Brauer trees for spin characters of S_n , $13 \le n \le 20$ modulo p=13

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Abstract :

In this paper we calculate the Brauer trees for the spin characters of the symmetric groups S_n , $13 \le n \le 20$ for the prime p =13

Introduction :

The covering group of the symmetric group S_n has characters[1] called spin characters , Decomposition matrix is the relation between ordinary (projective) and modular characters[2 ,3,4] The problem for finding modular characters from ordinary characters is very difficult in general[5,6] , for symmetric group see[3,4] , and for covering group of S_n see [7,8,9] , In this paper we find the 13-mudular spin characters for S_n $13 \le n \le 20$.

Notation :

≡	equivalence mod 13
deg <β>	the degree of the spin character <β>
i.m.	irreducible modular spin character
<β> , <β>′	the associate spin characters for S_n
<β>*	the double spin character S_n
<β>$\uparrow^{(r,\bar{r})}$	the $(r,\bar{r})$- inducing of <β>
<β>$\downarrow_{(r,\bar{r})}$	the $(r,\bar{r})$- restriction of <β>

Section 1: 13- Brauer tree for S_{13}

S_{13} has block of defect one B=$L_1 \cup L_2$ where L_1={<12,1> ,<12,1>′ , <10,3>, <10,3>′ , <8,5> , <8,5>′} and L_2={<13>*, <11,2> , <11,2> ′ , <9,4>, <9,4>′, <7,6> ,<7,6>′} such that deg $L_1 \equiv 1$, deg $L_2 \equiv -1$ .The other spin characters are of defect zero . S_{13} has 26 (13,α)-regular classes . so, B has 12 i.m.

<12>$\uparrow S_{13}$ = <13>* + <12,1> = C_1

<12>′$\uparrow S_{13}$ = <13>* + <12,1>′ = C_2

<11,1>$^*\uparrow S_{13}$ = <12,1> + <12,1>′ + <11,2> + <11,2>′ = K_1 = C_3+C_4 (10)

<10,2>$^*\uparrow^{(11,3)}S_{13}$ = <11,1> + <11,1>′ + <10,3> + <10,3>′ = K_2 = C_5+C_6

<9,3>* $\uparrow^{(10,4)}S_{13}$ = <10,3> + <10,3>′ +<9,4> + <9,4>′ =K_3 = C_7+C_8

<8,4>* $\uparrow^{(9,5)}S_{13}$ = <9,4> + <9,4>′ + <8,5> + <8,5>′ = K_4 = C_9+C_{10}

<7,5>$^*\uparrow^{(8,6)}S_{13}$ = <8,5> + <8,5>′ + <7,6> + <7,6>′ =K_5 = C_{11}+C_{12}

Since <11,2> ≠ <11,2>′ , <10,3> ≠ <10,3>′ , <9,4> ≠ <9,4>′ and <8,5> ≠ <8,5>′ on (13,α)-regular classes ,so the **Brauer** tree is :

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            <12,1>  – <11,2> –  <10,3> – <9,4> – <8,5> – <7,6>
          /
<13>*
          \
            <12,1>′ – <11,2>′ –  <10,3>′ –  <9,4>′ – <8,5>′ – <7,6>′
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Section 2: 13- Brauer tree for S_{14} :

S_{14} has block of defect one B=$L_1 \cup L_2$ where L_1={<13,1>* , <10,3,1> ,<10,3,1>′ , <8,5,1> , <8,5,1>′} and L_2={<14>, <14>′ , <11,2,1>, <11,2,1>′ , <9,4,1> ,<9,4,1>′ , <7,6,1> , <7,6,1>′} such that deg $L_1 \equiv 1$ and deg $L_2 \equiv -1$. The other spin characters are of defect zero . S_{13} has 32 (13,α)-regular classes . So, B has 12 i.m.

$C_1\uparrow^{(1,0)}S_{14}$ = <14> + <14>′ +2 <13,1>* = K = D_1 +D_2 (10)

$C_3\uparrow^{(1,0)}S_{14}$ = <13,1>* + <11,2,1> = D_3

$C_4\uparrow^{(1,0)}S_{14}$ = <13,1>* + <11,2,1>′ = D_4

$C_5\uparrow^{(1,0)}S_{14}$ = <11,2,1> + <10,3,1> = D_5

$C_6\uparrow^{(1,0)}S_{14}$ = <11,2,1>′ + <10,3,1>′ = D_6

$C_7\uparrow^{(1,0)}S_{14}$ = <10,3,1> + <9,4,1> =D_7

$C_8\uparrow^{(1,0)}S_{14}$ = <10,3,1>′ + <9,4,1>′ =D_8

$C_9\uparrow^{(1,0)}S_{14}$ = <9,4,1> + <8,5,1> = D_9

$C_{10}\uparrow^{(1,0)}S_{14}$ = <9,4,1>′ + <8,5,1>′ = D_{10}

$C_{11}\uparrow^{(1,0)}S_{14}$ = <8,5,1> + <7,6,1> = D_{11}

$C_{12}\uparrow^{(1,0)}S_{14}$ = <8,5,1>′ + <7,6,1>′ = D_{12}

so the Brauer tree is :

$<14>$ $<11,2,1> - <10,3,1> - <9,4,1> - <8,5,1> - <7,6,1>$
\ /
$<13,1>^*$
/ \
$<14>'$ $<11,2,1>' - <10,3,1>' - <9,4,1>' - <8,5,1>' - <7,6,1>'$

Section 3 : 13- Brauer tree for S_{15} :

S_{15} has block of defect one $B=L_1 \cup L_2$ where $L_1=\{(<13,2> + <13,2>')\ ,\ <10,3,2>^*\ ,\ <8,5,2>^*\}$ and $L_2=\{<15>^*\ ,\ <12,2,1>^*\ ,\ <9,4,2>^*\ ,<7,6,2>^*\}$ such that deg $L_1 \equiv 2$ and deg $L_2 \equiv -2$ .The other spin characters are of defect zero . S_{15} has 38 $(13,\alpha)$-regular classes , So B has 6 i.m.

$D_1\uparrow^{(12,2)}S_{15} = <15>^* + <13,2> + <13,2>' = E_1$

$D_3\uparrow^{(12,2)}S_{15} = <13,2> + <13,2>' + <12,2,1>^* = E_2$

$D_5\uparrow^{(12,2)}S_{15} = <12,2,1>^* + <10,3,2>^* = E_3$

$D_7\uparrow^{(12,2)}S_{15} = <10,3,2>^* + <9,4,2>^* = E_4$

$D_9\uparrow^{(12,2)}S_{15} = <9,4,2>^* + <8,5,2>^* = E_5$

$D_{11}\uparrow^{(12,2)}S_{15} = <8,5,2>^* + <7,6,2>^* = E_6$

From above we have the Brauer tree :

$<15>^* - <13,2> = <13,2>' - <12,2,1>^* - <10,3,2>^* - <9,4,2>^*$ ┐
$<7,6,2>^* - <8,5,2>^*$ ┘

Section 4 : 13- Brauer trees for S_{16} :

S_{16} has two blocks of defect one $B_1=L_1 \cup L_2$ where $L_1=\{\ <15,1>^*\ ,\ (<13,2,1> + <13,2,1>')\ ,\ <9,4,2,1>^*\ ,\ <7,6,2,1>^*\}$, $L_2=\{\ <14,2>^*\ ,\ <10,3,2,1>^*\ ,<8,5,2,1>^*\ \}$ such that deg $L_1 \equiv -2$, deg $L_2 \equiv 2$ and $B_2=L_3 \cup L_4$ where $L_3=\{<16>\ ,\ <16>'\ ,<12,3,1>\ ,<12,3,1>'\ ,\ <9,4,3>\ ,\ <9,4,3>',\ <7,6,3>\ ,\ <7,6,3>'\}$, $L_4=\{<13,3>^*,\ <11,3,2>\ ,\ <11,3,2>'\ ,<8,5,3>\ ,<8,5,3>'\}$ such that deg $L_3 \equiv -2$, deg $L_4 \equiv 2$, the other spin characters are of defect zero . S_{16} has 45 $(13,\alpha)$-regular classes , so B_1 and B_2 have 18 i.m.

(4:1) : 13- Brauer tree for B_1 :

$<14,1>\uparrow^{(12,2)}S_{16} = <15,1>^* + <14,2>^* = F_{13}$

$E_2\uparrow^{(1,0)}S_{16} = 2<14,2>^* + 2<13,2,1> + 2<13,2,1>'$
$= 2\ F_{14}$

$E_3\uparrow^{(1,0)}S_{16} = <13,2,1> + <13,2,1>' + <10,3,2,1>^* = F_{15}$

$E_4\uparrow^{(1,0)}S_{16} = <10,3,2,1>^* + <9,4,2,1>^* = F_{16}$

$E_5\uparrow^{(1,0)}S_{16} = <9,4,2,1>^* + <8,5,2,1>^* = F_{17}$

$E_6\uparrow^{(1,0)}S_{16} = <8,5,2,1>^* + <7,6,2,1>^* = F_{18}$

From above we have the Brauer tree :

$<15,1>^* - <14,2>^* - <13,2,1> = <13,2,1>' - <10,3,2,1>^*$ ┐
$<7,6,2,1>^* - <8,5,2,1>^* - <9,4,2,1>^*$ ┘

(4:2) : 13- Brauer tree for B_2 :

The block B_1 has 6 m.i. then B_2 has 12 i.m.

$E_1\uparrow^{(11,3)}S_{16} = <16> + <16>' + 2<13,3>^*$
$= K_1 = F_1+F_2$ **(10)**

$<12,3>\uparrow^{(1,0)}S_{16} = <13,3>^* + <12,3,1> = F_3$

$<12,3>'\uparrow^{(1,0)}S_{16} = <13,3>^* + <12,3,1>' = F_4$

$E_3\uparrow^{(11,3)}S_{16} = <12,3,1> + <12,3,1>' + <11,3,2> + <11,3,2>' = K_2 = F_5 + F_6$

$E_4\uparrow^{(11,3)}S_{16} = <11,3,2> + <11,3,2>' + <9,4,3> + <9,4,3>' = K_3 = F_7 + F_8$

$E_5\uparrow^{(11,3)}S_{16} = <9,4,3,> + <9,4,3>' + <8,5,3> + <8,5,3>' = K_4 = F_9 + F_{10}$

$E_6\uparrow^{(11,3)}S_{16} = <8,5,3> + <8,5,3>' + <7,6,3> + <7,6,3>' = K_5 = F_{11} + F_{12}$

Since $<12,3,1> \neq <12,3,1>'$, $<11,3,2> \neq <11,3,2>'$, $<9,4,3> \neq <9,4,3>'$ and $<8,5,3> \neq <8,5,3>'$ on $(13,\alpha)$-regular classes , then K_2 , K_3 , K_4 and K_5 are split (respectively) .

From above we have the Brauer tree :

$<16>$ $<12,3,1> - <11,3,2> - <9,4,3> - <8,5,3> - <7,6,3>$
\ /
$<13,3>^*$
/ \
$<16>^*$ $<12,3,1>' - <11,3,2>' - <9,4,3>' - <8,5,3>' - <7,6,3>'$

Section 5 : Brauer trees for S_{17} :

S_{17} has two blocks of defect one $B_1=L_1 \cup L_2$ where $L_1=\{(\ <13,4> + <13,4>'\ )\ ,\ <11,4,2>^*\ ,\ <8,5,4>^*\ \}$, $L_2=\{<17>^*\ ,\ <12,4,1>^*\ ,\ <10,4,3>^*\ ,\ <7,6,4>^*\ \}$ such that deg $L_1 \equiv 4$, deg $L_2 \equiv -4$ and $B_2=L_3 \cup L_4$ where $L_3=\{<14,3>\ ,\ <14,3>'\ ,\ <11,3,2,1>\ ,\ <11,3,2,1>'\ ,\ <8,5,3,1>\ ,<8,5,3,1>'\ \}$, $L_4=\{<16,1>\ ,\ <16,1>'\ ,<13,3,1>^*\ ,\ <9,4,3,1>\ ,\ <9,4,3,1>'\ ,\ <7,6,3,1>\ ,\ <7,6,3,1>'\ \}$ such that deg $L_3 \equiv 4$ and deg $L_4 \equiv -4$. The other spin characters are of defect zero . S_{17} has 54 $(13,\alpha)$-regular classes , so B_1 and B_2 have 18 i.m.

(5:1) : 13- Brauer tree for B_1 :

$F_1\uparrow^{(10,4)}S_{17} = <17>^* + <13,4> + <13,4>' = G_1$

$F_3\uparrow^{(10,4)}S_{17} = <13,4> + <13,4>' + <12,4,1>^* = G_2$

$F_5\uparrow^{(10,4}S_{17} = <12,4,1>^* + <11,4,2>^* = G_3$

$F_7\uparrow^{(10,4)}S_{17} = <11,4,2>^* + <10,4,3>^* = G_4$

$F_9\uparrow^{(10,4)}S_{17} = <10,4,3>^* + <8,5,4>^* = G_5$

$F_{11}\uparrow^{(10,4)}S_{17} = <8,5,4>^* + <7,6,4>^* = G_6$

From above we have the Brauer tree :

<17>*– <13,4> = <13,4>′– <12,4,1>*– <11,4,2>*–┐
<7,6,4>* – <8,5,4>*– <10,4,3>* –┘

(5:2): 13- Brauer tree for B_2 :

The block B_1 has 6 i.m. so B_2 has 12 i.m.

$F_1\uparrow^{(1,0)}S_{17}$ = <16,1> + <14,3> + <14,3>′ + <13,3,1>* = K_1

$F_{13}\uparrow^{(11,3)}S_{17}$ = <16,1> + <16,1>′ + <14,3> +<14,3>′ = K_2

$F_2\uparrow^{(1,0)}S_{17}$ = <16,1>′ + <14,3> + <14,3>′ + <13,3,1>* = K_3

$F_5\uparrow^{(1,0)}S_{17}$ = <13,3,1>* + <11,3,2,1> = G_{11}

$F_6\uparrow^{(1,0)}S_{17}$ = <13,3,1>* + <11,3,2,1>′ = G_{12}

$F_7\uparrow^{(1,0)}S_{17}$ = <11,3,2,1> + <9,4,3,1> = G_{13}

$F_8\uparrow^{(1,0)}S_{17}$ = <11,3,2,1>′ + <9,4,3,1>′ = G_{14}

$F_9\uparrow^{(1,0)}S_{17}$ = <9,4,3,1> + <8,5,3,1> = G_{15}

$F_{10}\uparrow^{(1,0)}S_{17}$ = <9,4,3,1>′ + <8,5,3,1>′ = G_{16}

$F_{11}\uparrow^{(1,0)}S_{17}$ = <8,5,3,1> + <7,6,3,1> = G_{17}

$F_{12}\uparrow^{(1,0)}S_{17}$ = <8,5,3,1>′ + <7,6,3,1>′ = G_{18}

<14,3,1>$\downarrow^{(1,0)}S_{17}$ = <13,3,1>* + <14,3> = G_9

since <14,3,1> i.m. in S_{18}

<14,3,1>′$\downarrow^{(1,0)}S_{17}$ = <13,3,1>* + <14,3>′ = G_{10}

since <14,3,1>′ i.m. in S_{18}

Since $K_2 = K_1 + K_3 – G_9 – G_{10}$ then $K_1 – G_{10} = G_7$ and $K_3 – G_9 = G_8$

From above we have the Brauer tree :

<16,1> – <14,3>   <11,3,2,1> – <9,4,3,1> – <8,5,3,1> – <7,6,3,1>
\ /
<13,3,1>*
**/ **
<16,1> – <14,3>   <11,3,2,1>′ – <9,4,3,1>′ – <8,5,3,1>′ – <7,6,3,1>′

Section 6 : 13- Brauer trees for S_{18} :

S_{18} has three blocks of defect one , $B_1=L_1\cup L_2$ where L_1={<16,2>* , (<13,3,2> + <13,3,2>′) , <9,4,3,2>, <7,6,3,2> }, L_2={ <15,3>*, <12,3,2,1>*, <8,5,3 ,2 >*} such that deg $L_1 \equiv 5$, deg $L_2 \equiv –5$ and $B_2=L_3\cup L_4$ where L_3={<17,1>* , (<13,4,1> + <13,4,1>′) , <10,4,3,1>* , <7,6,4,1>* }, L_4={<14,4>* ,<11,4,2,1>* , <8,5,4,1>* } such that deg $L_3 \equiv 1$, deg $L_4 \equiv –1$ and $B_3 = L_5 + L_6$ where L_5 ={<13,5>* ,<11,5,2> , <11,5,2>′ , <9,5,4> , <9,5,4>′ } , L_6 ={<18> ,<18>′ ,<12,5,1> , <12,5,1>′ , <10,5,3> , <10,5,3>′ ,<7,6,5> , <7,6,5>′ } such that deg $L_5 \equiv 4$, deg $L_6 \equiv – 4$

The other spin characters are of defect zero . S_{18} has 64 (13,α)-regular classes , so B_1 , B_2 and B_3 have 24 i.m.

(6:1) : 13- Brauer tree for B_1 :

$G_7\uparrow^{(12,2)}S_{18}$ = <16,2>* + <15,3>* =H_{19}

$G_9\uparrow^{(12,2)}S_{18}$ = <15,3>* + <13,3,2> + <13,3,2>′ = H_{20}

$G_{11}\uparrow^{(12,2)}S_{18}$ = <13,3,2> + <13,3,2>′ + <12,3,2,1>* = H_{21}

$G_{13}\uparrow^{(12,2)}S_{18}$ = <12,3,2,1>* + <9,4,3,2>* = H_{22}

$G_{15}\uparrow^{(12,2)}S_{18}$ = <9,4,3,2>* + <8,5,3,2>* = H_{23}

$G_{17}\uparrow^{(12,2)}S_{18}$ = <8,5,3,2>* + <7,6,3,2>* = H_{24}

From above we have the Brauer tree for B_1

<16,2>* – <15,3>* – <13,3,2> = <13,3,2>′ – <12,3,2,1>* –┐
<7,6,3,2>* – <8,5,3,2>* – <9,4,3,2>*–┘

(6:2) : 13- Braure tree for B_2 :

$G_7\uparrow^{(10,4)}S_{18}$ = <17,1>* + <14,4>* = H_{13}

$G_2\uparrow^{(1,0)}S_{18}$ = 2 <14,4>* +2 <13,4,1> + 2 <13,4,1>′ = $2H_{14}$

$G_3\uparrow^{(1,0)}S_{18}$ = <13,4,1> + <13,4,1>′ + <11,4,2,1>* = H_{15}

$G_4\uparrow^{(1,0)}S_{18}$ = <11,4,2,1>* + <10,4,3,1>* = H_{16}

$G_5\uparrow^{(1,0)}S_{18}$ = <10,4,3,1>* + <8,5,4,1>* = H_{17}

$G_6\uparrow^{(1,0)}S_{18}$ = <8,5,4,1>* + <7,6,4,1>* = H_{18}

From above we have the Brauer tree for B_2

<17,1>* – <14,4>* – <13,4,1> = <13,4,1>′ – <11,4,2,1>* –┐
<7,6,4,1>* <8,5,4,1>* – <10,4,3,1>* –┘

(6:3) : 13- Brauer tree for B_3 :

Sinse B_1 and B_2 have 12 i.m. then B_3 has 12 i.m.

$G_1\uparrow^{(9,5)}S_{18}$ = <18> + <18>′ + 2<13,5>* = K_1 = H_1+H_2 **(10)**

<12,5>$\uparrow^{(1,0)}S_{18}$ = <13,5>* + <12,5,1> = H_3

<12,5>$\uparrow^{(1,0)}S_{18}$ = <13,5>* + <12,5,1>′ =H_4

$G_3\uparrow^{(9,5)}S_{18}$ = <12,5,1> + <12,5,1>′ + <11,5,2> + <11,5,2>′ = K_2 = H_5+ H_6

$G_4\uparrow^{(9,5)}S_{18}$ = <11,5,2> + <11,5,2>′ + <10,5,3> + <10,5,3>′ = K_3 = H_7+ H_8

$G_5\uparrow^{(9,5)}S_{18}$ = <10,5,3> + <10,5,3>′ + <9,5,4> + <9,5,4>′ = K_4 = H_9 + H_{10}

$G_6\uparrow^{(9,5)}S_{18} = \langle 9,5,4\rangle + \langle 9,5,4\rangle' + \langle 7,6,5\rangle + \langle 7,6,5\rangle' = K_5 = H_{11} + H_{12}$

Since $\langle 12,5,1\rangle \neq \langle 12,5,1\rangle'$, $\langle 11,5,2\rangle \neq \langle 11,5,2\rangle'$, $\langle 10,5,3\rangle \neq \langle 10,5,3\rangle'$ and $\langle 9,5,4\rangle \neq \langle 9,5,4\rangle'$ on (13,α)-regular classes , then K_2 , K_3 , K_4 and K_5 are splits (respectively) .

From above we have the Brauer tree

$\langle 18\rangle \diagdown \qquad \diagup\ \langle 12,5,1\rangle - \langle 11,5,2\rangle - \langle 10,5,3\rangle - \langle 9,5,4\rangle - \langle 7,6,5\rangle$

$\qquad \langle 13,5\rangle^*$

$\langle 18\rangle' \diagup \qquad \diagdown\ \langle 12,5,1\rangle' - \langle 11,5,2\rangle' - \langle 10,5,3\rangle' - \langle 9,5,4\rangle' - \langle 7,6,5\rangle'$

<u>Section 7</u> : 13- Brauer trees for S_{19} :

S_{19} has four blocks of defect one $B_1 = L_1 \cup L_2$ where $L_1=\{\langle 19\rangle^*, \langle 12,6,1\rangle^*, \langle 10,6,3\rangle^*, \langle 8,6,5\rangle^*\}$, $L_2 =\{(\langle 13,6\rangle + \langle 13,6\rangle'), \langle 11,6,2\rangle^*, \langle 9,6,4\rangle^*\}$ such that deg $L_1 \equiv 5$, deg $L_2 \equiv -5$ and $B_2=L_3 \cup L_4$ where $L_3=\{\langle 14,5\rangle\ \langle 14,5\rangle', \langle 11,5,2,1\rangle, \langle 11,5,2,1\rangle', \langle 9,5,4,1\rangle, \langle 9,5,4,1\rangle'\}$, $L_4=\{\langle 18,1\rangle, \langle 18,1\rangle', \langle 13,5,1\rangle^*, \langle 10,5,3,1\rangle, \langle 10,5,3,1\rangle', \langle 7,6,5,1\rangle, \langle 7,6,5,1\rangle'\}$ such that deg $L_3 \equiv 3$, deg $L_4 \equiv -3$ and $B_3=L_5 \cup L_6$ where $L_5=\{\langle 16,2,1\rangle^*, \langle 14,3,2\rangle^*, \langle 9,4,3,2,1\rangle^*, \langle 7,6,3,2,1\rangle^*\}$, $L_6= \{\langle 15,3,1\rangle^*, (\langle 13,3,2,1\rangle + \langle 13,3,2,1\rangle'), \langle 8,5,3,2,1\rangle^*\}$ such that deg $L_5 \equiv 5$, deg $L_6 \equiv -5$ and $B_4= L_7 \cup L_8$ where $L_7= \{\langle 17,2\rangle, \langle 17,2\rangle', \langle 13,4,2\rangle^*, \langle 10,4,3,2\rangle, \langle 10,4,3,2\rangle', \langle 7,6,4,2\rangle, \langle 7,6,4,2\rangle'\}$, $L_8= \{\langle 15,4\rangle, \langle 15,4\rangle', \langle 12,4,2,1\rangle, \langle 12,4,2,1\rangle', \langle 8,5,4,2\rangle, \langle 8,5,4,2\rangle^*\}$ such that deg $L_7 \equiv 6$, deg $L_8 \equiv -6$ The other spin characters are of defect zero . S_{19} has 75 (13,α)-regular classes . So B_1 , B_2 , B_3 and B_4 have 36 i.m.

<u>(7:1)</u> : 13- Brauer tree for B_1 :

$H_1\uparrow^{(8,6)}S_{19} = \langle 19\rangle^* + \langle 13,6\rangle + \langle 13,6\rangle' = I_1$
$H_3\uparrow^{(8,6)}S_{19} = \langle 13,6\rangle + \langle 13,6\rangle' + \langle 12,6,1\rangle^* = I_2$
$H_5\uparrow^{(8,6)}S_{19} = \langle 12,6,1\rangle^* + \langle 11,6,2\rangle^* = I_3$
$H_7\uparrow^{(8,6)}S_{19} = \langle 11,6,2\rangle^* + \langle 10,6,3\rangle^* = I_4$
$H_9\uparrow^{(8,6)}S_{19} = \langle 10,6,3\rangle^* + \langle 9,6,4\rangle^* = I_5$
$H_{11}\uparrow^{(8,6)}S_{19} = \langle 9,6,4\rangle^* + \langle 8,6,5\rangle^* = I_6$

From above we have the Brauer tree for B_1

$\langle 19\rangle^* - \langle 13,6\rangle = \langle 13,6\rangle' - \langle 12,6,1\rangle^* - \langle 11,6,2\rangle^* -$

$\qquad \langle 8,6,5\rangle^* - \langle 9,6,4\rangle^* - \langle 10,6,3\rangle^* -$

<u>(7:2)</u> : 13- Brauer tree for B_2 :

$H_1\uparrow^{(1,0)}S_{19} = \langle 18,1\rangle + \langle 14,5\rangle + \langle 14,5\rangle' + \langle 13,5,1\rangle^* = K_1$
$H_{13}\uparrow^{(9,5)}S_{19} = \langle 18,1\rangle + \langle 18,1\rangle' + \langle 14\ 5\rangle + \langle 14,5\rangle' = K_2$
$H_2\uparrow^{(1,0)}S_{19} = \langle 18,1\rangle' + \langle 14,5\rangle + \langle 14,5\rangle' + \langle 13,5,1\rangle^* = K_3$
$H_5\uparrow^{(1,0)}S_{19} = \langle 13,5,1\rangle^* + \langle 11,5,2,1\rangle = I_{11}$
$H_6\uparrow^{(1,0)}S_{19} = \langle 13,5,1\rangle^* + \langle 11,5,2,1\rangle' = I_{12}$
$H_7\uparrow^{(1,0)}S_{19} = \langle 11,5,2,1\rangle + \langle 10,5,3,1\rangle = I_{13}$
$H_8\uparrow^{(1,0)}S_{19} = \langle 11,5,2,1\rangle' + \langle 10,5,3,1\rangle' = I_{14}$
$H_9\uparrow^{(1,0)}S_{19} = \langle 10,5,3,1\rangle + \langle 9,5,4,1\rangle = I_{15}$
$H_{10}\uparrow^{(1,0)}S_{19} = \langle 10,5,3,1\rangle' + \langle 9,5,4,1\rangle' = I_{16}$
$H_{11}\uparrow^{(1,0)}S_{19} = \langle 9,5,4,1\rangle + \langle 7,6,5,1\rangle = I_{17}$
$H_{12}\uparrow^{(1,0)}S_{19} = \langle 9,5,4,1\rangle' + \langle 7,6,5,1\rangle' = I_{18}$
$\langle 14,5,1\rangle\downarrow_{(1,0)}S_{19} = \langle 13,5,1\rangle^* + \langle 14,5\rangle = I_9$
since $\langle 14,5,1\rangle$ i.m. in S_{20}
$\langle 14,5,1\rangle'\downarrow_{(1,0)}S_{19} = \langle 13,5,1\rangle^* + \langle 14,5\rangle' = I_{10}$
since $\langle 14,5,1\rangle'$ i.m. in S_{20}

Since $K_2 = K_1 + K_3 - I_9 - I_{10}$ then $K_1 - I_9 = I_7$ and $K_3 - I_{10} = I_8$

From above we have the Brauer tree for B_2

$\langle 18,1\rangle - \langle 14,5\rangle \diagdown \qquad \diagup\ \langle 11,5,2,1\rangle - \langle 10,5,3,1\rangle - \langle 9,5,4,1\rangle - \langle 7,6,5,1\rangle$

$\qquad \langle 13,5,1\rangle^*$

$\langle 18,1\rangle' - \langle 14,5\rangle' \diagup \qquad \diagdown\ \langle 11,5,2,1\rangle' - \langle 10,5,3,1\rangle' - \langle 9,5,4,1\rangle' - \langle 7,6,5,1\rangle'$

<u>(7:3)</u> : 13- Brauer tree for B_3 :

$H_{19}\uparrow^{(1,0)}S_{19} = \langle 16,2,1\rangle^* + \langle 15,3,1\rangle^* = I_{31}$
$\langle 14,3,1\rangle\uparrow^{(12,2)}S_{19} = \langle 15,3,1\rangle^* + \langle 14,3,2\rangle^* = I_{32}$
$H_{21}\uparrow^{(1,0)}S_{19} = 2\langle 14,3,2\rangle^* + 2\langle 13,3,2,1\rangle + 2\langle 13,3,2,1\rangle' = 2I_{33}$
$H_{22}\uparrow^{(1,0)}S_{19} = \langle 13,3,2,1\rangle + \langle 13,3,2,1\rangle' + \langle 9,4,3,2,1\rangle^* = I_{34}$
$H_{23}\uparrow^{(1,0)}S_{19} = \langle 9,4,3,2,1\rangle^* + \langle 8,5,3,2,1\rangle^* = I_{35}$
$H_{24}\uparrow^{(1,0)}S_{19} = \langle 8,5,3,2,1\rangle^* + \langle 7,6,3,2,1\rangle^* = I_{36}$

From above we have the Brauer tree for B_3

$\langle 16,2,1\rangle^* - \langle 15,3,1\rangle^* - \langle 14,3,2\rangle^* - \langle 13,3,2,1\rangle = \langle 13,3,2,1\rangle' -$

$\qquad \langle 7,6,3,2,1\rangle^* - \langle 8,5,3,2,1\rangle^* - \langle 9,4,3,2,1\rangle^* -$

<u>(7:4)</u> : 13- Brauer tree for B_4 :

Since B_1 , B_2 and B_3 have 24 i.m. then B_4 has 12 i.m.

$H_{13}\uparrow^{(12,2)}S_{19} = \langle 17,2\rangle + \langle 17,2\rangle' + \langle 15,4\rangle + \langle 15,4\rangle' = K_4 = I_{19} + I_{20}$
$H_{14}\uparrow^{(12,2)}S_{19} = \langle 15,4\rangle + \langle 15,4\rangle' + 2\langle 13,4,2\rangle^* = K_5 = I_{21} + I_{22}$
$\langle 12,4,2\rangle\uparrow^{(12,2)}S_{19} = \langle 13,4,2\rangle^* + \langle 12,4,2,1\rangle = I_{23}$
$\langle 12,4,2\rangle'\uparrow^{(12,2)}S_{19} = \langle 13,4,2\rangle^* + \langle 12,4,2,1\rangle' = I_{24}$

$H_{16}\uparrow^{(12,2)}S_{19} = \langle 12,4,2,1\rangle + \langle 12,4,2,1\rangle' \langle 10,4,3,2\rangle + \langle 10,4,3,2\rangle' = K_6 = I_{25} + I_{26}$
$H_{17}\uparrow^{(12,2)}S_{19} = \langle 10,4,3,2\rangle + \langle 10,4,3,2\rangle' + \langle 8,5,4,2\rangle + \langle 8,5,4,2\rangle' = K_7 = I_{27} + I_{28}$
$H_{18}\uparrow^{(12,2)}S_{19} = \langle 8,5,4,2\rangle + \langle 8,5,4,2\rangle' + \langle 7,6,4,2\rangle + \langle 7,6,4,2\rangle' = K_8 = I_{29} + I_{30}$
Since $\langle 17,2\rangle \neq \langle 17,2\rangle'$, $\langle 15,4\rangle \neq \langle 15,4\rangle'$, $\langle 12,4,2,1\rangle \neq \langle 12,4,2,1\rangle'$, $\langle 10,4,3,2\rangle \neq \langle 10,4,3,2\rangle'$ and $\langle 8,5,4,2\rangle \neq \langle 8,5,4,2\rangle'$ on (13,α)-regular classes , then K_4 , K_5 , K_6 , K_7 and K_8 are splits (respectively) .
From above we have the Brauer tree for B_4

$$\begin{array}{ccccc} \langle 17,2\rangle - \langle 15,4\rangle & & & & \langle 12,4,2,1\rangle - \langle 10,4,3,2\rangle - \langle 8,5,4,2\rangle - \langle 7,6,4,2\rangle \\ & \diagdown & & \diagup & \\ & & \langle 13,4,2\rangle^* & & \\ & \diagup & & \diagdown & \\ \langle 17,2\rangle' - \langle 15,4\rangle' & & & & \langle 12,4,2,1\rangle' - \langle 10,4,3,2\rangle' - \langle 8,5,4,2\rangle' - \langle 7,6,4,2\rangle' \end{array}$$

Section 8 : 13- Brauer tree for S_{20} :

S_{20} has five block of defect one $B_1 = L_1 \cup L_2$ where $L_1 = \{ \langle 15,4,1\rangle, \langle 15,4,1\rangle , \langle 13,4,2,1\rangle^* , \langle 8,5,4,2,1\rangle , \langle 8,5,4,2,1\rangle' \}$, $L_2 = \{\langle 17,2,1\rangle , \langle 17,2,1\rangle' ,\langle 14,4,2\rangle , \langle 14,4,2\rangle', \langle 10,4,3,2,1\rangle , \langle 10,4,3,2,1\rangle', \langle 7,6,4,2,1\rangle, \langle 7,6,4,2,1\rangle' \}$ such that $\deg L_1 \equiv 2$, $\deg L_2 \equiv -2$ and $B_2 = L_3 \cup L_4$ where $L_3 = \{\langle 14,6\rangle^* ,\langle 11,6,2,1\rangle^* , \langle 9,6,4,1\rangle^*\}$, $L_4 = \{ \langle 19,1\rangle^* , (\langle 13,6,1\rangle + \langle 13,6,1\rangle') , \langle 10,6,3,1\rangle^* , \langle 8,6,5,1\rangle^* \}$ such that $\deg L_3 \equiv 1$, $\deg L_4 \equiv -1$ and $B_3 = L_5 \cup L_6$ where $L_5 = \{\langle 18,2\rangle^*, (\langle 13,5,2\rangle + \langle 13,5,2\rangle') ,\langle 10,5,3,2\rangle^*, \langle 7,6,5,2\rangle^*\}$, $L_6 = \{\langle 15,5\rangle^*,\langle 12,5,3,2\rangle^* , \langle 9,5,4,2\rangle^*\}$ such that $\deg L_5 \equiv 6$, $\deg L_6 \equiv -6$ and $B_4 = L_7 \cup L_8$ where $L_7 = \{\langle 16,4\rangle^* , \langle 12,4,3,1\rangle^*,\langle 8,5,4,3\rangle^* \}$, $L_8 = \{\langle 17,3\rangle^* , (\langle 13,4,3\rangle + \langle 13,4,3\rangle') , \langle 11,4,3,2\rangle^*, \langle 7,6,4,3\rangle^*\}$ such that $\deg L_7 \equiv 1$, $\deg L_8 \equiv -1$ and $B_5 = L_9 \cup L_{10}$ where $L_9 = \{\langle 20\rangle , \langle 20\rangle' , \langle 12,7,1\rangle , \langle 12,7,1\rangle' , \langle 10,7,3\rangle , \langle 10,7,3\rangle' ,\langle 8,7,5\rangle , \langle 8,7,5\rangle' \}$, $L_{10} = \{\langle 13,7\rangle^* , \langle 11,7,2\rangle ,\langle 11,7,2\rangle' ,\langle 9,7,4\rangle , \langle 9,7,4\rangle' \}$ such that $\deg L_9 \equiv 5$, $\deg L_{10} \equiv -5$. The other spin characters are of defect zero . S_{20} has 85 (13,α)-regular classes . So B_1 , B_2 , B_3 , B_4 and B_5 have 42 i.m.

(8:1) :13- Brauer tree for B_1 :

$I_{19}\uparrow^{(1,0)}S_{20} = \langle 17,2,1\rangle + \langle 15,4,1\rangle = J_1$
$I_{20}\uparrow^{(1,0)}S_{20} = \langle 17,2,1\rangle' + \langle 15,4,1\rangle' = J_2$
$I_{32}\uparrow^{(10,4)}S_{20} = \langle 15,4,1\rangle + \langle 15,4,1\rangle' + \langle 14,4,2\rangle + \langle 14,4,2\rangle' = K_1$
$I_{21}\uparrow^{(1,0)}S_{20} = \langle 15,4,1\rangle + \langle 14,4,2\rangle + \langle 14,4,2\rangle' + \langle 13,4,2,1\rangle^* = K_2$
$I_{22}\uparrow^{(1,0)}S_{20} = \langle 15,4,1\rangle' + \langle 14,4,2\rangle + \langle 14,4,2\rangle' + \langle 13,4,2,1\rangle^* = K_3$
$\langle 14,4,2,1\rangle\downarrow_{(1,0)}S_{20} = \langle 13,4,2,1\rangle^* + \langle 14,4,2\rangle = J_5$
since $\langle 14,4,2,1\rangle$ i.m. in S_{21}
$\langle 14,4,2,1\rangle'\downarrow_{(1,0)}S_{20} = \langle 13,4,2,1\rangle^* + \langle 14,4,2\rangle' = J_6$
since $\langle 14,4,2,1\rangle'$ i.m. in S_{21}
$I_{25}\uparrow^{(1,0)}S_{20} = \langle 13,4,2,1\rangle^* + \langle 10,4,3,2,1\rangle = J_7$
$I_{26}\uparrow^{(1,0)}S_{20} = \langle 13,4,2,1\rangle^* + \langle 10,4,3,2,1\rangle' = J_8$
$I_{27}\uparrow^{(1,0)}S_{20} = \langle 10,4,3,2,1\rangle + \langle 8,5,4,2,1\rangle = J_9$
$I_{28}\uparrow^{(1,0)}S_{20} = \langle 10,4,3,2,1\rangle' + \langle 8,5,4,2,1\rangle' = J_{10}$
$I_{29}\uparrow^{(1,0)}S_{20} = \langle 8,5,4,2,1\rangle + \langle 7,6,4,2,1\rangle = J_{11}$
$I_{30}\uparrow^{(1,0)}S_{20} = \langle 8,5,4,2,1\rangle' + \langle 7,6,4,2,1\rangle' = J_{12}$
Since $K_1 = K_2 + K_3 - J_5 - J_6$ then $K_2 - J_6 = J_3$ and $K_3 - J_5 = J_4$, from above we have the Brauer tree

$$\begin{array}{ccccc} \langle 17,2,1\rangle - \langle 15,4,1\rangle - \langle 14,4,2\rangle & & & & \langle 10,4,3,2,1\rangle - \langle 8,5,4,2,1\rangle - \langle 7,6,4,2,1\rangle \\ & \diagdown & & \diagup & \\ & & \langle 13,4,2,1\rangle & & \\ & \diagup & & \diagdown & \\ \langle 17,2,1\rangle' - \langle 15,4,1\rangle' - \langle 14,4,2\rangle' & & & & \langle 10,4,3,2,1\rangle' - \langle 8,5,4,2,1\rangle' - \langle 7,6,4,2,1\rangle' \end{array}$$

(8:2) : 13- Brauer tree for B_2 :

$I_2\uparrow^{(1,0)}S_{20} = 2\langle 14,6\rangle^* + 2\langle 13,6,1\rangle + 2\langle 13,6,1\rangle' = 2 J_{13}$
$I_3\uparrow^{(1,0)}S_{20} = \langle 13,6,1\rangle + \langle 13,6,1\rangle' + \langle 11,6,2,1\rangle^* = J_{14}$
$I_4\uparrow^{(1,0)}S_{20} = \langle 11,6,2,1\rangle^* + \langle 10,6,3,1\rangle^* = J_{15}$
$I_5\uparrow^{(1,0)}S_{20} = \langle 10,6,3,1\rangle^* + \langle 9,6,4,1\rangle^* = J_{16}$
$I_6\uparrow^{(1,0)}S_{20} = \langle 9,6,4,1\rangle^* + \langle 8,6,5,1\rangle^* = J_{17}$
$I_7\uparrow^{(1,0)}S_{20} = \langle 19,1\rangle^* + \langle 14,6\rangle^* = J_{18}$
From above we have the Brauer tree

$$\langle 19,1\rangle^* - \langle 14,6\rangle^* - \langle 13,6,1\rangle = \langle 13,6,1\rangle' - \langle 11,6,2,1\rangle^* - \langle 10,6,3,1\rangle^* - \langle 9,6,4,1\rangle^* - \langle 8,6,5,1\rangle^*$$

(8:3) : 13- Brauer tree for B_3 :

$I_7\uparrow^{(12,2)}S_{20} = \langle 18,2\rangle^* + \langle 15,5\rangle^* = J_{19}$
$I_9\uparrow^{(12,2)}S_{20} = \langle 15,5\rangle^* + \langle 13,5,2\rangle + \langle 13,5,2\rangle' = J_{20}$
$I_{11}\uparrow^{(12,2)}S_{20} = \langle 13,5,2\rangle + \langle 13,5,2\rangle' + \langle 12,5,2,1\rangle^* = J_{21}$
$I_{13}\uparrow^{(12,2)}S_{20} = \langle 12,5,2,1\rangle^* + \langle 10,5,3,2\rangle^* = J_{22}$
$I_{15}\uparrow^{(12,2)}S_{20} = \langle 10,5,3,2\rangle^* + \langle 9,5,4,2\rangle^* = J_{23}$
$I_{17}\uparrow^{(12,2)}S_{20} = \langle 9,5,4,2\rangle^* + \langle 7,6,5,2\rangle^* = J_{24}$
From above we have the Brauer tree

$$\langle 18,2\rangle^* - \langle 15,5\rangle^* - \langle 13,5,2\rangle = \langle 13,5,2\rangle' - \langle 12,5,2,1\rangle^* - \langle 10,5,3,2\rangle^* - \langle 9,5,4,2\rangle^* - \langle 7,6,5,2\rangle^*$$

(8:4) : 13- Brauer tree for B_4 :

$I_{19}\uparrow^{(11,3)}S_{20} = \langle 17,3\rangle^* + \langle 16,4\rangle^* = J_{25}$
$I_{21}\uparrow^{(11,3)}S_{20} = \langle 16,4\rangle^* + \langle 13,4,3\rangle + \langle 13,4,3\rangle' = J_{26}$
$I_{23}\uparrow^{(11,3)}S_{20} = \langle 13,4,3\rangle + \langle 13,4,3\rangle' + \langle 12,4,3,1\rangle^*$

$= J_{27}$

$I_{25}\uparrow^{(11,3)}S_{20} = \langle 12,4,3,1\rangle^{*} + \langle 11,4,3,2\rangle^{*} = J_{28}$

$I_{27}\uparrow^{(11,3)}S_{20} = \langle 11,4,3,2\rangle^{*} + \langle 8,5,4,3\rangle^{*} = J_{29}$

$I_{29}\uparrow^{(11,3)}S_{20} = \langle 8,5,4,3\rangle^{*} + \langle 7,6,4,3\rangle^{*} = J_{30}$

From above we have the Brauer tree

$$\langle 17,3\rangle^{*} - \langle 16,4\rangle^{*} - \langle 13,4,3\rangle = \langle 13,4,3\rangle' - \langle 12,4,3,1\rangle^{*}$$

$$\langle 7,6,4,3\rangle^{*} - \langle 8,5,4,3\rangle^{*} - \langle 11,4,3,2\rangle^{*}$$

(8:5) : 13- Brauer tree for B_5 :

Since B_1 , B_2 , B_3 and B_4 have 30 i.m. then B_5 has 12 i.m.

$I_1\uparrow^{(7,7)}S_{20} = \langle 20\rangle + \langle 20\rangle' + 2\langle 13,7\rangle^{*} = K_4 = J_{31} + J_{32}$ **(10)**

$\langle 12,7\rangle\uparrow^{(1,0)}S_{20} = \langle 13,7\rangle^{*} + \langle 12,7,1\rangle = J_{33}$

$\langle 12,7\rangle'\uparrow^{(1,0)}S_{20} = \langle 13,7\rangle^{*} + \langle 12,7,1\rangle' = J_{34}$

$I_3\uparrow^{(7,7)}S_{20} = \langle 12,7,1\rangle + \langle 12,7,1\rangle' + \langle 11,7,2\rangle + \langle 11,7,2\rangle' = K_5 = J_{35} + J_{36}$

$I_4\uparrow^{(7,7)}S_{20} = \langle 11,7,2\rangle + \langle 11,7,2\rangle' + \langle 10,7,3\rangle + \langle 10,7,3\rangle' = K_6 = J_{37} + J_{38}$

$I_5\uparrow^{(7,7)}S_{20} = \langle 10,7,3\rangle + \langle 10,7,3\rangle' + \langle 9,7,4\rangle + \langle 9,7,4\rangle' = K_7 = J_{39} + J_{40}$

$I_6\uparrow^{(7,7)}S_{20} = \langle 9,7,4\rangle + \langle 9,7,4\rangle' + \langle 8,7,5\rangle + \langle 8,7,5\rangle' = K_8 = J_{41} + J_{42}$

Since $\langle 12,7,1\rangle \neq \langle 12,7,1\rangle'$, $\langle 11,7,2\rangle \neq \langle 11,7,2\rangle'$, $\langle 10,7,3\rangle \neq \langle 10,7,3\rangle'$ and $\langle 9,7,4\rangle \neq \langle 9,7,4\rangle'$
on $(13,\alpha)$-regular classes , then K_5 , K_6 , K_7 and K_8 are split (respectively) .

From above we have the Brauer tree

$\langle 20\rangle$ \ / $\langle 12,7,1\rangle - \langle 11,7,2\rangle - \langle 10,7,3\rangle - \langle 9,7,4\rangle - \langle 8,7,5\rangle$

$\langle 13,7\rangle^{*}$

$\langle 20\rangle'$ / \ $\langle 12,7,1\rangle' - \langle 11,7,2\rangle' - \langle 10,7,3\rangle' - \langle 9,7,4\rangle' - \langle 8,7,5\rangle'$

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شجرة براورللمشخصات الاسقاطية لـ Sn ، $13 \le n \le 20$ قياس p= 13

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الخلاصة

في هذا البحث تم حساب شجرة براورللمشخصات الاسقاطية للزمرة التناظرية لـ Sn، $13 \le n \le 20$ للعدد الاولي p=13.