A New Signal De-Noising Method UsingAdaptive Wavelet Threshold based on PSO Algorithm and Kurtosis Measuring for Residual Noise

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الخلاصة:

ان عملية تقليل الضوضاء بالاعتماد على طريقة قياسالعتبة في تحويلة المويجات للاشارة المستلمةيخضع الى قيمة العتبة وطريقة اختيارها. وهذا يجعل قيمة العتبة كالفاصل الذي يميز بين الضجيج والإشارة. لهذا الوقت توجد هناكعدة طرققد طورت لتخمين قيمة العتبة والتي تعتمد على الحسابات الاحصائية للاشارة المشوشة حيثتفرض ان توزيع الاشارة الاصلية وتوزيع الضوضاء معلوم مسبقا. في الحقيقة وفي اي نظام عملي تكون الاشارة المستلمة (المشوشه) هي المعلومة فقط . ولهذا - في هذا العمل - قد تم تصميم نظام ذكي يبحث قيمة العتبة بدون اي معلومات سابقة لتلك التوزيعات الاحصائيةوذلك عن طريق تنفيذخوارزميةتجمع الجسيماتلـ (مقياس تفلطح متبقي الضوضاء المخمن) لايجاد قيمة العتبة المثالية والتي تكون عندها قيمة دالة التفلطح اعظم مايمكن. ويمكن تخمينمتبقي الضوضاءوذلك باستخدام دالة العتبة العكسية للعوامل التفصيلية لتحويلة المويجة المتقطعة للاشارة.ان مصداقية هذا الموديل قد تمت بمقارنة النتائج مع الحسابات الاحصائية وقد بينت النتائج قبول جيد لقيمة العتبة المستحصلة.

تم تنفيذ النظام المقترحبواسطة المحاكاة عن طريقبرنامج MATLAB2011كالاتي: اولا:تم قياس معامل التفلطح للضوضاء المتبقي لثلاث إشارات مختلفة بأربعة مستويات مختلفه للضوضاء.وقد وجد ان هناك قيمة واحدة فقط يكون عندهامعامل التفلطح للضوضاء المتبقيةأعظممايمكن. وبعد ذلك تم تنفيذخوارزميةتجمع الجسيماتللعثور على هذه القيمة المثالية.وفي نفس الوقت لوحظ أن خوارزمية تجمع الجسيمات بـ عشرة جسميات فقطتوفر سرعة تقارب حوالي من 20 الى 30 تكرار لأي توزيع إشارة ولأي مستوىضوضاء مستخدم .وبالاخرتم تقييم الاداء باستعمال معدل مربع الخطأمعمستويان و خمسة مستوياتتحلللكل إشارة اختبار.

الكلمات ألمفتاحيه: تقليل ضوضاء الاشارة، العتبة العكسيه، الضوضاء المتبقيه، خوارزمية تجمع الجسيمات ، معامل التفلطح.

Abstract

The signal de-noising based on waveletthresholdingis subjected to the value of threshold and how the way selection for it. Thismadea threshold value acts as an oracle which distinguishes between noise and signal.To date, there havebeen several methodsdeveloped to predictthe value of thresholddepending on statistical calculationsfor the noisy signalassuming that there is some priori knowledge for original signal and noise distributions.In fact, in any practical issues, only theobserved noisy signal that wehold.Therefore, in this work, an intelligent modelis developed to estimate the value of thresholdwithout any priority of knowledge for these distributions. This is done by implementingthe Particle Swarm Optimization (PSO) algorithm for kurtosis measuring of the residual noise signal to find an optimum threshold value at which the kurtosis function be maximum. These residual noise signal can be estimated by applying an inverse threshold function to the detail coefficients of the DWT. This model has been validated by comparison the results with the statisticalmodels and shows strong agreement for the obtained threshold parameter.

Computer simulation for proposed model was implemented using MATLAB2011 as follows:At first, the kurtosis measuring for residual noise was analyzedusing three different signals with four SNR levels. Through this step we found that: there's a single value for the threshold that maximizes the kurtosis of residual noise. Then, the PSO algorithm was implemented to find this optimumvalue.At the same time, it's noticed that the PSO algorithm with ten swarmprovidesa convergence speed about (20~ 30)iterationfor any signal distribution at any SNR level and for each decomposition level.Finally, the mean square error (MSE) was used to evaluate the performance oftwo and five decomposition levels for each tested signal.

Keywords:Signal de-noising, inverse threshold, residual noise, PSO, kurtosis.

1-Introduction

In recent years signal de-noising became one of the most important issues in the area of digital signal processing. The corruptions of signal by noise are common occurs during its processing, compression, transmission, and reproduction. Signal de-noising algorithm aimsto recover the cleaned version ofa signalfrom its noisy one by removing the noise and retaining the maximum possible signal information.

The capability of the wavelet transform inprecisely separation the high and low frequency components in the signal made it works as a dominant technology in the fields of signal de-noising. Because the noise are frequently localized at high frequency components in the signal, therefore, it becomes very useful to use a wavelet transform for decomposing the signal into its different frequency components and then get rid of the noise by thresholding it by a suitable threshold value. It should be noted here ifa large value of the threshold used, this will lead to destroy the signal data, while the small value of the threshold retains the noisy data.However, the threshold process modifies the signal data as the per selected threshold value. Therefore, thevalue of threshold plays an important part in signal de-noising and should be carefully selected (*KS Thyagarajan, 2006, Gao, R.X. and Yan, R.,2011*).

In the works like (S. Grace Chang *et.al., 2000)*and (Pankaj Hedaoo and Swati S Godbole, 2011) the threshold selection has been derived in a Bayesian method usinggeneralized Gaussian distribution (GGD) as a probabilistic model of the signal wavelet coefficients.In last few years, optimization algorithms have been made arevolutionarydevelopment in thethreshold selectionissue.In(Xing *et.al.,* Siwei *et.al., 2014*), (Dinesh *et.al.,2015*) and(V.Gopinath*, et.al., 2014*) MSE or signal statistics is used as a fitness functions for the optimization algorithms that made these worksneed a prior knowledge for signal statistical properties.

In this paper, we used PSO algorithm as an optimizationtechniquethat depends ona novel criteria for fitness function that rely on the kurtosis measuring for the estimated residual noise signal. ***Inverse threshold function*** was innovated to estimate these***residual noise***from the detail coefficients of the DWT of the noisy signal.*Our proposed algorithm suppose that there is a single value for the threshold called optimum threshold that maximizes kurtosis value of the residual noise which is then discovered by PSO algorithm*.The robust points in this criteria that it's no need for any prior knowledge for originalstatistical properties of the noisy signal. In other words, our proposed algorithm can be used for any signal at any SNR level.

This paper organized as follows: Section (2) and (3) survey the methods related to the traditional wavelet de-noising and traditional threshold selection respectively. Section (4)outlinesour proposed algorithm. In section (5) the PSO algorithm is presented. Section (6) discusses the results and performance analysis of the proposed model corresponding to other methods. FinallySection (7) concludes our paper.

2- Traditional Wavelet De-Noising Methods

A noisy signal with additive noise is modeled by:

$$nSig = Sig + Noise \quad (1)$$

Where ,$nSig$:denotes to the observed noisy signal,Sig: is the unknown original signal and $Noise$: isan independent identically distributed (*iid*) random Gaussian noise with zero mean and finite varianceσ^2.The goalis toremove the noise, or "de-noise" the observed{ $nSig$ }, to obtain an estimated { $eSig$ } ofthe original { Sig}with minimum mean square error (MSE):

$$MSE = \frac{1}{N}\sum_i^N (Sig_i - eSig_i)^2 \quad (2)$$

Where N is the signal length (should beinteger power of 2)(S. Grace Chang *et.al., 2000*).

Many de-noising techniques are proposed to overcome this problem, but the most powerfulonethat using a wavelet transform.Wavelet transform is a well-known tool for signal analysis.It can decompose the signal into many segments which belongto different frequency components. This is accomplished by comparing the signal with a group ofwavelet basis functions and then looking for their similarities in frequency contents (Gao *et.al., Yan et.al., 2011*).

Let Wdenotesto the orthogonal Discrete Wavelet Transform (DWT) matrix, then the wavelet coefficientsis:

$$W \times nSig = W \times Sig + W \times Noise$$
$$W_{nSig} = W_{Sig} + W_{Noise} \quad (3)$$

Where, $W_{Noise}$is also (*iid*) noise since the transformation is orthogonal(*S. Grace Chang et.al., 2000*).

Practically, the discrete wavelet transform is implemented by using a perfect reconstruction filter bankeach of which represent an orthogonal wavelet basis function. The result of this process is amultilevel decomposition, in which the signal is dividedat each levelinto sub bands called ***approximation*** and ***detail***coefficients as shown in Figure (1).If wedenote to the detail coefficientsof the multilevel transform by cD then $cD = [cD_J \dots\ , cD_k \dots\ , cD_2, cD_1]$where$k$ is the scalar, with$J$ being thelargest (or coarsest) scale in the decomposition, eachsubbandat scale$k$ has a length equal to$\left(\frac{N}{2^k}\right)$. The subbandcA_Jdenotes to the approximate coefficients (cA)and representsthe lowresolution details of the signal(*KS Thyagarajan, 2006*).

(a)

(b)

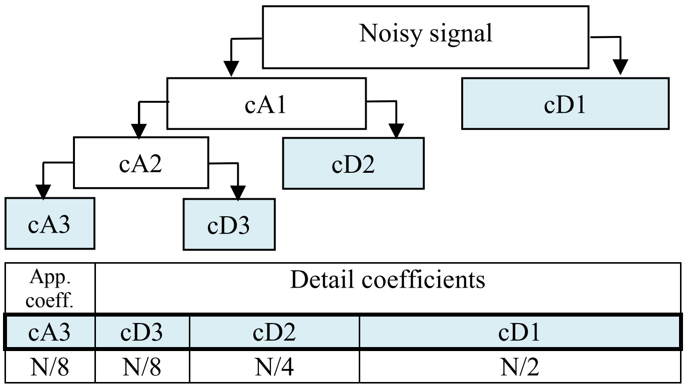


Figure (1): a) Subbands of 3 levels wavelet decomposition. b) Vectorrepresentation of the decomposed signal.

The traditional way in wavelet de-noising method start with trimming each coefficient from the detail subbands (cD) with a certain threshold to obtainthreshold version of detail sub bands (Z)as shown in Fig. (2). Then (Z)reconstructed with the approximation coefficients(cA) to produce the estimatedor de-noised signal where:

$$eSig = W^{-1} \times [cA_J\ , Z] \quad (4)$$

Where,W^{-1}: referred to the Inverse Discrete Wavelet Transform (IDWT) operator(*KS Thyagarajan, 2006*).

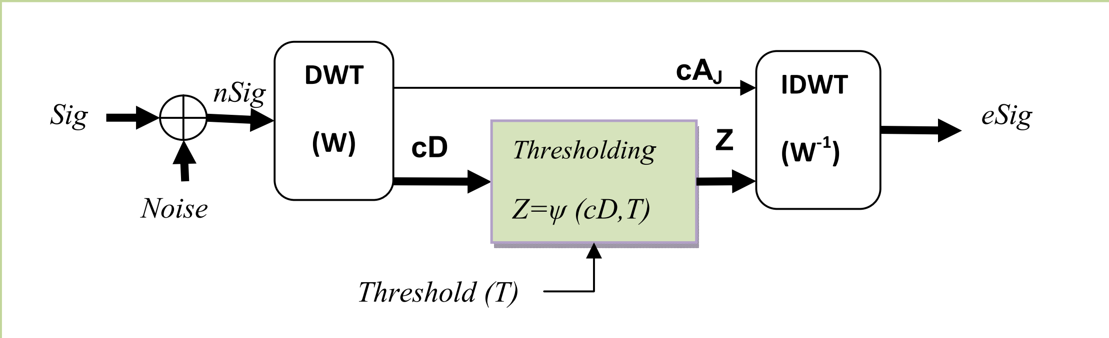
There are two main thresholdfunction that frequently used. The ***soft-threshold function*** (also called the shrinkage function), which is defined as:

$$Z = \psi(cD, T) = sign(cD).\times\ max\{(|cD| - T), 0\} \quad (5)$$

It takes the argument and shrinks it toward zero by the threshold *T*. The other popular alternative is the ***hard-threshold function***, which is defined by:

$$Z = \psi(cD, T) = cD.\times\ L(|cD| \geq T) \quad (6)$$

Where $L(\ )$ is a logic function (0 or 1), this function keeps the input if its value larger than threshold *T* otherwise, setit to zero (*S. Grace Chang et al, 2000*).



Figure(2):Traditional threshold wavelet de-noisingmodel.

3-TraditionalThreshold Selection Methods

The main deference between all existing wavelet de-noising methods is how to choose the way in which the threshold value is selected.There are many threshold selection methods that have been developed over the years such asVisu-Shrink, Sure-Shrink, and Bayes-Shrink.The Visu-Shrink threshold is evaluated by the following expression:

$$T_{Visu} = \sigma_N \sqrt{2log(L)} \qquad (7)$$

Where $\sigma_N$represents a noise variance and L is a length of signal.

This method results in an estimate asymptotically optimal in the minimax sense (minimizing the maximumerror over all possible L-sample signals) (S. Grace Chang *et.al., 2000*).

Another notablethreshold isSure-Shrink threshold which is definedby:

$$T_{Sure} = min\{t_J , \sigma\sqrt{2\log{(L)}}\} \qquad (8)$$

Wheret_J represents the threshold value at J_{th}decomposition level in wavelet domain *(Mantosh Biswas and HariOm, 2013*).

One of the most popular methods namely, BayesShrink was proposed by (*S. Grace Chang et.al.,2000*) in which the threshold has been derived from Bayesian method. This method has a better performance than the Sure-Shrink in terms of meansquare error (MSE). The BayesShrink threshold for every subband is given by:

$$T_{Bayes} = \frac{\sigma^2}{\sqrt{\sigma_{Sig}^2}} \qquad (9)$$

Where σ^2noise variance and σ_{Sig}^2 is the variance of original signal.

4-Proposed Algorithm

Through section (3) all these methodsof signal de-noising assume that there is some priori knowledgefor signal and noise distributions with known parameters to determine the value of threshold. It is known that in any practical issues, only the noisy signal that observed is determined, therefore, in order to propose a new and effective wavelet de-noising method without depending on this priority of knowledge,in this paper,we proposed a model (Figure (3)) thatfirstly usesthe ***Kurtosis statistic*** of the ***residual noise***signal to discoverthe optimum value for threshold at which the Kurtosis function becomes maximum,and then uses the PSO algorithmto reach this value after some iterations.

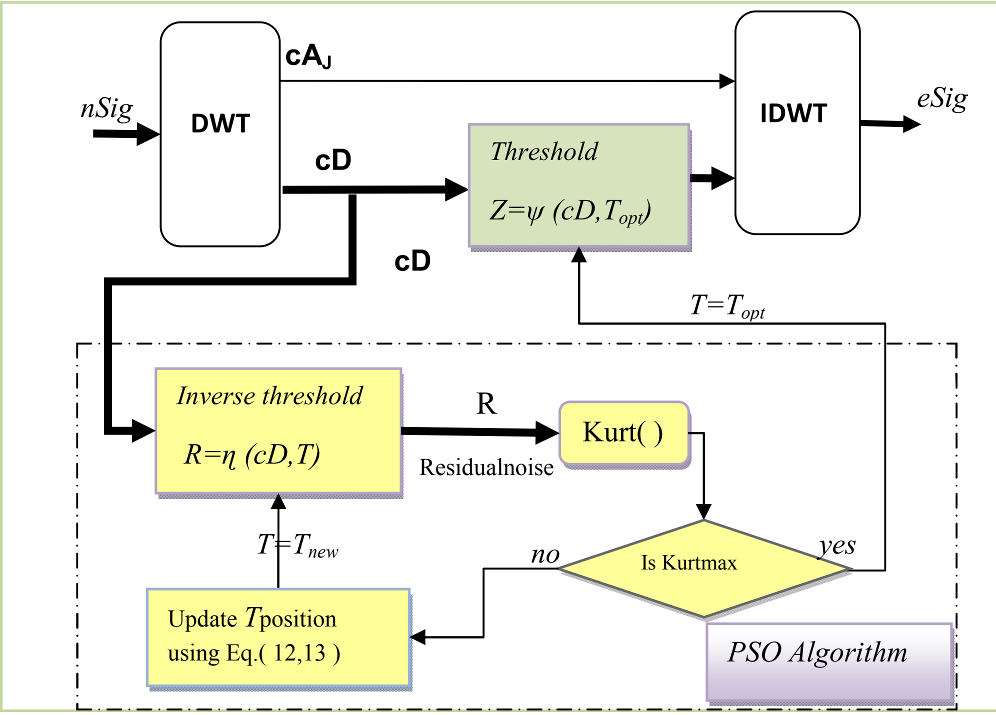


Figure (3): The proposed de-noising model based onPSO algorithm and Kurtosismeasuring for residual noise.

The normalized ***Kurtosisfunction*** for any random variable x is defined as:

$$kurt(x) = \frac{E((x-m_x)^4)}{\left(E((x-m_x)^2)\right)^2} - 3 \quad (10)$$

Where:$E(x)$ is the expected value of x.The kurtosis function provides an effective means to probe the statistical properties of random variables. For instance, if x is a Gaussian distribution vector, its kurtosis is always approach to zero (*Andrzej and AMARI, 2002*).

To estimate the noise added to the signal, the algorithm starts with applying DWT to noisy signal (*nSig*) to decompose it into approximation and detail coefficients.Then anew function is innovated toextract this noise (residual noise ***R***) from thedetail coefficients. Thisproposed functionis nominatedas***inverse threshold***function and works to shrinkthe input by T if its absolute value smaller than 2T, otherwise, set itto T.

$$R = \eta(cD, T) = sign(cD).\times \min\{|cD| - T\ , T\} \quad (11)$$

However, to improve the performance of our proposed algorithm,successive approximation techniquescan be used.The successive approximationmethod usesa sequence of thresholds$[T_J\ , \ldots.., T_k, \ldots.., T_2, T_1]$ for eachsub bandof the detailcoefficients$[cD_J, \ldots\ , cD_k \ldots\ \ , cD_2, cD_1]$.Usually threshold values are halved successively as follows$T_k = \frac{T_{k-1}}{2}$, (*KSThyagarajan, 2006*). But in our problem this is not strictly true, since the amount of noise in each sub band are random.Hence,the accepted valuesare: $T_1 > T_2 > \cdots > T_J$.

5-Particle Swarm Optimization (PSO)

The PSO algorithm is an evolutionary computation algorithm which has been developed by Eberhard and Kennedy in 1995. It simulatesthesocial behavior of birdflocking or fish schoolingwhile searching for the food. In PSO, each particle in the swarm represents a possible solution of the optimization problem, which is defined by its ***fitness function***. Ateach iteration, a new***location***for the particles is evaluated based on its last ***position*** and ***velocity***. In other words, every particle has a one chance to move for each iteration by a magnitude of velocity. So if the velocity is very high the particle will take bigger steps, and if the velocity is very small the particle will move in small steps making the convergence very slowly.Initially, the PSO algorithm deploys the particles randomly within the search space, and then it simply uses the objective function to estimate the fitness of each particle. Therefore, every particle will have a position, fitness value, and velocity.In this case, the best fitness valuedefined as a best particle or a best individual solution. Finally, the PSO algorithm estimates the global best solution (particle position which gives maximum orminimumfitness value among all particles in the population).The following equations depict the concept of the standard PSO algorithmwhich uses both the current global best$g_i^*$and the current individual bestx_i^* to reach the desired value after some iteration *t*:

$$x_i^{t+1} = x_i^t + v_i^{t+1} \quad (12)$$

$$v_i^{t+1} = wv_i^t + c_1\varepsilon_1(x_i^* - x_i^t) + c_2\varepsilon_2(g_i^* - x_i^t) \quad (13)$$

Where w is the inertia weight, ε_1 and ε_2 are uniform random numbers usually chosen between [0,1], $c_1$is a positive constant called as coefficient of the self-recognition component, c_2 is a positive constant called as coefficient of the social component(*Xin-She Yang , 2010, Dinesh K. Gupta 2015*).

6-Results and Performance Analysis

In order to analyze the performance of our proposed de-noising method, MATLAB2011 program have been used to implement the system shown in Fig.(3). In our algorithm three differentsignals(sine, rectangular andtriangle)are used to test the proposed model, each of which have *N*=32000 symbol lengthwith different frequency range as shown in Figure(4). These signals are contaminated with Gaussian noise with *SNR*= 10,15, 20, and25 togeta noisy signal$nSig$ from each one.

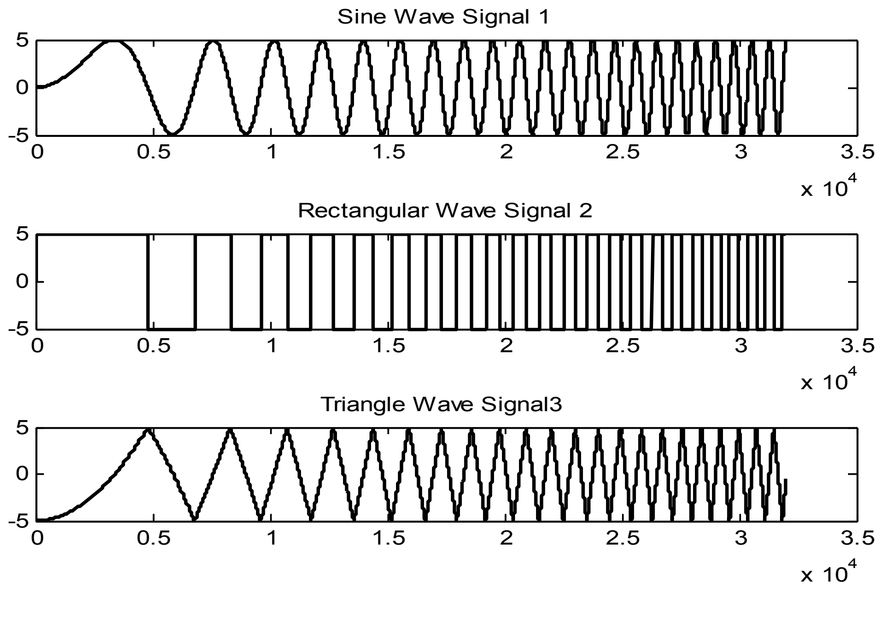


Figure (4):The tested signals in simulation.

6-1 Kurtosis Statistics of Residual Noise

In this section aone level Haar DWT has beenused to decomposeeach noisysignalinto approximation and detail coefficients each of which with 1600 samples. The kurtosis function evaluated for detail coefficientsafterthresholdingthem by the inverse soft threshold function (Eq. (11)). It is noticed from Figure (5) for each tested signal with different *SNR* that there is a single value for threshold called optimumthreshold (T_{opt}) at which the kurtosis measuringfunction of residual noise (***R***)be maximum.

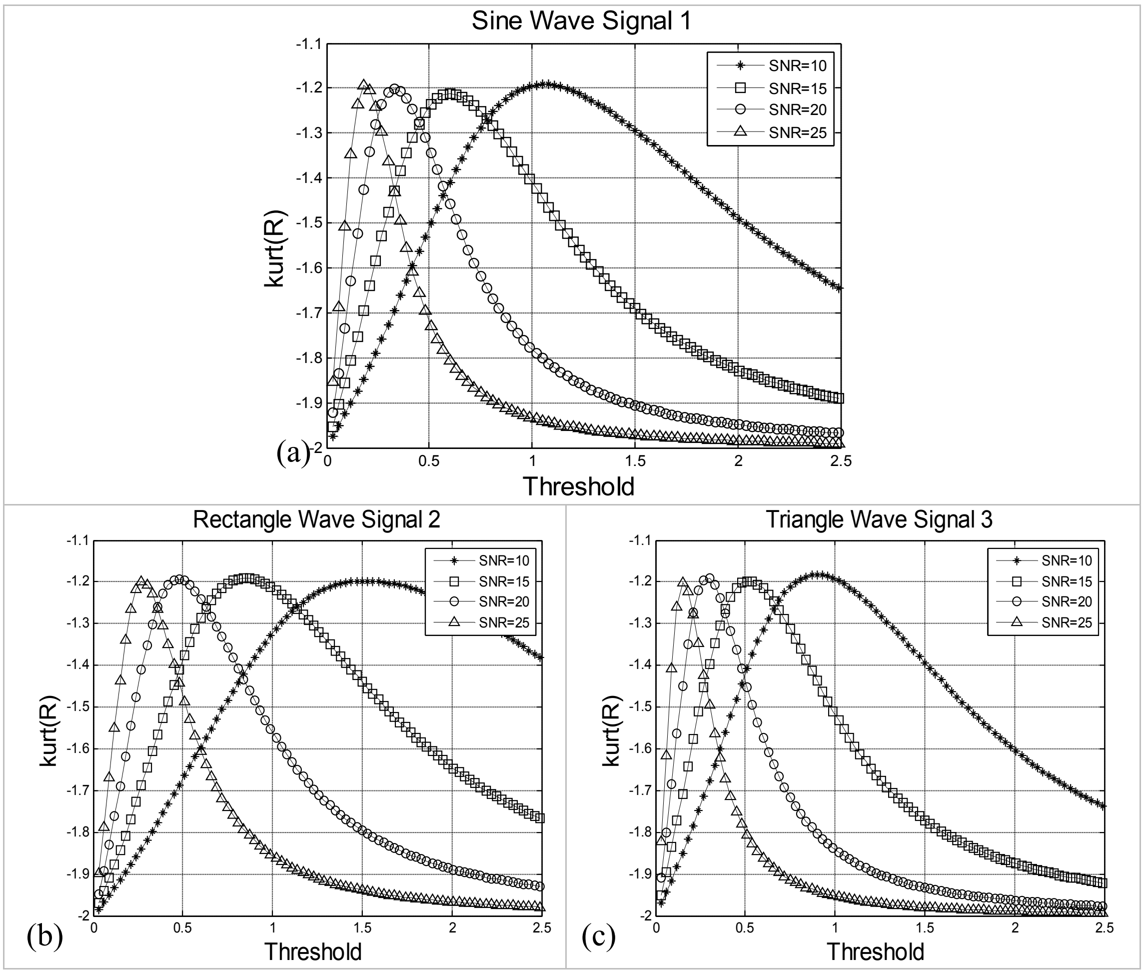


Figure (5):Kurtosis measuring of residual noise at adifferent SNR levelsfor (a) Sine wave.(b) Rectangular wave. (c) Triangle wave.

To validate our proposed method, we compare the optimum threshold value that obtained from kurtosis measuring with the well-known one, The BayesShrink threshold (Eq. (9)),the results in Table (1) shows a full agreement between the two threshold values.

Table (1) Comparison between Bayes Shrink and optimum thresholds

SNR dB	Sine wave		Rectangle wave		Triangle wave	
	T_{opt}	T_{Bayes}	T_{opt}	T_{Bayes}	T_{opt}	T_{Bayes}
10	1.08	1.0934	1.53	1.5709	0.9	0.9373
15	0.6	0.6148	0.84	0.8834	0.51	0.5270
20	0.33	0.3457	0.48	0.4967	0.27	0.2964
25	0.18	0.1944	0.27	0.2793	0.15	0.1666

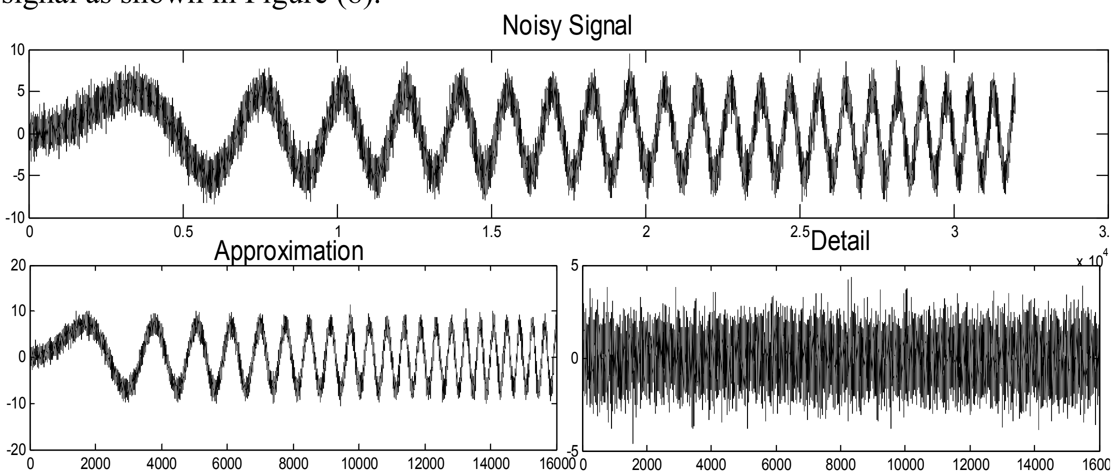
6-2 Optimum Threshold value Using PSO Algorithm

PSO algorithm is an effective tool to find these optimum threshold values (T_{opt}) in Table (1). For this purpose, the number of particles in the swarm has been considered equal to 10, each particle represents a possible threshold value. Maximum number of iteration assumes to be 100.It should be noted here that the number of iterations that needed to locate maximum kurtosis value depends on time neededfor particlesto converge into optimum position.

6-2-1 PSO Algorithm for One Level DWT

Although one leveldecomposition is rarely used in practical applications, it is used her to check the accuracy of our proposed algorithm. A one level decompositionof a sine wave noisy signal with SNR=10dB has been considered here as an example case.The signal decomposed into approximation and detailcoefficients each of which with 16000 samplesas shown in Figure(6).

After applying the proposed algorithm, the optimum threshold value was $T_{PSO} = 1.0602$ with maximum kurtosis= -1.2051 for the residual noise. The convergence behavior of PSO algorithm (swarm position at each iteration) is shown in Figure (7).Finally T_{PSO} value used to threshold the detail coefficients and the resultant is reconstructed with approximation coefficients using IDWT to obtain de-noised signal as shown in Figure (8).



Figure(6):One level DWT decomposition for noisy sine wave at SNR=10dB.

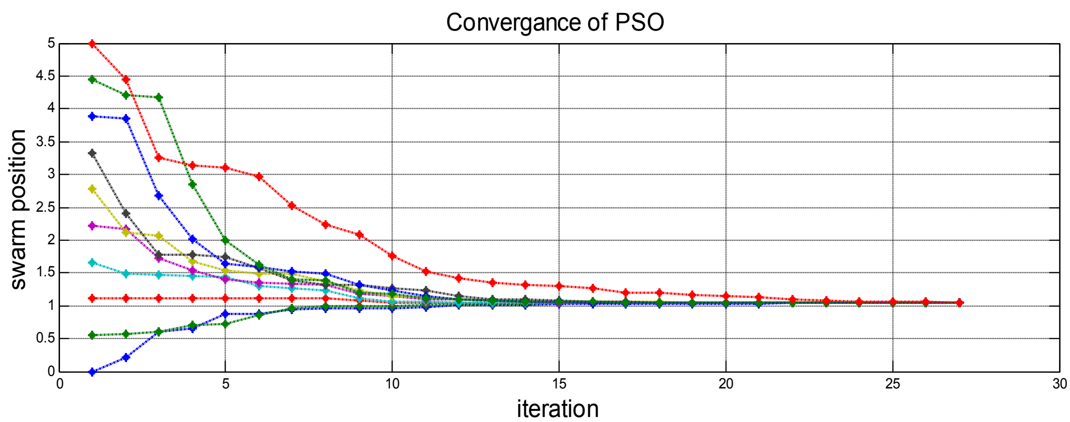
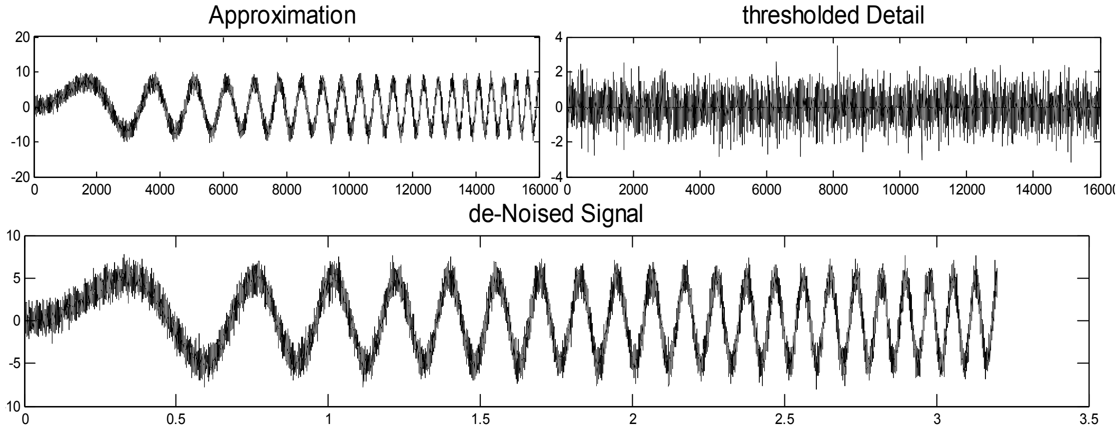


Figure (7):Convergence behavior of PSOin case of one level DWT decomposition for noisy sine wave at SNR=10dB.



Figure(8):De-noised signal using proposed algorithmin case of one level DWT with noisy sine wave at *SNR*=10dB.

It's obvious that the de-nosing performance is so awful due to using only one level DWT. The same procedure used for the three signals at four different*SNR* level to obtain 12 results recorded in Table (2).

Table (2):Threshold values and number of iteration (n_{iter}) forPSO algorithm whenone level DWT is used.

SNR dB	Sine wave		Rectangle wave		Triangle wave	
	T_{PSO}	n_{iter}	T_{PSO}	n_{iter}	T_{PSO}	n_{iter}
10	1.0602	22	1.538	27	0.91216	25
15	0.59661	23	0.82026	24	0.51365	23
20	0.33285	24	0.48165	23	0.29531	28
25	0.19106	28	0.27301	22	0.16519	29

Another validation for the proposed model has been done by comparing T_{PSO} in Table(2) with the corresponding values (T_{bayes}) in Table (1). It's obvious that the PSO algorithm provides a strong agreement results with acceptable computational complexity for all signal types at any SNR level. For that reason our algorithm used such asmethod thatnot depending on any statistical properties of the signal to extract the desired threshold value.

6-2-2 PSO Algorithm for Multilevel DWT

In this sectiona DWT with five levelsdecompositionhave beenused. The detailcoefficients ofa five sub bands have been chosen (Figure (9))for a sine wave noisy signal with SNR=10dB as an example case from the three cases for each tested signal.

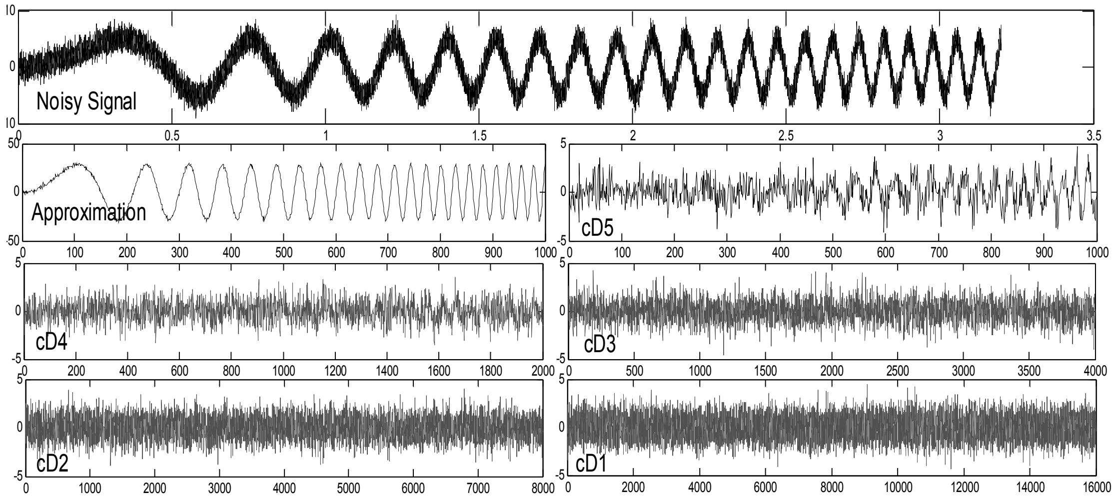
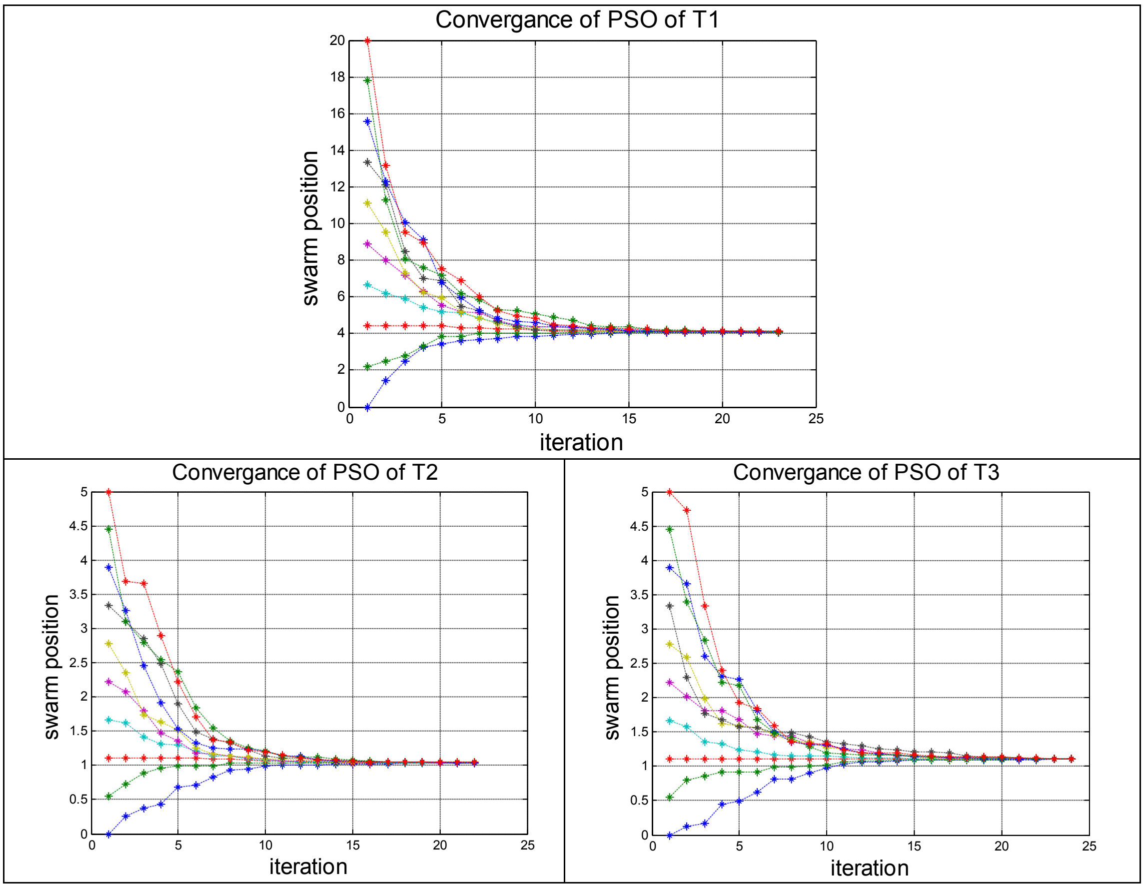


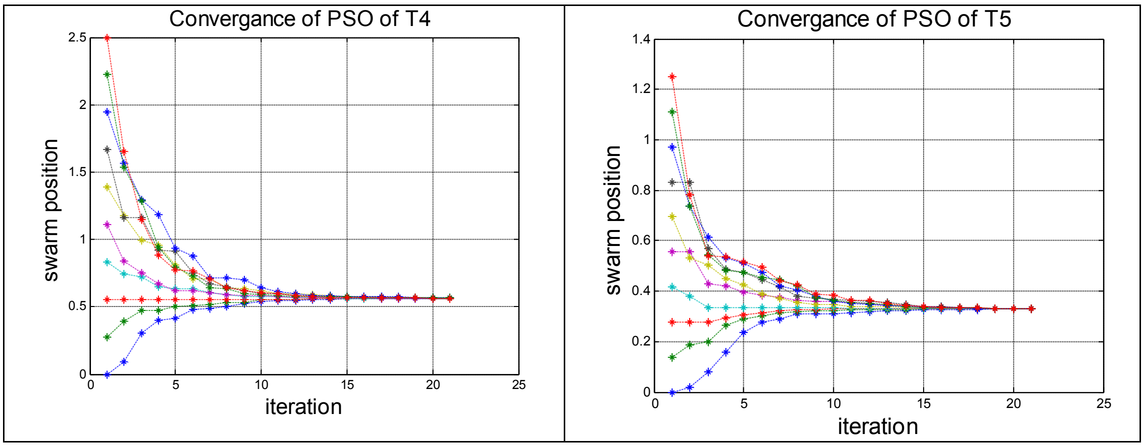
Figure: (9)Five level decomposition for sine wave signal with SNR=10.

After applying the proposed algorithm,the obtained values of maximum kurtosis,threshold and number of iteration for each detail subbands are:

Detail coefficient: cD1:T1 = 4.0987 |n_{iter}= 23 |Kurtmax1 = -1.1784
Detail coefficient: cD2:T2 = 2.0799 |n_{iter}= 22 | Kurtmax2 = -1.1971
Detail coefficient: cD3:T3 = 1.1036 |n_{iter}= 24 |Kurtmax3 = -1.1794
Detail coefficient: cD4:T4 = 0.5634 | n_{iter}= 21 |Kurtmax4 = -1.1665
Detail coefficient: cD5:T5 = 0.3305|n_{iter}= 21|Kurtmax5 = -1.2034

The convergence behavior of PSO algorithm (swarm position at each iteration) for each detail subbands is shown in Figure (7).





Figure(10):Convergence behavior of PSO in case of five level DWT with noisy sine wave at *SNR*=10dB.

Finally threshold values of $T_{PSO}$are used to threshold the detail coefficients and the resultant is reconstructed with approximation coefficients using IDWT to obtain de-noised signal as shown in Figure(11) .

It's obvious that the five levels DWT provides a powerful performance as compared with one level DWT. For more illustration Table (3) contain mean square error (MSE)of these two cases.

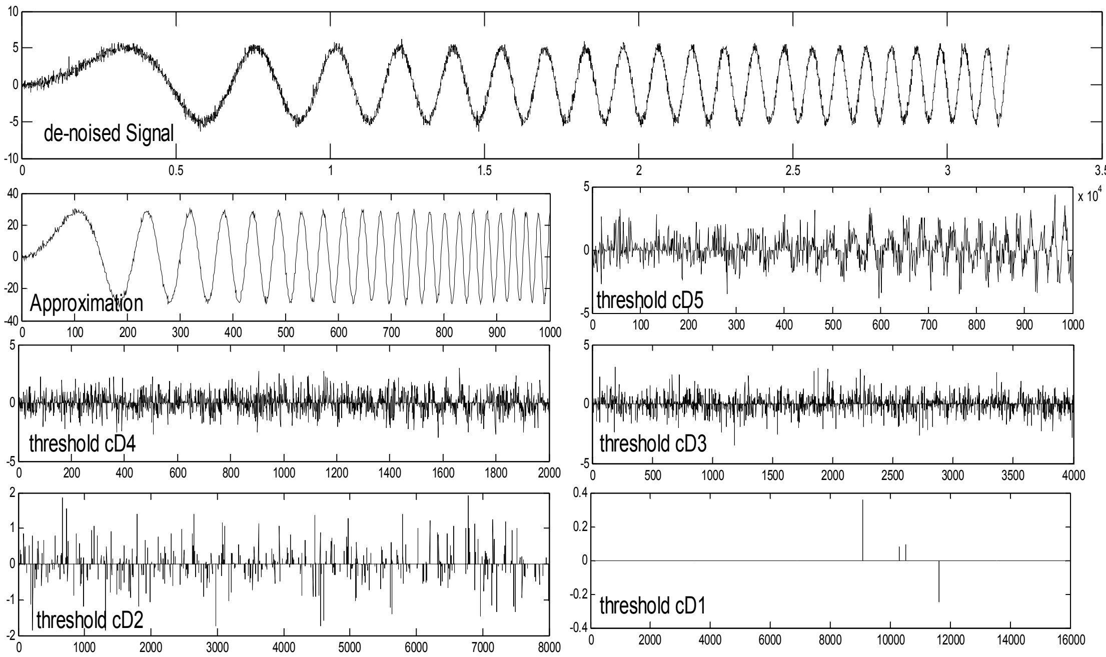


Figure (11): De-noised signal and threshold detail coefficientsusing proposedalgorithm in case of five level DWT with noisy sine wave at *SNR*=10dB.

Table (3): MSE ofproposed algorithm in case ofone and five levels DWT

SNR	Sine wave		Rectangle wave		Triangle wave	
	MSE_{L5}	MSE_{L1}	MSE_{L5}	MSE_{L1}	MSE_{L5}	MSE_{L1}
10	0.023984	0.05364	0.016733	0.037642	0.027161	0.063036
15	0.013636	0.030792	0.0094562	0.02206	0.015628	0.036331
20	0.0082766	0.017496	0.0052221	0.012292	0.015628	0.020508
25	0.0054104	0.00983	0.0028875	0.0069232	0.0069915	0.011278

7-Conclusions

This paper proposed a PSO based multilevel adaptivethresholding technique for signal de-noising.Through our work we noticed the following: In Figure (5)it's appeared that: there is a single value for threshold (optimum threshold) which maximizes the kurtosis measuring of residual noise for any signal at any SNR level. This point led us to ensure that our proposed algorithm can be used for any signal without any prior knowledge for its original statistical properties. In section 6-2 PSO algorithmused forsearching optimum threshold value.By comparing threshold valuesthat obtained by PSO algorithm in Table (2) with the corresponding values in Table (1) it's obvious that PSO algorithm provides an exultant result with acceptable computational complexity for all signals type at any SNR level. The second important feature for the PSO algorithm that strongly noticed here is the number of iterationsthat needed for the algorithm to convergence tothe optimum value is always about (20 ~30) iteration regardless the signal type and SNR level and decomposition level, see Table(2) and Figure(7) and Figure (10). This point made our proposed algorithm providesa constantprocessing time for any signal and this feature is very important in practical applications. In multi-level DWT threshold values always$T_1 > T_2 > \cdots > T_J$. Also, this point can be used to reduce processing time by evaluation only odd value of threshold $(T_1, T_3, \ldots.)$ and even values $(T_2, T_4, \ldots.)$can be calculated intuitively. Finally the clearconclusionthat the five level DWT provides a more powerful performance than one level decomposition as given in Table (3).

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