Calculation the Electromagnetic Properties of ^{134}Sn Nucleus

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Abstract

In this study, we calculated the energy levels of ^{134}Sn nucleus, which have two neutrons outside closed-shell core nucleus ^{132}Sn. This study is performed by using the *Particle –Vibration Coupling Model(PVCM)* which adopt the *Modified Surface Delta Interaction (MSDI)*, as two neutron interaction, and 0-2 quadrupole phonons for collective core vibrations. The reduced electric transition probability *B(E2*) for transitions between states and the electric quadrupole moment Q are calculated within this model also. To improve all our calculations of *B(E2*) and Q we introduced the concept of core polarization effect to obtain the best description for the available experimental data .

Keywords :*particle-vibration coupling model PVCM, Core-Polarization Effect CP, quadrupole phonon,^{134}Sn Nucleus.*

1. INTRODUCTION

Particle –Vibration Coupling Model(PVCM) is one of the ways to study the properties of nucleus. This model mixes the macroscopic collective model with microscopic shell model. The development of this model began with a few particles or holes coupled to a closed-shell vibrating core . Bohr and Motelson are taking a first idea of this model that applied to nuclei by Fierz in 1924 then Bohr in 1952, Jakeli, Feenberg and others [1,2,3,4]. ^{134}Sn nucleus (Z=50, N=84) consist of two neutrons outside the vibrating core (closed shell) of ^{132}Sn (Z=50, N=82), which is doubly magic nucleus. The structure of this nucleus was studied by Coraggio and team, Sarkar and Sarkar, and Gargano [5,

[6, 7, 8] in the framework of shell model that consider ^{132}Sn as inert core and distributing the two extra neutrons over the harmonic oscillator shells beyond the magic number 82. The effective charge was included to compensate the effect of the truncation of model space on the calculation of electromagnetic properties.

The PVCM has enjoyed some success in the A≈100 region of the periodic table in describing the level spectra of odd-A nuclei and also in calculated transitions and moments[9]. In this work we study the structure of the even–even nucleus ^{134}Sn by using the PVC model. In the calculation of electromagnetic properties of ^{134}Sn, the microscopic core polarization effects are taken into account.

2. THEORY

2.1 Hamiltonian and basis function

The total Hamiltonian operator of the three body system which consists of core plus two nucleons is given by:

$$\hat{H} = \hat{H}_{coll} + \sum_{i=1}^{2} \hat{H}_{s.p.}(i) + \sum_{i=1}^{2} \hat{H}_{int}(i) + \hat{V}(1,2) \qquad (1)$$

where the collective Hamiltonian of the vibrating core is given in terms of boson creation and annihilation operators by,

$$\hat{H}_{coll} = \sum_{\mu} \hbar\omega_2 \left[b_{2\mu}^{\dagger} b_{2\mu} + \frac{1}{2} \right] \qquad (2)$$

with $\hbar\omega_2$ represents the quadrupole phonon energy. $\hat{H}_{s.p.}(i)$, is that for a particle in a harmonic oscillator potential well, and the particle-core interaction Hamiltonian, $\hat{H}_{int}(i)$, is that for the *i*th particle in the field caused by the vibrating core which is supposed to follow the density vibrations adiabatically. This coupling Hamiltonian is given in terms of the scalar products of boson creation and

annihilation operators and spherical harmonics, by,

$$\widehat{H}_{int}(i) = \sqrt{\frac{\pi}{5}}\xi_2^{(i)}\hbar\omega_2\left[b_2^{\dagger}\cdot Y_2(\hat{r}_i) + b_2\cdot Y_2^{*}(\hat{r}_i)\right] \quad (3)$$

where the strength of this interaction is given by the parameters $\xi_2^{(i)}$. The last term of $\widehat{H}$ is the two-particle potential, V (1,2).The space of the PVCM is defined by the basis function $|N_2R_2,(j_1j_2)J;IM\rangle$. Where N_2 represents the number of quadrupole phonons coupled to angular momentum R_2. The single particle states j_1 and j_2 are coupled to J, then both R_2 and J are coupled to total angular momentum I in the order $\vec{I} = \vec{R}_2 + \vec{J}$, with $|R_2 - J| \leq I \leq R_2 + J$. Diagonalization of the Hamiltonian in the configuration mixing space of the PVCM states $|N_2R_2,(j_1j_2)J;IM\rangle$, yields energy eigenvalues, $E^{(\alpha)}$, and eigenvectors, $C_\alpha(N_2R_2,(j_1j_2)J;I)$.

2.2 Electromagnetic transitions

The electromagnetic transition operator can be written in the framework of PVCM as:

$$\hat{O}^{\eta}_{LM} = \hat{O}^{\eta}_{LM}(core) + \sum_{i=1}^{2}\hat{O}^{\eta}_{LM}(i) \quad (4)$$

where $\hat{O}^{\eta}_{LM}(i)$ and $\hat{O}^{\eta}_{LM}(core)$ are the single–particle and the core parts, respectively, and η stands for the type of the operator (η = M or E for magnetic or electric operator, respectively,), L is the transition multipolarity. By using the standard Racah Algebra, the reduced matrix element of the electromagnetic transition operator between the PVCM states can be written as:-

$$\left\langle N_2'R_2', (j'_1 j'_2)J', I' \middle|\middle| \hat{O}_L^{\eta} \middle|\middle| N_2R_2, (j_1 j_2)J, I \right\rangle = (-1)^{I+L}\sqrt{(2I+1)(2I'+1)} \times$$

$$\left[(-1)^{R_2+J} \begin{Bmatrix} J' & J & L \\ I & I' & R'_2 \end{Bmatrix} \sum_{i=1}^{2} \left\langle J' \middle|\middle| \hat{O}_L^{\eta}(i) \middle|\middle| J \right\rangle \delta_{N_2 N_2'} \delta_{R_2 R_2'} + \right.$$

$$\left. (-1)^{R'_2+J'} \begin{Bmatrix} R'_2 & R_2 & L \\ I & I' & J \end{Bmatrix} \left\langle N_2'R_2' \middle|\middle| \hat{O}_L^{\eta}(core) \middle|\middle| N_2R_2 \right\rangle \delta_{JJ'} \delta_{j_1 j'_1} \delta_{j_2 j'_2} \right] \quad (5)$$

where the states $|\mathrm{J}\rangle$ and $|\mathrm{R}_2\rangle$ represent, respectively, the two-particle and collective model space wave functions. The operator $\hat{O}_{LM}^{\eta}(i)$ acts on a single –particle initial state $|\mathrm{j}\rangle$ and transit it to final state $|\mathrm{j}'\rangle$ within the single-particle model space.

Introducing the core polarization (CP) effects, which based on a microscopic theory that combines shell model wave functions and configurations with higher energy as first order perturbations, on the single particle operator $\hat{O}_{LM}^{\eta}(i)$ corrects its matrix elements as follow [9]:

$$\left(j' \middle\| \hat{O}_L^{\eta}(i) \middle\| j\right) = \left\langle j' \middle|\middle| \hat{O}_L^{\eta}(i) \middle|\middle| j \right\rangle + \left\langle j' \middle|\middle| \hat{O}_L^{\eta}(i) \frac{\hat{Q}}{E_j - H^{(0)}} V_{res} \middle|\middle| j \right\rangle +$$

$$\left\langle j' \middle|\middle| V_{res} \frac{\hat{Q}}{E_{j'} - H^{(0)}} \hat{O}_L^{\eta}(i) \middle|\middle| j \right\rangle \quad (6)$$

The first term is the model space contribution, while the second and third terms appear due to the core-polarization effect, i.e., that outside the model space. The operator $\hat{Q}$ is the projection operator onto the space outside the model space. E_j and $E_{j'}$ are the energies of the initial and final single-particle states, respectively, and $H^{(0)}$ is the unperturbed Hamiltonian. The CP terms are given by:

$$\sum_{j_p j_h J} (2J+1)(-1)^{j+j_h+J} \begin{Bmatrix} j' & j & L \\ j_h & j_p & J \end{Bmatrix} \sqrt{\left(1+\delta_{j_p j'}\right)\left(1+\delta_{j_h j}\right)} \frac{\left\langle j' \middle|\middle| \hat{O}_L^{\eta}(i) \middle|\middle| j \right\rangle}{e_j - e_{j'} - e_{j_p} + e_{j_h}}$$

$$\langle j' j_p | V_{res} | j j_h \rangle_J +$$

terms with $\mathrm{j_p}$ *and* $\mathrm{j_h}$ *exchanged with an overall minus sign* (7)

where j_p and j_h runs over particle and hole states outside the model space .In proton-neutron formalism, the sum will be over all protons and neutrons particle-hole pairs. The single-particle energy $e_{n\ell j}$ can be calculated from [10]:

$$e_{n\ell j} = \left(2n + \ell - \tfrac{1}{2}\right)\hbar\omega + \begin{cases} -\frac{1}{2}(\ell+1)\langle f(r)\rangle & \text{for } j = \ell - \frac{1}{2} \\ \frac{1}{2}\ell\langle f(r)\rangle & \text{for } j = \ell + \frac{1}{2} \end{cases} \quad (8)$$

Where,

$\hbar\omega = 45\mathrm{A}^{-1/3} - 25\mathrm{A}^{-2/3}$ MeV, and $\langle f(r)\rangle_{n\ell} = -20\ \mathrm{A}^{-2/3}$.

The reduced transition probability for electromagnetic transition in the framework of PVCM is[9, 11],

$$B(\eta L, I \to I') = \frac{1}{2I+1}\left|\sum_{\alpha\beta} C_\alpha C_\beta \left\langle \alpha; I' \middle|\middle| \hat{\mathrm{O}}^{\eta}_{\mathrm{L}} \middle|\middle| \beta; I \right\rangle\right|^2 \quad (9)$$

with $\alpha \equiv N_2' R_2' j'_1 j'_2 J'$ and $\beta \equiv N_2 R_2 j_1 j_2 J$. The eigenvectors C_α and C_β are obtained from the diagonalization of the Hamiltonian within configuration mixing of PVCM states, while the reduced matrix elements of $\hat{\mathrm{O}}^{\eta}_{\mathrm{L}}$ are separated in eq.(6) in terms of the reduced matrix elements of single-particle and collective operators.

The single particle contribution in the electric quadrupole operators is given by[11]:

$$\hat{\mathrm{O}}^{\mathrm{E}}_{2}(i) = er^2\ \mathrm{Y}_2(\hat{r}_i) \quad (10)$$

and the core contribution is given by[10]:

$$\hat{\mathrm{O}}^{\mathrm{E}}_{2}(core) = \frac{3}{4\pi}\ \mathrm{Z\,e}\ R_0^2\ \alpha_2 \quad (11)$$

where $\mathrm{Y}_2(\hat{r}_i)$ represents the spherical harmonics, Z is the atomic number, and R_0 is the equilibrium radius of the core nucleus, which is given in term of the mass number A by $R_0 = 1.2\,A^{1/3}$ fm . And the deformation parameter $\hat{\alpha}_{\lambda\mu}$ is given in terms of boson creation and annihilation operators by[10]:

$$\hat{\alpha}_{\lambda\mu} = \sqrt{\frac{\hbar\omega_\lambda}{2C_\lambda}}\ \left[\hat{b}_{\lambda\mu} + (-1)^\mu\ \hat{b}^\dagger_{\lambda-\mu}\ \right] \qquad (12)$$

The electric quadrupole moment Q can be related with the reduced transition probability through[9]:

$$Q = \sqrt{\frac{16\pi}{5}}\ \sqrt{\frac{I(2I-1)}{(I+1)(2I+3)}}\ \sqrt{\mathrm{B(E2;I \rightarrow I)}} \qquad (13)$$

3. RESULTS AND DISCUSSION

The ^{134}Sn nucleus consists of N=84 and Z=50, so it considered as ^{132}Sn core nucleus plus two neutrons, which are assumed to occupy the harmonic oscillator shells (shell model space) $2f_{7/2}$ and $1h_{9/2}$. The unperturbed energy of each neutron state is 0.0 MeV and 1.561 MeV, respectively [8,6] . The allowed J^π values for two neutrons in the configurations $(2f_{7/2})^2$, $(1h_{9/2})^2$, and $(2f_{7/2})(1h_{9/2})$ are restricted by the generalized Pauli exclusion principle. The first 2^+state in ^{132}Sn is found experimentally at 4.04 MeV [12], which represent the unperturbed energy of the quadrupole phonon, $\hbar\omega_2$. For calculations including 0-2 quadrupole phonons, the allowed collective states of the core are , $R_2^\pi = 0^+, 2^+, 0^+, 2^+, and\ 4^+$. All I^πvalues of ^{134}Sn nucleus are obtained from the coupling of R_2 and J.

The calculated and experimental values of the energy levels of ^{134}Sn are presented in Fig. (1). The strength parameters of MSDI adopted in these calculations are those of following equation[9],

$$A_0 \approx A_1 \approx B \approx \frac{25}{A}\ ,\quad C \approx 0 \qquad (15)$$

which are A_0=A_1=B=0.1866 MeV and C=0.0.

The strength parameters of the particle-core interaction are $\xi_2^{(1)} = \xi_2^{(2)} = 0.1$, which are chosen so that the best agreement with experimental levels is obtained. The calculated energy levels by using PVCM with core excitations up to one quadrupole phonon are denoted as PVCM1, while that with core excitations up to two quadrupole phonons are denoted as PVCM2. However, in PVCM2, the position of 8_1^+ state is enhanced slightly as compared with that of PVCM1. On the other hand, the energy eigenvalue of 2_1^+ state in PVCM1 is better than that obtained in PVCM2.

Our results are comparable with that obtained by Coraggio *et al* [5], where they performed shell model calculations by considering ^{132}Sn as closed (inert) core and letting the valence neutrons occupy the six single particle states of the 82 to 126 shells. In their calculations, a realistic effective interaction derived from the CD-Bonn NN potential, were used. Our calculations predict a first 1^+, 3^+, 5^+, and 7^+ states at 2.589, 2.575, 2.569, and 2.587 MeV, respectively. While Coraggio *et al* [5], found these states at 2.304, 1.89, 1.944, 2.744 MeV, respectively. However, using of a more realistic effective interaction in PVCM, may enhance our predictions as compared with that of Coraggio *et al* [5],

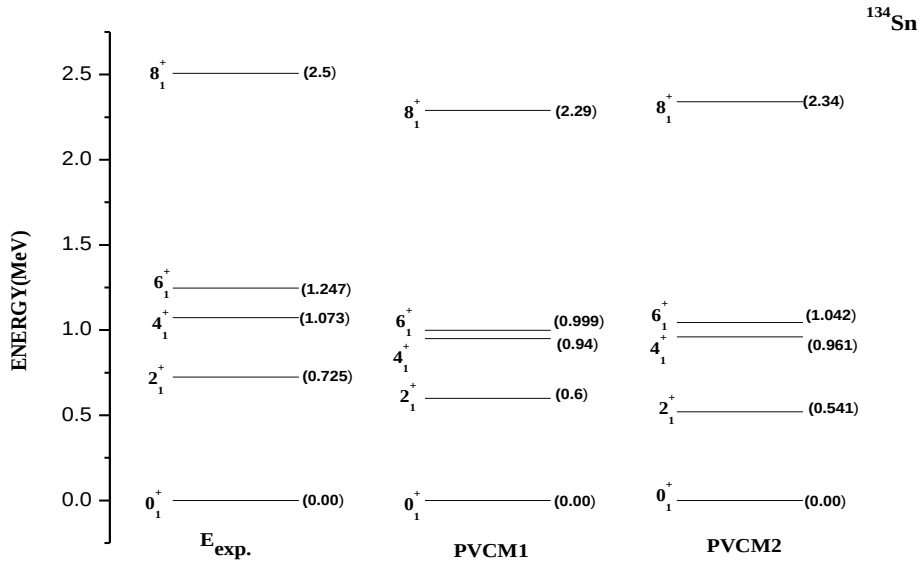

Figure (1):- Comparison of the experimental and calculated positive parity energy levels of [134]Sn. The numbers 1 and 2 attached to PVCM denote the maximum number of phonon excited.

The structure of PVCM wave functions for ^{134}Sn levels of Fig. (1), is dominant by the configurations $|00,\frac{7}{2}\frac{7}{2}\rangle$, $|00,\frac{7}{2}\frac{9}{2}\rangle$, *and* $|00,\frac{9}{2}\frac{9}{2}\rangle$, which reflect the weak coupling ($\xi_2^{(1)} = \xi_2^{(2)} = 0.1$) of the neutrons with the core. The wave functions of PVCM are important for calculation of electromagnetic properties.

The calculated electromagnetic transition probabilities $B(E2; I \rightarrow \acute{I})$ in the framework of PVCM are given in 3rd and 4th columns of table (1), for one-quadrupole phonon and two-quadrupole phonons, respectively, of collective core vibrations. Since the neutron has no charge, these values of B(E2) are due to core vibration degrees of freedom only. However, these values are far from the available experimental one. To reproduce the experimental values of B(E2), the valence neutron was given an effective charge equal to 0.7e by Coraggio *et al* [(5)], A Covello [(13)] and 0.72e by S. Sarkar [(7)] , which are listed in the 7th column of table (1). To enhance our calculations for B(E2), we use the microscopic counterpart for the

effective charge, which is the core polarization effect CPE. According to CPE, a proton is excited from the sd-pf-$1g_{9/2}$ core orbits to all higher energy orbits outside the core with $2\hbar\omega$, leaving a hole within the core. Two residual interactions are adopted, MSDI and M3Y. The results are given in the 5th and 6th columns of table (1) as PVCM+CP, which are reproduce the available experimental values very well. However, $B(E2;\ 2_1^+ \rightarrow 0_1^+)$ reproduced excellently by including CP with M3Y as residual interaction, while the inclusion of CP with MSDI as residual interaction reproduces the value of $B(E2;\ 6_1^+ \rightarrow 4_1^+)$ excellently. Table (2), represent two prediction value of electric quadrupole moment Q*(e fm²)* of ^{134}Sn nucleus, where 2_1^+ state value is compared with A Covello study [13].

Table(1):The calculated and experimental values of B(E2) in ^{134}Sn nucleus

Transition	$B(E2)_{exp.}$ (W.u)	$B(E2)_{the}$(W.u.) PVCM		PVCM+CP		Other Calcul.
		PVCM1	PVCM2	PVCM1+CP MSDI/M3Y	PVCM2+CP MSDI/M3Y	
$2^+_1 \rightarrow 0_1^+$	1.4±0.20[a] 1.42[b]	0.00204	0.00583	1.1106/1.742	0.9307/1.485	1.72[c] 1.64[a]
$4^+_1 \rightarrow 2^+_1$	---	0.0007	0.00000189	1.457/2.2112	1.3133/2.0086	1.71[c] 1.66[a]
$6^+_1 \rightarrow 4_1^+$	0.89 [d] 0.89±0.17[a]	0.00123	0.00059	0.722/1.0953	0.6959/1.0611	0.88[c] 0.82[a] 0.662[e]
$8^+_1 \rightarrow 6_1^+$	---	0.0011	0.00068	0.0126/0.011	0.01204/0.01002	0.12[c]

Table (2) :The calculated values of electric quadrupole moment Q(e fm^2) *of* ^{134}*Sn nucleus*

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State	Q exp. (e fm^2)	Q the. (e fm^2) PVCM		PVCM+CP		Other calcul.
		PVCM1	PVCM2	PVCM1+CP MSDI/M3Y	PVCM2+CP MSDI/M3Y	
2^+_1	----	1.79	1.997	12.17/14.82	10.99/13.36	-1.6 [a]
4^+_1	---	0.046	0.222	1.3	1.516/1.825	----

a=(13), b=(14), c=(6), d=(5), e=(7)

4. CONCLUSIONS

The energy levels, electromagnetic transition probabilities $B(E2)$and the electric quadrupole moment Q(e.fm^2) of ^{134}Sn nucleus, are investigated by the particle vibration coupling model (PVCM), which showing a successful calculations of these nuclear properties for heavy nuclei around ^{132}Sn nucleus. The calculated energy levels are in agreement with the available experimental data. The concept of the Core- Polarization Effects (CPEs), which inserted in the electromagnetic properties calculations were added more accuracy in the results, especially for the $B(E2, 2^+_1 \rightarrow 0^+_1)$ and $B(E2, 6^+_1 \rightarrow 4^+_1)$ transitions.

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حساب الخواص الكهرومغناطيسية لنواة ^{134}Sn

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الخلاصة

في هذه الدراسة، قمنا بحساب مستويات الطاقة لنواة ال ^{134}Sn ، التي تتكون من اثنين من النيوترونات الواقعة خارج قلب مغلق هو نواة ^{132}Sn. يتم تنفيذ هذه الدراسة باستخدام نموذج اقتران جسيم - ذبذبه (PVCM) والتي اعتمدنا فيها على جهد دلتا السطحي المعدل (MSDI) لوصف تفاعل اثنين من النيوترونات، و 2-0 من الفونونات رباعية القطب. تم حساب احتمالية انتقال رباعي القطب الكهربائي B (E2) للانتقالات بين الحالات النوويه المختلفة و عزم رباعي القطب الكهربائي Q . كما ادخل مفهوم تأثير استقطاب اللب على حسابات كل من B(E2) و Q والذي يتضمن حركة النيوكلونات الى خارج اللب وخارج حدود فضاء النموذج وضمن طاقه قدرها $2\hbar\omega$، والذي اظهر تحسينا واضحا لقيم النتائج النظرية مقارنتا بالقيم العملية المتوفرة.