

INTERACTING BOSON MODEL CALCULATIONS FOR EVEN-EVEN ${}^{74}_{34}\text{Se}_{40}$ NUCLEUS

Ibrahim Jasim Abdullah

College of Education / University of Anbar



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ABSTRACT

In the present work, the interacting boson model (IBM-1) has been used in the calculation of the energy levels as a function of angular momentum $E(L)$ for even-even ${}^{74}_{34}\text{Se}_{40}$ deformed nuclei. The calculated results are compared with the available experimental data and found to be in a good agreement, especially at low-lying states, while at high angular momentum, some theoretical values are somehow larger than the experimental values.

Introduction

When high resolution spectroscopy is applied to study for atomic spectral lines emitted as electric dipole radiation in the visible and ultraviolet ray, many lines are found to consist of several closely spaced components. This splitting of atomic nuclei is called “hyperfine structure” (hfs).

Pauli (1924)[1] was the first to advocate the idea that the (hfs) owes its existence to a magnetic coupling between nucleus and electrons and he predicted on this basis the fundamental features of Zeeman and Paschen – Back effects of the (hfs).

The magnitude of the (hfs) splitting is proportional to the product of the nuclear magnetic moment μ_L and

the magnetic field produced at the nucleus by the (orbital and spin) magnetic moments of the orbital electrons. The determination of μ_L was firstly, carried out by Fermi (1930)[2]. He pointed out that the magnetic moment μ_L of the nucleus as a whole depends on the strength of the magnetic field. The spin of the nucleus and angular momentum of orbital electrons are tightly coupled or decoupled depending upon the value of the magnetic field strength.

Schmidt and Schuler (1935)[3] were the first to show the existence of nuclear electric quadrupole moments (QL) from deviations of the selection rule of the separation between (hfs) levels. The energy values of the (hfs) levels may be increased or decreased further by the additional interaction of a nuclear quadrupole moment (QL) with the outer electrons[4].

—————* Corresponding author at: College of Education / University of Anbar, Iraq.E-mail address:

There are many models which are used to describe the nuclear properties one of them is “Interacting Boson Model” (IBM) . This model was proposed, early by Feshbach and Iachello (1973)[5], and it was developed after that by Arima and Iachello (1974)[6], which is based on the following assumptions:

I-The pairs of active nuclear particles or holes near closed shells are treated as "bosons" i.e pairs of fermions [7].

II-This model depends on the total number of bosons (N).

III-The multitude of shell which appears in the shell model is reduced to the simple s-shell(L=0) and d-shell(L=2)only.

There are four versions of the (IBM), called IBM-1,2,3 and 4.This paper is concerned with version one of this model i.e"IBM-1" to study the nuclear structure of $^{74}_{34}\text{Se}_{40}$ nucleus.

Nurettin and Ismail(2007)[8]studied the energy levels and B(E2)values of cerium Ce(A=128-138)isotopes using IBM-1 and compared the results with experimental and theoretical(PTSM)model data and it was observed that they are in good agreement.

Theoretical part

Arima and Iachello[9]showed that the low-lying collective states can be described in terms of monopole boson with angular momentum $L\pi=0+$ (s-

boson) and a quadrupole boson with angular momentum $L\pi=2+$ (d-boson), where s-bosons are coupled to L=0 state, and d-bosons having five single boson states[10]:

$$L=0,\pm 1, \pm 2 \dots\dots\dots(1)$$

The operators $\hat{s}, \hat{s}^\dagger, \hat{d}, \hat{d}^\dagger$ satisfy the following “Bose Commutation Relations” as[11]:

$$\left. \begin{aligned} [\hat{s}, \hat{s}^\dagger] &= 1; [\hat{s}, \hat{s}] = [\hat{s}^\dagger, \hat{s}^\dagger] = 0 \\ [\hat{d}_\mu, \hat{d}_\mu^\dagger] &= \delta_{\mu\mu'}; [\hat{d}_\mu, \hat{d}_\mu] = [\hat{d}_\mu^\dagger, \hat{d}_\mu^\dagger] = 0 \\ [\hat{s}, \hat{d}_\mu^\dagger] &= [\hat{s}, \hat{d}_\mu] = [\hat{s}^\dagger, \hat{d}_\mu^\dagger] = [\hat{s}^\dagger, \hat{d}_\mu] = 0 \end{aligned} \right\} \dots\dots\dots(2)$$

Where:

$\hat{s}, \hat{d}_\mu \equiv$ annihilation operators

$\hat{s}^\dagger, \hat{d}_\mu^\dagger \equiv$ creation operators

The operators $\hat{\tilde{s}}, \hat{\tilde{d}}_\mu$ are given by the following relation $\hat{\tilde{s}} = \hat{s}; \hat{\tilde{d}}_\mu = (-1)^\mu \hat{d}_\mu \dots\dots\dots(3)$ [10]:

I-Transitional regions in the IBM-1

There are three transitional regions in the IBM-1:

1- O(6)-SU(5) region:

In this transitional region, the Hamiltonian of the operators $\hat{s}, \hat{s}^\dagger, \hat{d}, \hat{d}^\dagger$ in equations(2,3)) can be written as[11,12]:

$$\hat{H} = \epsilon \hat{n}_d + a_0 (\hat{P} \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) \dots\dots\dots(4)$$

The solution of this equation depends on the ratio (ϵ/a_0) when this ratio is large the eigenfunction of \hat{H} is appropriate to SU(5) symmetry, but when it is

small, the eigenfunction is appropriate to O(6) symmetry. The branching ratios (R) of the electric transition probability B(E2) for this region remains zero in SU(5) and O(6) [11,12].

2-SU(3)-SU(5) region:

In this region the rotational SU(3) symmetry has to be broken with $\epsilon \hat{n}_d$ term. The general form of the Hamiltonian operator (\hat{H}) of this region can be written as [11]:

$$\hat{H} = \epsilon \hat{n}_d + a_1 \hat{L}^2 + a_2 \hat{Q}^2 \dots \dots \dots (5)$$

The solution of above equation depends only on the ratio (ϵ / a_2) when it is large the eigenfunction of (\hat{H}) are appropriate to the SU(5) symmetry, while if it is small the eigenfunction of (\hat{H}) are appropriate to SU(3) symmetry.

The branching ratios (R) of B(E2) of this region are [11,12].

$$R = \frac{B(E2; 2_2^+ \rightarrow 0_1^+)}{B(E2; 2_2^+ \rightarrow 2_1^+)} = \begin{cases} 0 & \text{in SU(5) region} \\ 0.7 & \text{in SU(3) region} \end{cases} \dots \dots \dots (6)$$

3- SU(3)-O(6) region:

In this region the SU(3) symmetry has to be broken with $\hat{P}^+ \cdot \hat{P}$ term. The general form of the Hamiltonian can be written as [11]:

$$\hat{H} = a_0 (\hat{P} \cdot \hat{P}) + a_1 (\hat{L} \cdot \hat{L}) + a_2 (\hat{Q} \cdot \hat{Q}) + a_3 (\hat{T}_3 \cdot \hat{T}_3) \dots \dots \dots (7)$$

The solution of above equation depends on (a_0/a_2) when it is large, the eigenfunctions are appropriate to

O(6) symmetry, while when it is small, the eigenfunctions are appropriate to SU(3) symmetry. In this region the γ -band is below the β -band, and the branching ratios (R) of B(E2) are [11,12]:

The Electric Transition Probability B(E2):

The electric transition probability B(E2) can be written in three dynamical symmetries[11]:

i) SU(5) symmetry:

$$B(E2; L+2 \rightarrow L) = \alpha_2^2 \left(\frac{L+2}{2} \right) \left(\frac{2N-L}{2} \right) \dots \dots \dots (9)$$

where α_2^2 is a boson effective charge

According to equation (9) the basic condition for the observation SU(5) symmetry is [11,12]:

$$R^{(I)} = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} = 2 \left(\frac{N-1}{N} \right) < 2 \dots \dots \dots (10)$$

ii) SU(3) symmetry:

B(E2) for the ground state and of the first excited state 2_1^+ are given by [11,12]:

$$B(E2; L+2 \rightarrow L) = \alpha_2^2 \frac{3}{4} \frac{(L+2)(L+1)}{(2L+3)(2L+5)} (2N+L+3)(2N-L) \dots (11)$$

The basic condition for this symmetry is:

$$R^{(II)} = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} = \frac{10(N-1)(2N+5)}{7N(2N+3)} < \frac{10}{7} \dots \dots \dots (12)$$

iii) O(6) symmetry:

The electric transition probability $B(E2; L+2 \rightarrow L)$ can be written as [13]:

$$B(E2; L+2 \rightarrow L) = \frac{1}{4} \alpha_2^2 \frac{(L+2)}{2(L+5)} (2N-L)(2N+L+8) \dots \dots \dots (13)$$

The basic condition for this symmetry is [11,12]:

$$R^{(III)} = \frac{B(E2; 4_1^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} = \frac{B(E2; 2_2^+ \rightarrow 2_1^+)}{B(E2; 2_1^+ \rightarrow 0_1^+)} = \frac{10(N-1)(N+5)}{7N(N+4)} < \frac{10}{7} \dots \dots (14)$$

Results and Discussions:

In order to find the dynamical symmetry of the understudy nucleus we must calculate the theoretical and the experimental [14] energy ratios of $E(4_1^+)/E(2_1^+)$, $E(6_1^+)/E(2_1^+)$ and $E(8_1^+)/E(2_1^+)$ and comparing them with ideal experimental ratios [11] and ideal typical spectrum in reference [12] these calculations shows the $^{74}_{34}\text{Se}_{40}$ nucleus belonging to the dynamical symmetry SU(5)-O(6)-SU(3).

The parameters of IBM-1 Hamiltonian have been fixed by fitting them with energy levels of experimental values for low lying states. These parameters are shown in table (1).

Energy Bands Spectrum

The general known arrangement is the appearance of g-band with sequence $(0_1^+, 2_1^+, 4_1^+, \dots)$, β -band $(0_2^+, 2_2^+, 4_2^+, \dots)$ and γ -band $(2_3^+, 3_1^+, 4_3^+, 5_1^+, \dots)$

i.e 0_2 is below 2_2^+ . In this case the dynamical symmetry is either SU(3) or SU(5), whereas the appearance of γ -band before β -band (i.e 2_2^+ is below 0_2^+) this means occurrence of "breaking symmetry" to which, the nucleus belongs to it, In other words, if the state 2_2^+ comes before 0_2^+ the dynamical symmetry is O(6). leading to appearance of γ -unstable band, within the general behavior SU(3).

The energy bands arrangement of $^{74}_{34}\text{Se}_{40}$ nucleus are classified to (g, $\beta_1, \gamma_1, \beta_2, \beta_3$, and β_4) bands as shown in figure (1).

Figure (1) shows the energy band arrangement and energy levels for $^{74}_{34}\text{Se}_{40}$ nucleus in comparison with available experimental data. The comparison shows quite well agreement between the calculated IBM-1 energy spectrum (pw) of even-parity levels with available experimental results belonging to the dynamical symmetries SU(5)-O(6)-SU(3). The energy spectra and the spacing of these nuclei were found to fit well with experimental data. From this figure we can see a good reasonable agreement between the values of energy ground state (g-band) of sequence $(0_1^+, 2_1^+, 4_1^+, \dots)$ and their experimental state best than other bands, also the SU(3), SU(5), O(6)-Hamiltonian give a very reasonable description of the $^{74}_{34}\text{Se}_{40}$ energy spectrum.

The Electric Quadrupole Moments(QI):

Studying the values of nuclear electric quadrupole moment (QI) is one of the great features for measuring the nuclear deformations. The parameters (α_2, β_2) in table(1) have been obtained by fitting them with the available experimental values of electric quadrupole moments. The present values of (QI) show that these values are depending on β_2 -value more than (α_2) and both of them depend on total number of bosons (N).

Figure (2) shows the relation between the values of electric quadrupole moments and angular momentum (L_i) for selected nucleus. The results have been compared with experimental data [14] with good agreement.

The comparison shows very perfect agreement of the first excited (2_1^+) state, and somehow small in second excited state (2_2^+).

The $^{74}_{34}\text{Se}_{40}$ nucleus of the dynamical symmetry SU(5)-O(6)-SU(3) has an "oblate shape" at states ($2_1^+, 2_2^+, 2_3^+, 2_4^+, 2_7^+, 2_8^+$, and 2_9^+). This is agreeing with

experimental[14] shape in(21)state. While it has an "prolate shape" at states ($2_5^+, 2_6^+$, and 2_{10}^+).

The Electric transition probability B(E2):

The electric transition probabilities B(E2) are one of the important basic properties of the deformed nuclei. In order to evaluate B(E2), we must determine the parameters (α_2, β_2) by fits them with previous experimental results. These parameters are shown in table(1).

Figure(5) shows the relation between the probability of the electric transitions B(E2) for the transitions ($2_1^+ \rightarrow 0_1^+$), ($4_1^+ \rightarrow 2_1^+$), ($6_1^+ \rightarrow 4_1^+$), $8_1^+ \rightarrow 6_1^+$, ($10_1^+ \rightarrow 8_1^+$), ($12_1^+ \rightarrow 10_1^+$) as a function of angular momentum $L+2 \rightarrow L$. It is noticed in this figure that the highest point happened at the transition ($6_1^+ \rightarrow 4_1^+$) and increases with increase $L+2$ before high point ($6_1^+ \rightarrow 4_1^+$) and decreases after that. This means that the strong B(E2) happened at the transition ($6_1^+ \rightarrow 4_1^+$).

Conclusions

- 1-The fitted parameters which are used to the present calculation succeeded in studying the nuclear structure of even-even $^{74}_{34}\text{Se}_{40}$ nucleus of the dynamical symmetry SU(5)-O(6)-SU(3).
- 2- This model gives a good agreement with experimental data specially, in the ground state than other bands.

3- The electric quadrupole moments (Q_{21}) give a very good agreement than (Q_{22}).with the available experimental data in a value and sign.

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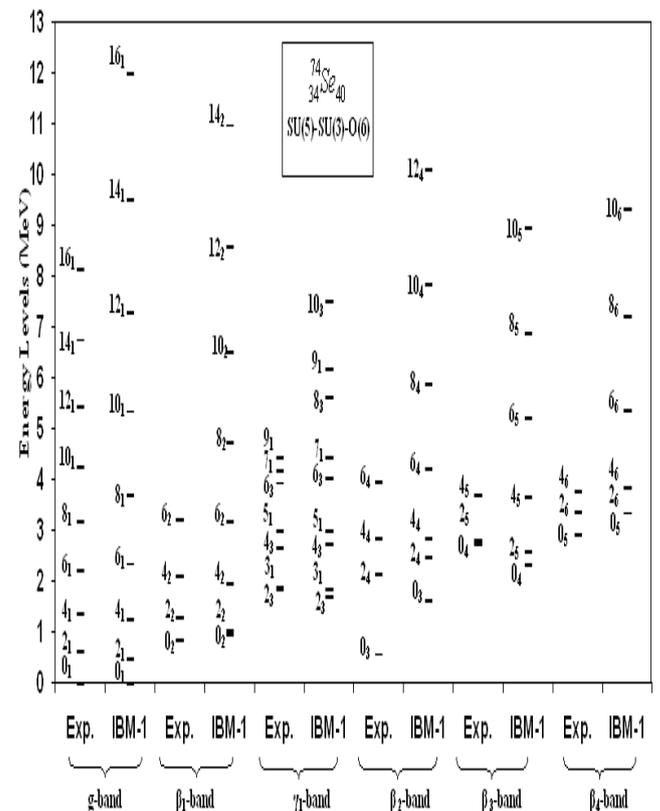


Figure (1):The comparison of the experimental[14]and the calculated IBM-1 energy for $^{74}_{34}\text{Se}_{40}$ nucleus .The parameters are used for calculations ain table(1).

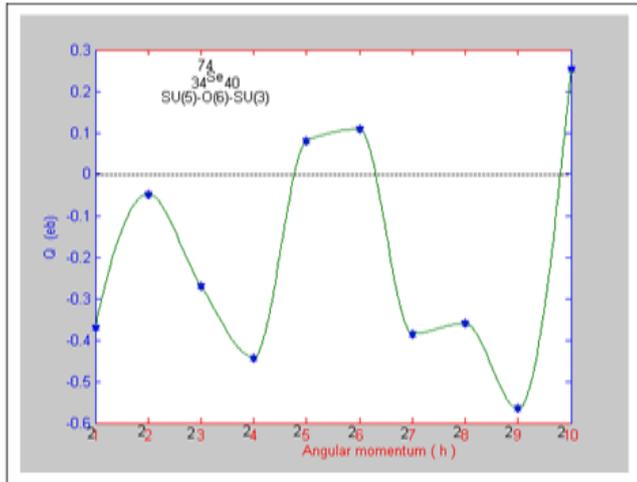


Figure (2): The electric quadrupole moment as a function of angular momentum $L=2_1$ to 2_{10}

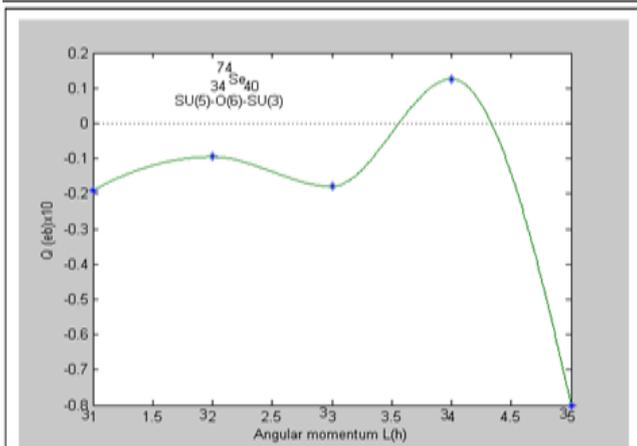
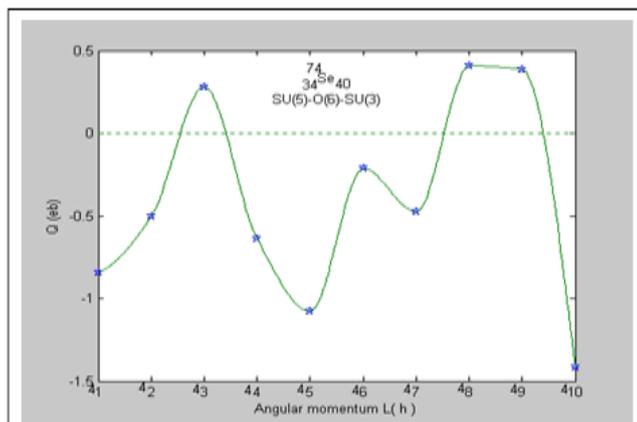


Figure (3): Same as in fig. (2) but for $L=3_1$ to 3_5 .



Figure(4):The electric quadrupole moment as a function of angular momentum $L=4_1$ to 4_{10}

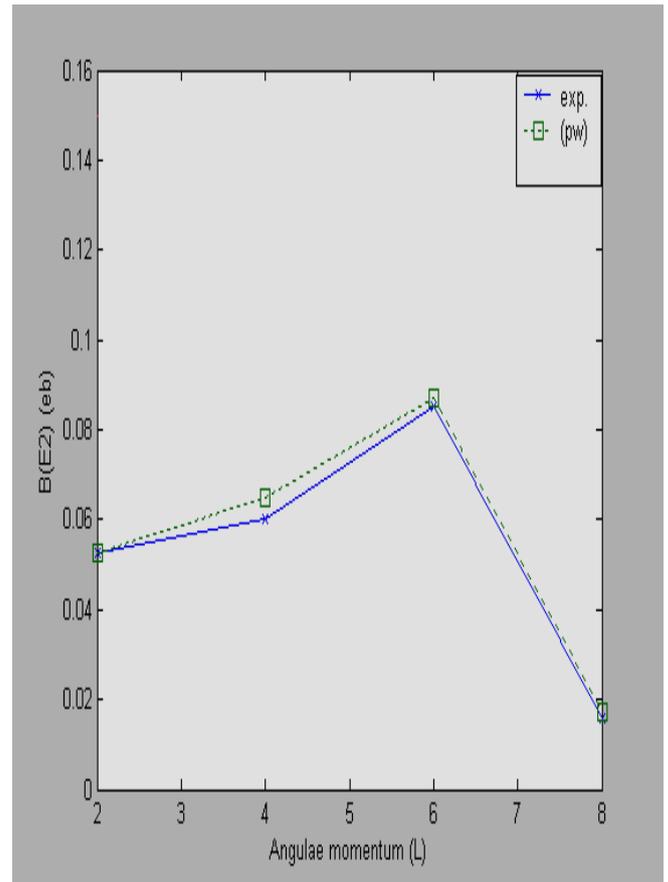


Figure (5): Electric transition probability $B(E2)$ with angular momentum (L)for g-band

Table (1): The values of the Hamiltonian operators and parameters (α_2, β_2) for even-even $^{74}\text{Se}_{40}$ nucleus.

N_π	N_ν	N	$\alpha_2(\text{eb})$	$\beta_2(\text{eb})$	$E(\text{MeV})$	$\hat{P}^\dagger \cdot \hat{P}(\text{MeV})$	$\hat{L} \cdot \hat{L}(\text{MeV})$	$\hat{Q} \cdot \hat{Q}(\text{MeV})$	$\hat{T}_3 \cdot \hat{T}_3(\text{MeV})$	$\hat{T}_4 \cdot \hat{T}_4(\text{MeV})$	CHI (MeV)
3	5	8	0.0830	-0.1960	0.2662	0.1100	0.0314	0.0068	0.1020	0.0660	0.0800

حسابات نموذج البوزونات المتفاعلة -الأول لنواة $^{74}_{34}Se_{40}$ الزوجية- الزوجية

إبراهيم جاسم عبد الله

الخلاصة

في البحث الحالي تم استخدام نموذج البوزونات المتفاعلة الاول (IBM-1) لحساب مستويات الطاقة كدالة للزخم الزاوي لبعض لنواة $^{74}_{34}Se_{40}$ المشوهة الزوجية- الزوجية وقد تم مقارنة هذه النتائج مع النتائج العملية المتوفرة وكانت النتائج متطابقة بشكل جيد خصوصا عند مستويات الطاقة الواطئة. أما عند المستويات العالية فكانت القيم النظرية اكبر من القيم العملية.