\pi}{N}$ rad. For example, for FIR filter with N=8 and K=4, the sampled version vector is expressed as

$A^t = [1 \;\; 1 \;\; 1 \;\; 1 \;\; 0.5 \;\; 0 \;\; 0 \;\; 0 \;\; 0]$

Solving using such design method, the resulting coefficients are

$h[0] = 0.5$, $h[1] = 0.314904$,

$h[2] = 0$, $h[3] = -0.0935379$,

$h[4] = 0$, $h[5] = 0.0417612$, $h[6] = 0$,

$h[7] = -0.012432$, $h[8] = 0$.

In the above example, the number of zero coefficients is $N/2$. The corresponding frequency response for this digital FIR filter is shown in Fig. 4. The choice of N determines the number of zero coefficients. In this form of Eqn. (25), even length (N even) gives $N/2$ zero coefficients, while odd length (N odd) gives only $h[N]$ be equal to zero, and this is the worst case.

For double transition-band samples, the sampled version vector A is:

$$A^t = [\underbrace{1 \;\; \dots \;\; 1}_{\text{Pass-band } K \text{ samples}} \;\; \underbrace{a_{t1} \;\; a_{t2}}_{\text{Double transition-band samples}} \;\; \underbrace{0 \;\; \dots \;\; 0}_{\text{Stop-band } (N\text{-}K\text{-}1) \text{ samples}}] \tag{26}$$

where $a_{t1} + a_{t2} = 1$, with $a_{t1} > a_{t2}$, $\omega_c = \left(\left(\frac{K-1}{N}\right)\pi\right)$ rad., and $2\pi/N$ rad. is the transition-band interval. The above choice will give rise to K zero coefficients. For the case, with $N=9$, $K=4$, $a_{t1}=0.8$, and $a_{t2}=0.2$, the filter coefficients are:

$h[0]=0.5$, $h[1]=0.312214$,

$h[2]=0$, $h[3]=-0.088889$,

$h[4]=0$, $h[5]=0.0384762$,

$h[6]=0$, $h[7]=-0.0173569$,

$h[8]=0$, $h[9]=0.0055556$,

Fig. 5 shows the frequency response of this FIR filter.

From the last two simple examples, the number of zero coefficients may be increased to be $N/2$ or K. This means that a big reduction in degree of complexity can be obtained. This in turn reduces the number of multipliers in the filter realization by a number U where $1 \leq U \leq N/2$.

4- Efficient Design and Realization

An improvement in the above mentioned design is to be considered, here. The double transition-band samples (a_{t1} and a_{t2}) with ($a_{t1}+a_{t2}=1$), and ($a_{t1}>a_{t2}$) give some zero coefficients, but this arbitrary choice means a wide range for a_{t1} and consequently a_{t2} values. Therefore, optimization process can be employed to reduce both peak stop-band ripples, and pass-band average deviation from ideal filter. So, the new designed FIR filter can be achieved with $N/2$ zero coefficients, minimum stop-band ripples, and reduced pass-band average deviation level. As an example, we choose $N=11$, $K=5$, $a_{t1}=0.9$, and $a_{t2}=0.1$. The total number of zero coefficients is five as listed below:

$h[0]=0.500012$, $h[1]=0.316814$,

$h[2]=0$, $h[3]=-0.101866$,

$h[4]=0$, $h[5]=0.0575044$,

$h[6]=0$, $h[7]=-0.0387368$,

$h[8]=0$, $h[9]=0.0299282$,

$h[10]=0$, $h[11]=-0.0136364$.

The frequency response for this 11-tap filter is shown in Fig. 6 with minimum stop-band peak ripple at -33dB, and average deviation level of 1.3% in the pass-band. Optimizing double transition-band samples (a_{t1}, a_{t2}) values, by using Golden section search and Steepest Descent algorithm together, results in optimized sample values of (0.779858, 0.220142), respectively. In this case, the resulting FIR filter coefficients are:

$h[0]=0.500003$, $h[1]=0.313696$,

$h[2]=0$, $h[3]=-0.092792$,

$h[4]=0$, $h[5]=0.0431997$,

$h[6]=0$, $h[7]=-0.0203601$,

$h[8]=0$, $h[9]=0.00896901$,

$h[10]=0$, $h[11]=-0.00271436$,

The frequency response for this filter is also shown in Fig. 6 with minimum stop-band peak ripple being reduced to -53dB, and average deviation level of 0.14% from the ideal filter in the pass-band. It should be noted here, that the number of zero coefficients is still maintained at five.

5- A General Design Example

Different design methods are examined for different cases. *Case* 1 illustrates a 15-tap low-pass FIR digital filter with no optimization, while *Case*

2 represents the same FIR filter with arbitrary single transition-band sample. The same FIR filter is redesigned in *Case* 3 with optimized single transition-band sample. Finally, *Case* 4 illustrates the above FIR filter with double optimized transition-band samples. All the above cases can be realized with no zero coefficients. The frequency responses of the above cases are shown in Fig. 7.

A 15-tap low-pass FIR filter is also designed in *Case* 5 with fixed single transition-band sample and realized with single zero coefficient value. The same FIR filter of *Case* 5, is redesigned in *Case* 6 with unity sum double transition-band samples and *N*/2 zero coefficient values. The corresponding frequency responses of these two cases are shown in Fig. 8.

The example in *Case* 7 is a 15-tap low-pass FIR filter designed by the method of reduced realization (many zero coefficients). The design is accomplished by using a comparative optimization process to reduce the pass-band average deviation level, while having as many as possible zero coefficients. Figure 9 shows this FIR filter frequency response.

Table 2 shows the FIR filter coefficient values for different cases. Table 3 summarizes the resulting parameters of different cases. From this table it can be seen that in the *Cases* from 1 to 4, the percentage pass-band average deviation (Δ%) is reduced from 4.78% in *Case* 1 to 1. 20% in *Case* 4. In addition to that, the stop-band peak ripple is reduced from -14dB in *Case* 1 to -73dB in *Case* 4. The advantages of the method used in *Case* 5 and *Case* 6 are the effective reduction in realization because there are some zero coefficients resulted in these cases which means a realization with reduced number of multipliers (without 1 multiplier in *Case* 5, and without 7 multipliers in *Case* 6).

Our goals of minimum realization and optimal response are reached in *Case* 7 to reduce the percentage pass-band average deviation to 0.12% and having a good stop-band peak ripple of -55dB, while 6 zero coefficients will lead to minimization in the number of required multipliers by six.

6-Conclusion

One of drawbacks of FIR digital filter is that a fast-roll-off can be only achieved with high complexity. In this paper, a novel design method, which leads to many zero coefficients (complexity reduction), has been presented. The method depends on Frequency Sampling of the desired frequency response. Many examples have been given to illustrate the simple design with the reduced realization complexity. In order to have a perfect design, optimal methods have been used to obtain a frequency response with minimized stop-band ripple levels, and reduced pass-band average deviation from ideal FIR filter. A final FIR filter has been designed having both properties optimal frequency response and complexity. This complexity reduction leads to high speed operation, less energy consumption, low cost, and small implementation size.

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Table –1 stop-band and pass-band improvement for a 19 tap FIR filter

Case	Without transition-band sample	Within arbitrary transition-band sample a_U=0.5	Optimized single transition-band sample a_U= 0.404639	Optimized double transition-band samples a_U= 0.5668437, a_L=0.0904549
Stop-band peak ripple(dB)	-15	-29	-41	-73
Pass-band average deviation (Δ%)	5	3.52	2.20	1.32

Table –2 FIR filter coefficients for different cases

FIR coefficient	*Case 1*	*Case 2*	*Case 3*	*Case 4*	*Case 5*	*Case 6*	*Case 7*
h[0]	-0.0498159	-0.0052072	-0.0135713	-0.0065559	0.466667	0.50023	0.499788
h[1]	0.0412023	-0.0127322	-0.0026195	-0.0073765	0.315468	0.314502	0.315983
h[2]	0.0666667	0.033333	0.0395833	0.0261127	0.326049	0.00155332	-0.000146367
h[3]	-0.0364879	0.0244157	0.0129958	0.02442939	-0.0675684	-0.0956064	-0.0980053
h[4]	0.107869	0.0872678	-0.0911305	-0.0762969	-0.0304515	0.0	0.0
h[5]	0.034078	-0.0311318	-0.018905	-0.0407539	0.05	-0.0466481	0.0538416
h[6]	0.318892	0.311924	0.31325	0.306689	0.0269672	0.0	0.0
h[7]	0.466667	0.533333	0.520833	0.547775	-0.0227512	-0.0230514	-0.0299526
h[8]	0.318892	0.311924	0.31325	0.306689	-0.0273044	0.0	0.0
h[9]	0.034078	-0.0311318	-0.018905	-0.0407539	0.014255	0.0088416	0.017187
h[10]	0.107869	0.0872678	-0.0911305	-0.0762969	0.0166667	0.0	0.0
h[11]	-0.0364879	0.0244157	0.0129958	0.02442939	0.00603636	0.0	-0.00936995
h[12]	0.066667	0.033333	0.0395833	0.0261127	-0.103006	0.0	0.0
h[13]	0.0412023	-0.0127322	-0.0026195	-0.0073765	0.0014731	-0.0050478	0.00504243
h[14]	-0.0498159	-0.0052072	-0.0135713	-0.0065559	0.00348428	0.0	0.0
h[15]	0.0	0.0	0.0	0.0	0.0	0.003333	-0.001582463

Table –3 Different cases with several design methods for the FIR filter with N= 15, and K=7

Case No.	Transition-band samples value		Pass-band average deviation (Δ%)	No. of zero coefficients	Stop-band peak ripple (dB)
	a_{t1}	a_{t2}			
1	0	0	4.78	0	-14
2	0.5	0	3.52	0	-29
3	0.40625	0	2.09	0	-42
4	0.53766809	0.0706421	1.20	0	-73
5	0.5	0	0.71	1	-30
6	0.7	0.3	0.67	7	-32
7	0.777368	0.222632	0.12	6	-55

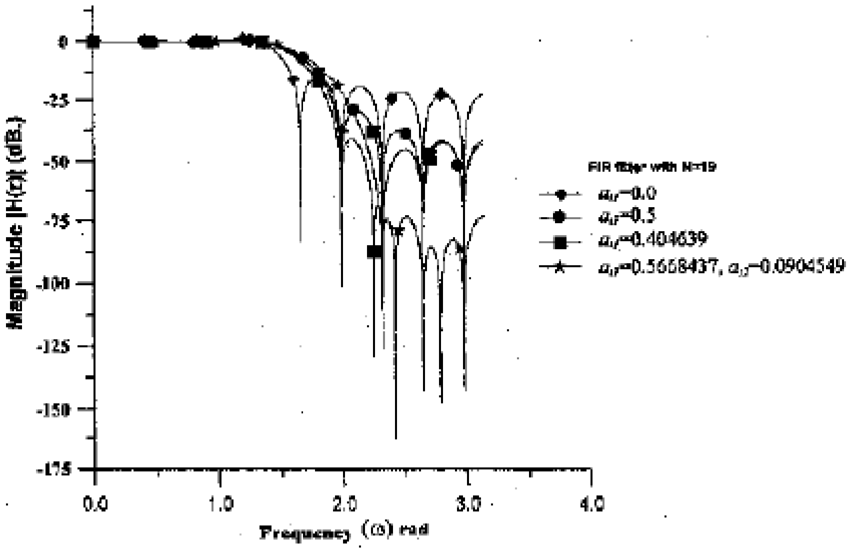

Fig. 2 Frequency response for a low-pass FIR filter with N=19, and K= 9

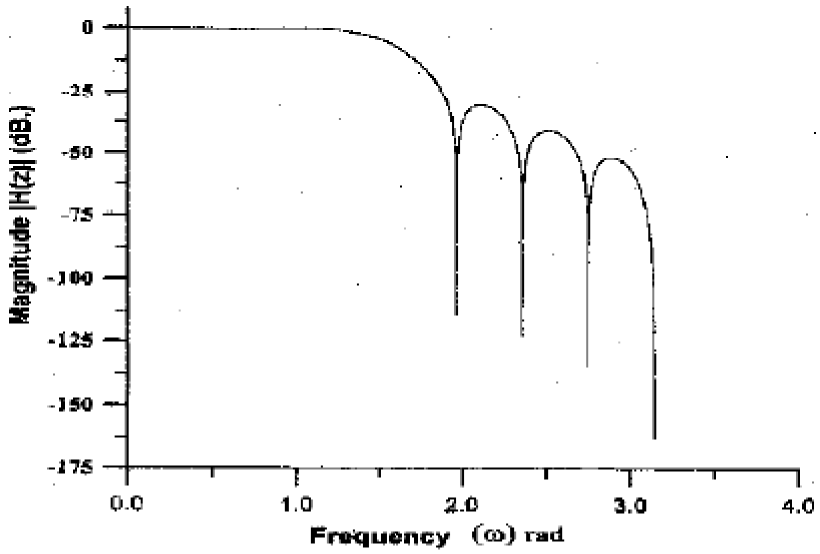

Fig. 4 Frequency response for a low-pass FIR filter with N=8, and K= 4

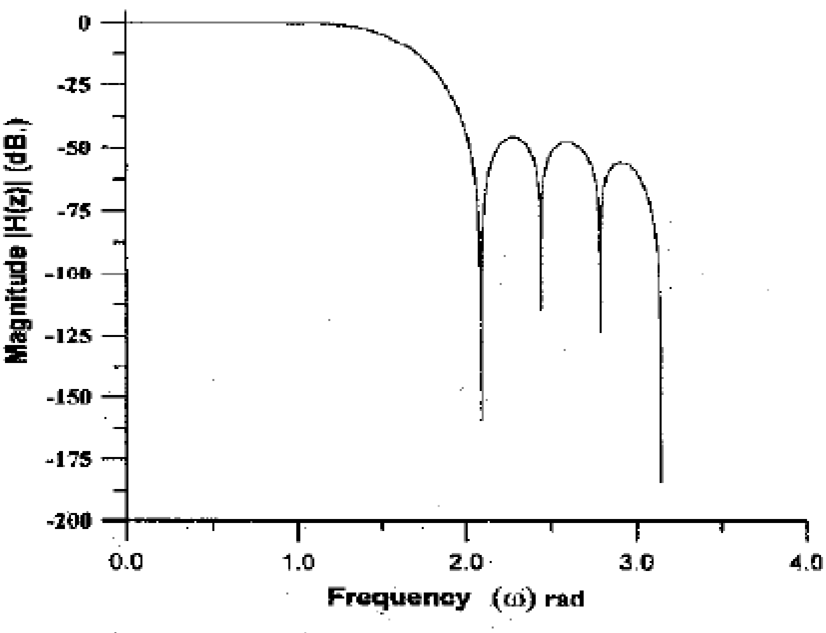

Fig. 5 Frequency response for a low-pass FIR filter with N=9, K= 4, a_{l1}=0.8, and a_{l2}=0.2

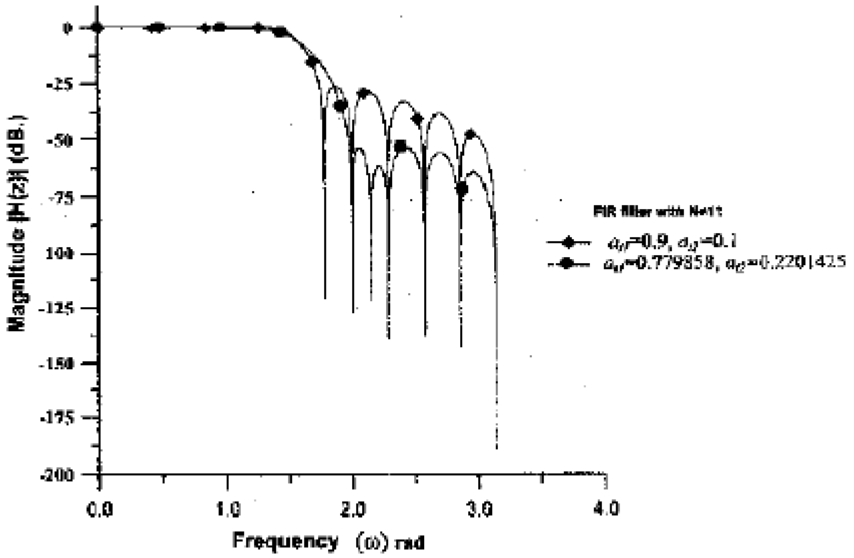

Fig. 6 Frequency response for a low-pass FIR filter with N=11, and K= 5

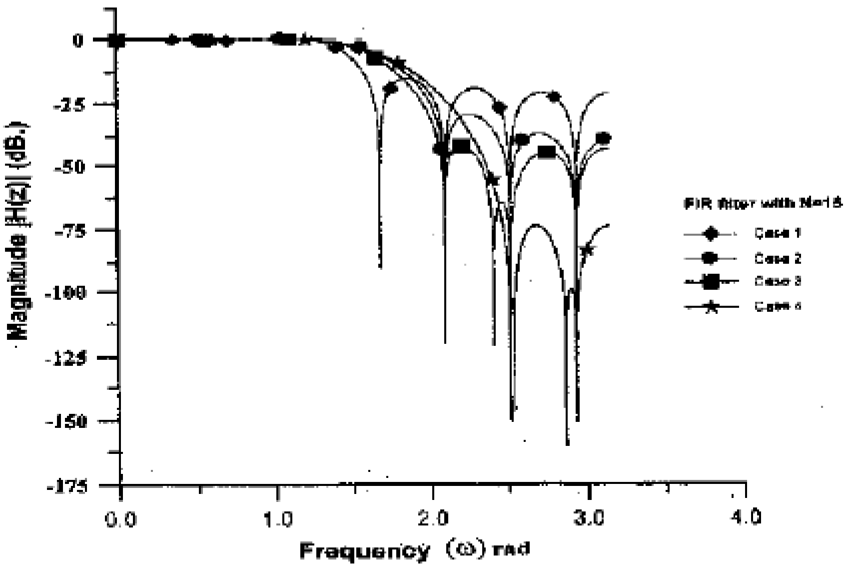

Fig. 7 Frequency response for a low-pass FIR filter with N=15, and K= 7

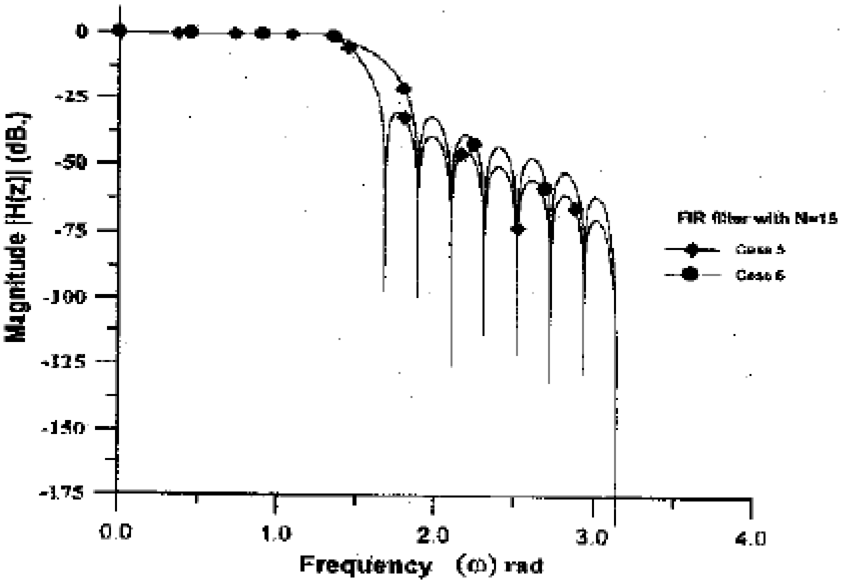

Fig. 8 Frequency response for a low-pass FIR filter with N= 15, and K= 7