# Theoretical study of the electromagnetic wave behavior in Josephson transmission line using MATLAB model

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## Abstract

This paper presented a detailed description of how the magnetic vector potential (**A**) can be evaluated for an infinite weak link Josephson junction with planar geometry. Utilizing the magnetic vector potential formulation, both electric and magnetic fields were computed , and by using MATLAB model we showed that we obtained the same dispersion results as other approaches have previously demonstrated. However, we first revisited the beginnings of the Josephson junction and the conventional formulation techniques used to describe the electromagnetism of layered superconducting structures and Josephson junctions. In addition, we derived the field equations, for a transverse magnetic, for a superconducting transmission line, and take an in depth look at what these electromagnetic field equations represent.

**Keywords;** Electromagnetic fields, Josephson junctions, Superconducting transmission lines, magnetic vector potential, and dispersion relation.

## 1. Introduction

Even before the advent of Josephson's initial paper "Possible new effects in superconductive tunneling" [1], much interest and excitement existed over the structure that would later be known by Josephson's name. Scientists such as P.W. Anderson [2], A. B. Pippard [3], and J. C. Swihart [4] among others were intently trying to unravel the mysteries of coupling and electromagnetic propagation in superconducting devices of different architectures in hopes of discovering a novel application, some new and exotic physics, or a physical realization of a past theory. While none of this flurry of work would be possible without the discovery of superconductivity by Heike Kamerlingh Onnes [5] in 1911 when he observed the disappearance of resistance in mercury at liquid Helium temperatures, the interpretation of superconductivity as a quantum phenomena by F. and H. London [6] in 1935, the modification of London's theory by Ginzburg and Landau [7] in 1950 giving us an equation by their names, or the microscopic (BCS) theory of superconductivity developed by Bardeen, Cooper, and Schrieffer [8] in 1957 .Much of the general and applied research being done on Josephson junctions depends greatly on how these junctions respond to internal electromagnetic fields. Consequently, an alternative formulation of the propagating fields in a Josephson junction may increase the quality of the production of more precise and effective devices. Many applications have already been studied and presented using different approaches to study the electromagnetic behavior of the Josephson junction. Bulaevskii et al.[9] have used Maxwell's equations and the Josephson relation to derive the equation for time-dependent phase differences for Josephson-coupled multilayer systems . Wu et al.[10] have used the dispersion relationship obtained by matching the tangential components of the fields to investigate the propagation of an electromagnetic wave in a stack of superconducting layers . Aziz and Saeed have used HTS strip line model to investigate some important parameters like pulse propagation and comparing their results with normal conductors [11]. Additionally, with the onset of high $T_c$ superconductors, it may one day be feasible to use superconducting transmission lines for low-loss power distribution, energy storage, and fault current limiters [12]. Finally, these structures are an essential component of today's and tomorrow's high-energy experimental physics labs [13]. In particular, the application that is addressed in this paper is basically superconductive layered structure that is a form of a superconducting transmission line. It should also be noted that a true Josephson junction, and related devices, do have much more complex behavior compared to what is addressed in this paper. In particular, a more detailed formulation of the currents involved is needed for fully extending the proposed approach to Josephson junctions and related devices.

## 2. Theory

We begin by looking at the fundamental approach of using Maxwell's equations to solve for the propagating fields in a Josephson junction, which we noted earlier was employed by Swihart [4] in his investigation of propagation. For simplicity, we assume that our Josephson junction consists of two identical superconducting layers and a thin insulator of thickness D. We also assume that the superconductors are much thicker and the insulating layer thinner than the London penetration depth, $\lambda$ [11], and that structure is infinite in both the y and z directions and centered at the origin. Fig. 1 illustrates the junction of interest.

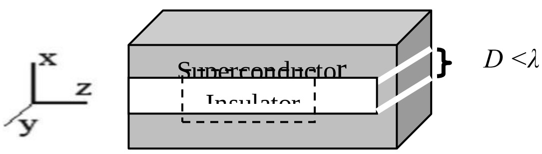


**Fig. 1.** ***A simple superconducting transmission line composed of two superconductors separated by a dielectric of thickness D***

We start by using Faraday's law[14],

$$\vec{\nabla} \times \vec{E} = -\frac{\partial \vec{B}}{\partial t} \quad (1)$$

Now, we can integrate this equation over the dashed surface (Fig.1).

$$\int_S \vec{\nabla} \times \vec{E} \cdot d\vec{s} = \int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} \quad (2)$$

The left hand side can then be rewritten using Stoke's law and the definition of a derivative with respect to z.

$$\int_s \vec{\nabla} \times \vec{E} \cdot d\vec{s} = \oint_{dashedsurface} \vec{E} \cdot d\vec{l} = \{E_x(z+dz) - E_x(z)\}D = D\frac{\partial E_x}{\partial z}dz \quad (3)$$

Here, Ex is the electric field polarized in the x-direction inside the insulator. The right hand side is then evaluated giving us:

$$\int_S -\frac{\partial \vec{B}}{\partial t} \cdot d\vec{s} = -\frac{\partial B_y}{\partial t} \cdot (2\lambda + D)dz \quad (4)$$

Again, By is the magnetic induction polarized in the y-direction inside the insulator and λ is the penetration depth into the superconductor.. Equating (3) and (4) now gives rise to

$$\frac{\partial E_X}{\partial z} = -\frac{\partial B_y}{\partial t}(1 + \frac{2\lambda}{D}) \quad (5)$$

Operating on (5) with d/dz yields the following relation

$$\frac{\partial^2 E_x}{\partial z^2} = -\frac{\partial^2 B_y}{\partial z \, \partial t}(1 + \frac{2\lambda}{D}) \quad (6)$$

If we now consider a similar surface but this time position it normal to the z-axis, it is easily shown that this results in

$$\frac{\partial^2 E_x}{\partial y^2} = -\frac{\partial^2 B_y}{\partial y \, \partial t}(1 + \frac{2\lambda}{D}) \quad (7)$$

This is similar to that seen for our original surface. Now using Maxwell's equation for Ampere's law we can derive another relation governing our Josephson junction.

$$\vec{\nabla} \times \vec{H} = \vec{J} + \varepsilon \frac{\partial \vec{E}}{\partial t} \quad (8)$$

Where J is the current density and ε is the dielectric constant of the layer under consideration. Expanding this expression using the constitutive relation for H and B, and keeping only the x-components we have

$$\frac{1}{\mu}(\vec{\nabla} \times \vec{B})_x = \frac{1}{\mu}\left(\frac{\partial B_z}{\partial y} - \frac{\partial B_y}{\partial z}\right) = J_x + \varepsilon \frac{\partial E_x}{\partial t} \quad (9)$$

Where μ is the permeability close to that of the vacuum. Now, taking the time derivative of this expression we obtain

$$\frac{1}{\mu}\left(\frac{\partial^2 B_z}{\partial t \, \partial y} - \frac{\partial^2 B_y}{\partial t \, \partial z}\right) = \frac{\partial J_x}{\partial t} + \varepsilon \frac{\partial^2 E_x}{\partial t^2} \quad (10)$$

If we add (6) and (7) and multiply by 1/μ we find that

$$\frac{1}{\mu}\left(\frac{\partial^2 B_z}{\partial t \, \partial y} - \frac{\partial^2 B_y}{\partial t \, \partial z}\right) = \frac{1}{\mu}\left(1 + \frac{2\lambda}{D}\right)^{-1}\left(\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2}\right) \quad (11)$$

The left hand side of (11) is then the same as the left hand side of (10). Now equating the right hand side of (10) with the right hand side of (11) and simplifying we have the following relation

$$\frac{\partial^2 E_x}{\partial y^2} + \frac{\partial^2 E_x}{\partial z^2} - \mu\varepsilon\left(1 + \frac{2\lambda}{D}\right)\frac{\partial^2 E_x}{\partial t^2} = \left(1 + \frac{2\lambda}{D}\right)\mu\frac{\partial J_x}{\partial t} \quad (12)$$

We can then rewrite this expression by factoring out Ex to get

$$\left\{\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \mu\varepsilon\left(1 + \frac{2\lambda}{D}\right)\frac{\partial^2}{\partial t^2}\right\}E_x = \left(1 + \frac{2\lambda}{D}\right)\mu\frac{\partial J_x}{\partial t} \quad (13)$$

We now define the phase velocity, v to be

$$v = \left\{\mu\varepsilon\left(1 + \frac{2\lambda}{D}\right)\right\}^{-\frac{1}{2}} \quad (14)$$

Rearranging this equation we find that

$$\mu\left(1 + \frac{2\lambda}{D}\right) = 1/\varepsilon v^2 \quad (15)$$

Substituting this result into equation (13) we now have

$$\left\{\frac{\partial^2}{\partial y^2} + \frac{\partial^2}{\partial z^2} - \frac{1}{v^2}\frac{\partial^2}{\partial t^2}\right\}E_x = \frac{1}{\varepsilon v^2}\frac{\partial J_x}{\partial t} \quad (16)$$

It is seen that for the steady state where $\partial J_x / \partial t = 0$, equation (16) well describes a transverse electromagnetic (TEM) wave propagating between parallel plates with properties differing from a normal metal system [10]. It is also observed that the phase velocity, v, is lowered by a factor equal to $(1 + 2\lambda / D)^{-1/2}$ .To write equation (16) in terms of the phase change, ϕ across the junction we use

$$\frac{\partial \phi}{\partial t} = \frac{2eV}{\hbar} = -\frac{2e}{\hbar}DE_x \quad (17)$$

Where ϕ is the phase difference across the barrier, $\hbar$ is Dirac's constant, e is the electron charge, and D is the barrier thickness, which is assumed in the present case to be smaller than the penetration depth $\lambda$. Solving for $E_x$ we find

$$E_x = -\frac{\hbar}{2eD}\frac{\partial \phi}{\partial t} \quad (18)$$

We see here that the electric field depends on the first time derivative of the phase change and is inversely proportional to the barrier thickness. Now if ϕ is taken to have the form $\phi = e^{(\omega t - \vec{k}.\vec{r})}$ [13], then the following dispersion relation holds:

$$\omega^2 = k^2 v^2 + \omega_J^2 \quad (19)$$

This is then the governing relation describing our junction, where k is the propagation constant, v is the propagation velocity, ωJ is the Josephson angular frequency, Here ω our angular operating frequency and ωJ is given by $\omega_J^2 = (v / \lambda_J)^2$ where $\lambda_J$ is the Josephson penetration depth. It is obvious from this result that no propagating waves exist below $\omega_J$. While equation (19) is a good approximation, more explicit equations may be obtained using an argument based on the current density in the junctions [9]. Since, we have taken the structure to be infinite in the y and z directions, there can only be current in the x direction, and therefore, $|J|=J_x$ . This method then gives us the following field equations for a superconducting media:

$$H_y = \frac{\omega\varepsilon}{k_z}\left(1 - \frac{1}{\omega^2 \mu\varepsilon\lambda^2}\right)E_x \qquad (20)$$

Where ω is the angular frequency of the electromagnetic wave.

$$E_z = \frac{-j}{k_z}\frac{\partial E_x}{\partial x} \qquad (21)$$

We can also develop a dispersion relation inside the superconducting layer using the Helmholtz equation [15], giving us

$$k_x^2 = k_z^2 - \omega^2 \mu\varepsilon + \frac{1}{\lambda^2} \qquad (22)$$

Here λ is the London penetration depth, $k_x$ is the wave number for the x dimension, and $k_z$ is the wave number for the z direction. We note that the only difference between the dispersion relation seen in (22) and the dispersion relation for vacuum is the term containing λ [16].

## 3. The magnetic vector potential

The use of the magnetic vector potential, **A** is a powerful tool for solving Maxwell's equations which has proven to be a mainstay approach for device level formulation including antenna [13], microwave [17], and optoelectronics [18] as opposed to the traditional approaches we reviewed in the previous section. While A is not a physically measurable quantity, it does mathematically lend itself as a useful tool to simplify possibly complicated mathematical systems. That is why it is the dominant approach for most of the electromagnetic-based devices [19].

As described by Balanis [20], there are two ways to specify an electromagnetic boundary-value problem. One way is the method we have just looked at, the current density method, which is a direct integration from **J** to **E** and **H**. The other method is the one that we will look at in this section, which is to determine **A** from **J** and then **E** and **H** from **A**. Utilization of the magnetic vector potential to solve Josephson Junction problems will enable analytical closed form solutions to more complicated superconducting geometries that otherwise would require numerical techniques to solve [20]. We now review an alternative approach that relies on the magnetic vector potential to mathematically simplify the problem further. Utilization of vector potential and resultant equations are not new to this field [9] and [21]. Consequently, almost all of the available formulations are based on the conventional and equations [4] and [22]. Consequently, this paper will examine viewing Josephson junctions as devices (superconducting transmission lines) and utilizing magnetic vector potential formulation to describe the theoretical study of the electromagnetic wave behavior in Josephson transmission line. In fact this approach has many benefits. In this paper, our aim is to show the basis of this formulation and show that it fundamentally provides the same answers as the conventional approaches do. We will develop a solution based on **A** and show that such a solution will provide the same **E** and **H** fields as the conventional approaches. In addition, we will show that the gauge properties can be understood based on the boundary conditions. We begin by looking at the initial structure that Josephson proposed, which was used in the conventional formulation, as depicted in Fig. 1. The magnetic vector potential stems from the absence of free magnetic poles. This is most commonly exemplified by the following Maxwell equation[14]:

$$\vec{\nabla}\cdot\vec{B} = 0 \qquad (23)$$

Due to this, **B** can then be written as the curl of another vector quantity since the divergence of a curl is zero.

$$\vec{\nabla}\cdot\vec{\nabla}\times\vec{A} = 0 \qquad (24)$$

This vector is defined as the magnetic vector potential. We can easily determine both magnetic and electric fields quite simply from **A** using:

$$\vec{B} = \vec{\nabla}\times\vec{A} \qquad (25)$$

$$\vec{E} = -\frac{\partial\vec{A}}{\partial t} \qquad (26)$$

The reason we introduce **A** is because both the electric and magnetic fields remain unchanged under a scalar transformation of **A**.

We now turn our attention to the calculation of the magnetic vector potential for a superconducting transmission line, going to Fig.1, we look at the TM case [16] . Based on the symmetry of the problem, we can choose a magnetic vector potential **A** such that $A_z$ is the only nonzero component of the vector potential given by:

$$\vec{A}_z = X(x)Y(y)Z(z)\hat{z} \qquad (27)$$

We then solve the following wave equation to obtain the general form of the magnetic vector potential in the three regions:

$$\vec{\nabla}^2\vec{A} + \omega^2\mu\varepsilon\vec{A} = 0 \qquad (28)$$

This equation holds provided that the divergence of **A** is zero. We use separation of variables to determine the general form of $A_z$. Earlier we had defined the current density traveling in the y-direction to be zero leading to $B_x = E_y = 0$. Using this and the relation between **B** and **A** given by (25) and (26) we have

$$B_x = \frac{\partial A_z}{\partial y} - \frac{\partial A_y}{\partial z} = 0 \qquad (29)$$

Since we have already defined $A_y = 0$ , we clearly see that

$$\frac{\partial A_z}{\partial y} = 0 \qquad (30)$$

We can then use separation of variables with $A_z = X(x)Z(z)$. If we choose

$$\frac{1}{Z}\frac{\partial^2 Z}{\partial z} = -\kappa^2, \frac{1}{X}\frac{\partial^2 X}{\partial x^2} = -K^2 \qquad (31)$$

Where $\kappa^2 > K^2$ ,and $K^2 + \omega^2\mu\varepsilon = \kappa^2$

we have the following general solutions

$$Z(z) = e^{\pm i\kappa z}, X(x) = e^{\pm Kx} \qquad (32)$$

Selecting a +z traveling solution and adding in the time dependence $e^{-i\omega t}$ we find the following solution for $A_z$

$$A_z = (ae^{k_x x} + be^{-k_x x})e^{i(k_z z - \omega t)} \quad (33)$$

This is obviously the vector potential given in the region of the insulating barrier, where $k_z^2 > k_x^2$ and leads to the dispersion relation

$$k_x^2 = k_z^2 - \omega^2 \mu\varepsilon \quad (34)$$

To keep our vector potential from diverging, the general form of the magnetic vector potential for the top and bottom superconductors is then given by

$$A_{z,top} = (be^{-k_x x})e^{i(k_z z - \omega t)},$$

$$A_{z,bottom} = (ae^{+k_x x})e^{i(k_z z - \omega t)} \quad (35)$$

However, the divergence of **A** must vanish for both regions if we want to work in the Coulomb gauge[23], but this is not the case for the general solutions we derived. We must now apply a gauge transformation [23]such that

$$\vec{A} \rightarrow \vec{A}' = \vec{A} + \vec{\nabla}\Lambda \quad (36)$$

Where Λ is a scalar function chosen to make the divergence of A vanish and is found to be of the form

$$\Lambda = \frac{ik_z}{k_x^2 - k_z^2}\left(ae^{k_x x} + be^{-k_x x}\right)e^{i(k_z z - \omega t)} \quad (37)$$

Now, A for the insulating layer becomes

$$\vec{A} = \begin{bmatrix} \frac{ik_x k_z}{\omega^2 \mu\varepsilon}\left(ae^{k_x x} + be^{-k_x x}\right)e^{i(k_z z - \omega t)}\hat{x} + \\ \left(1 - \frac{k_z^2}{\omega^2 \mu\varepsilon}\right)\left(ae^{k_x x} + be^{-k_x x}\right)e^{i(k_z z - \omega t)}\hat{z} \end{bmatrix}$$

(38)

Where a = 0 for the top superconductor and b = 0 for the bottom superconductor as we saw before. We can substitute this relation for **A** into (25) and (26) to determine the electric and magnetic fields in the dielectric region. Taking the curl of **A** we find

$$\vec{B} = -k_x\left(ae^{k_x x} - be^{-k_x x}\right)e^{i(k_z z - \omega t)}\hat{y} \quad (39)$$

$$\vec{E} = \begin{bmatrix} \frac{k_x k_z}{\omega\mu\varepsilon}\left(ae^{k_x x} - be^{-k_x x}\right)e^{i(k_z z - \omega t)}\hat{x} + \\ \frac{ik_x^2}{\omega\mu\varepsilon}\left(ae^{k_x x} - be^{-k_x x}\right)e^{i(k_z z - \omega t)}\hat{z} \end{bmatrix} \quad (40)$$

Equations (39) and (40) are the electromagnetic equations for the insulating region of the superconducting transmission line. For the superconducting region of the transmission line structure, we use the Helmholtz equation with a non-zero current since we have propagating currents in this region [15]. Equation (28) is rewritten as,

$$\nabla^2 A_z + \omega^2 \mu\varepsilon A_z = -\mu J \quad (41)$$

Where **J** is the current density in the superconducting region. The London equation relates the current density to the magnetic vector potential for a superconductor.

$$J = -\frac{1}{\mu\lambda^2}A \quad (42)$$

Here, λ is the London penetration depth given by

$$\lambda = \sqrt{\frac{\varepsilon m c^2}{n q^2}} \quad (43)$$

In this equation, m is the mass, n the concentration, and q the charge. The London penetration depth is the thickness of the surface layer in which currents and magnetic fields can exist. General values for the London penetration depth are calculated to be about 4 × $10^6$ cm for many elemental metals at absolute zero. If we may digress slightly from our field calculations, it is interesting to note at this point that the magnetic field in the superconducting region should decay exponentially as [24]

$$B(x) = B_o e^{-x/\lambda} \quad (44)$$

We will see that this relation is indeed found in our solutions using the magnetic vector potential. Resuming our prior calculation we find that by inserting (42) into (41) the following dispersion relation exists in the superconducting region.

$$k_x^2 = k_z^2 - \omega^2 \mu\varepsilon + \frac{1}{\lambda^2} \quad (45)$$

The dispersion relation found in (45) matches the dispersion relation found using current density method but using much simpler mathematics to do so. This relation is very similar to the dispersion relation found for the insulating region with the exception of the London penetration depth term. Still, this relation is simple enough that we can visualize the general form of it based on the result for the insulating region.

## 4. Results

Since we are working with a TMz mode of propagation, our Helmholtz equation in the Lorentz gauge reduces to

$$\nabla^2 A_z + (\omega^2 \mu\varepsilon - \frac{1}{\lambda^2})A_z = 0 \quad (46)$$

This is very similar to the Helmholtz equation we found in the insulating region with an extra term in the coefficient of $A_z$. Solving in a similar manner as previously, we determine that **A** at the top and bottom superconductors is given by

$$\vec{A}_{top} = \frac{-ik_z k_x}{\omega^2 \mu\varepsilon} de^{-k_x x} e^{i(k_z z - \omega t)}\left(\hat{x} + \left[1 - \frac{k_z^2}{\omega^2 \mu\varepsilon - \frac{1}{\lambda^2}}\right]\hat{z}\right)$$

(47)

$$\vec{A}_{bottom} = \frac{-ik_z k_x}{\omega^2 \mu\varepsilon} ce^{-k_x x} e^{i(k_z z - \omega t)}\left(\hat{x} + \left[1 - \frac{k_z^2}{\omega^2 \mu\varepsilon - \frac{1}{\lambda^2}}\right]\hat{z}\right)$$

(48)

where *d* and *c* can be written in terms of *a* and *b* using boundary conditions. We confirm that the divergence of both the top and bottom superconductor magnetic vector potentials goes to zero.

$$\nabla\cdot\vec{A}_{top}=\left[\begin{array}{l}\dfrac{\partial}{\partial x}\left(\dfrac{-ik_zk_x}{\omega^2\mu\varepsilon}be^{-k_xx}e^{i(k_zz-\omega t)}\right)+\\ \dfrac{\partial}{\partial z}\left[\left(1-\dfrac{k_z^2}{\omega^2\mu\varepsilon}-\dfrac{1}{\lambda^2\omega^2\mu\varepsilon}\right)be^{-k_xx}e^{i(k_zz-\omega t)}\right]\end{array}\right] \quad (49)$$

$$\nabla\cdot\vec{A}_{top}=\frac{-ik_z}{\omega^2\mu\varepsilon}be^{-k_xx}e^{i(k_zz-\omega t)}\left(k_x^2-k_z^2+\omega^2\mu\varepsilon-\frac{1}{\lambda^2}\right) \quad (50$$

Using (45) it is clear that the quantity on the final equation in parenthesis is equal to zero, confirming that the divergence of **A** does indeed go to zero. By a similar solution, the divergence of Abottom goes to zero. Using the relations found in (25)and (26) , the magnetic and electric fields can be determined in the same way as in the insulating layer. After taking the curl of the magnetic vector potential and rearranging terms, we find the following relation for the magnetic field for the top and bottom superconductors.

$$\vec{B}_{top}=\frac{k_zk_x}{\omega^2\mu\varepsilon}de^{-k_xx}e^{i(k_zz-\omega t)}\left[k_z-ik_x+\frac{ik_z^2k_x}{\omega^2\mu\varepsilon-\frac{1}{\lambda^2}}\right]\hat{y} \quad (51)$$

$$\vec{B}_{bottom}=\frac{k_zk_x}{\omega^2\mu\varepsilon}ce^{-k_xx}e^{i(k_zz-\omega t)}\left[\frac{ik_z^2k_x}{\omega^2\mu\varepsilon-\frac{1}{\lambda^2}}-ik_x-k_z\right]\hat{y} \quad (52)$$

We can also use the dispersion relation (45) to rewrite these equations in varying forms. From equations (51) and (52) we see that since $k_x$ is a function of $1/\lambda$, the magnetic field inherently decays as a function of the London penetration depth as discussed earlier.

In order to obtain a physical representation of the spatial and time dependence of the magnetic field, we look at the spatial dependence of the magnetic field at selected times. By symmetry, the top and bottom magnetic field equations should yield the same result, so we will focus on the magnetic field for the top of the structure. To get a realistic sense of time dependence we use a frequency of 100 G-rad/s, which is in the range of a typical Josephson frequency [10]. This frequency value gives rise to a Josephson length of 150 μm, which is equivalent to the London penetration depth, but for a Josephson junction[10] . For consistency we will use the same values of wave-numbers that we used in the insulating layer and set all other general constants equal to 1. The following shows the results of these simulations.

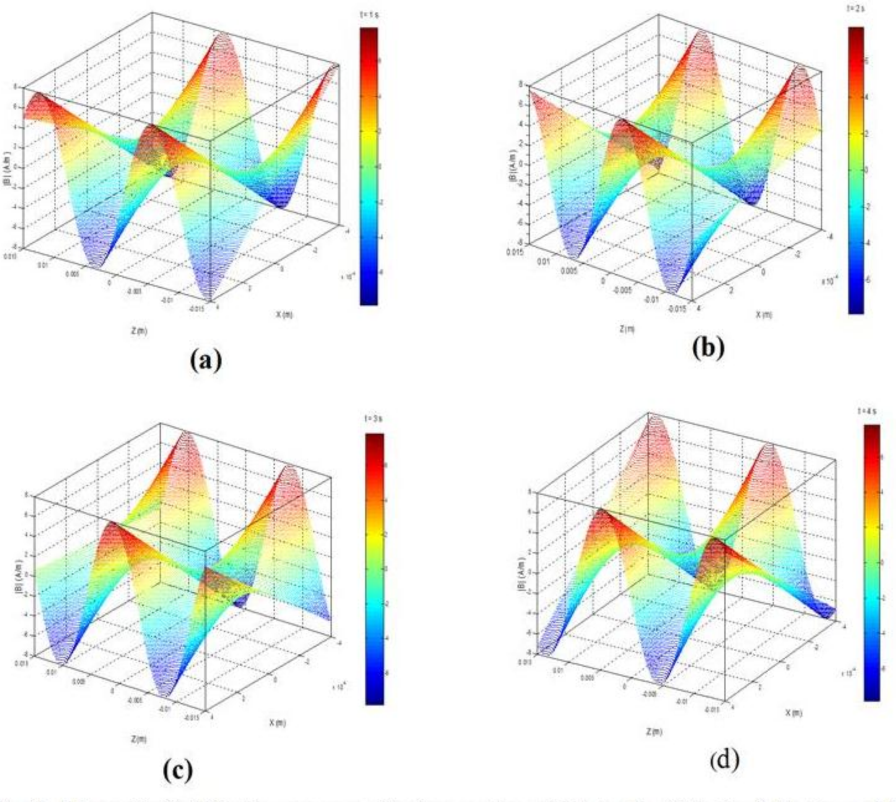

**Fig. 2.** ***Magnetic field in the superconducting region at (a) t = 1s, (b)t =2s, (c)t =3s and (d)t =4s.***

Now, we derive the Electric Field equations using the magnetic vector potential and equations (25)and (26). Taking the negative of the derivative of **A** with respect to time we find the following equations for the top and bottom superconductor electric fields.

$$\vec{E}_{top} = \frac{k_z k_x}{\omega\mu\varepsilon} d e^{-k_x x} e^{i(k_z z - \omega t)} \left( \hat{x} + \left[ 1 - \frac{k_z^2}{\omega^2 \mu\varepsilon - \frac{1}{\lambda^2}} \right] \hat{z} \right) \quad (53)$$

$$\vec{E}_{bottom} = \frac{-k_z k_x}{\omega\mu\varepsilon} c e^{k_x x} e^{i(k_z z - \omega t)} \left( \hat{x} + \left[ 1 - \frac{k_z^2}{\omega^2 \mu\varepsilon - \frac{1}{\lambda^2}} \right] \hat{z} \right) \quad (54)$$

In a similar manner we can obtain a visualization of the spatial and time dependence of the electric field equations using the same simplifications and values used to plot the magnetic field previously.

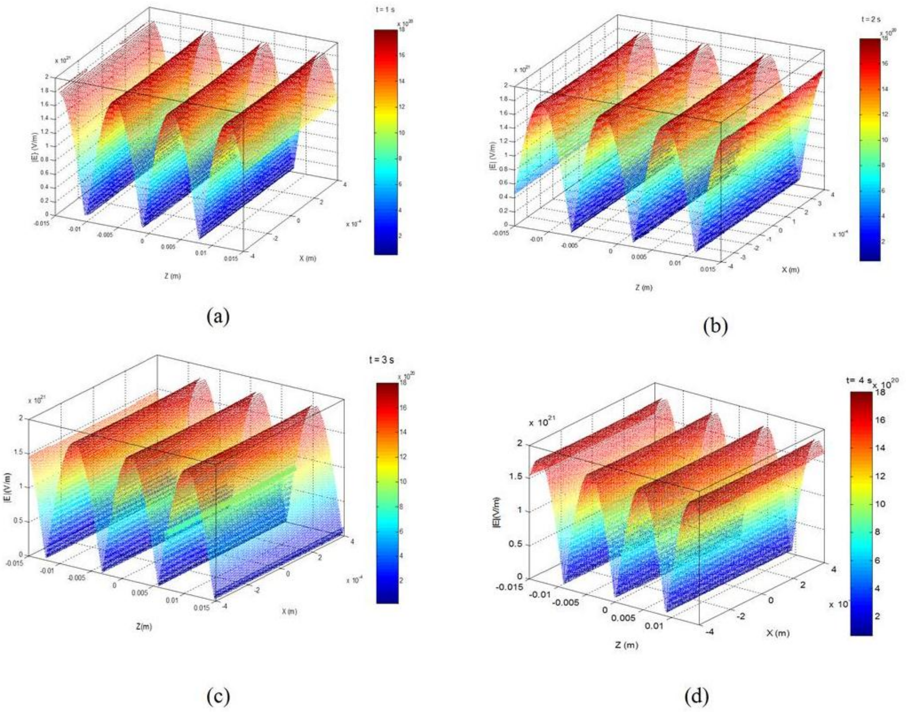


**Fig. 3.** ***Electric field in the superconducting region at (a) t = 1s, (b) t = 2s,(c) t =3s and (d) t =4s.***

## 5. Conclusion

We began by reviewing the work of Josephson and the events leading up to his discovery of the Josephson junction. It was this work on the coupling between superconductors separated by a thin non-superconducting layer that has led to an increasingly large body of research on superconducting devices. More recently, it was the work by Wu and others [10] that re-kindled the interest in the investigation of the electromagnetic fields of a superconducting layered device and was the starting point of the research in this paper. We have calculated the dispersion relation, magnetic vector potential, and electromagnetic field equations for the insulating and superconducting regions of the superconducting transmission line depicted in Figure 1. The results obtained are similar to those previously obtained by only considering the electric and magnetic fields [13]. We also found closed form solutions for the propagating fields of the superconducting transmission line structure. These fields were then simulated using a MATLAB script in order to obtain a realistic view of the behavior of these fields. Our results for the electromagnetic fields for the top and bottom superconductors are then summarized by equations (51-54). Figure 2 depict the magnetic fields in the top superconducting layer in the xz-plane for times of 1s, 2s, 3s, and 4s. The propagation of the magnetic field in the superconducting layer is consistent with that seen in the insulating layer as expected. We note that the minimum and maximum values do not align along the insulating / superconducting boundary as one would expect. We do note, however, that these fields were not plotted in the same manner. While we are looking at the magnitude of the magnetic field in the superconducting layer, we looked at the actual magnetic field in the insulating layer. We will get a better idea of the possible cause of this phenomenon when we look at the comparison of the electric fields in the two regions. Finally, we note that the magnitude of the magnetic field does indeed decrease as expected as we move in the x-direction. Turning now to the electric fields in the superconducting region, we look at figure 3. We see, as we have for all of the fields so far, that the electric field is propagating in the z-direction. In this case, we do not see a phase difference between the insulating and superconducting regions. This leads us to believe that the difference in representation of the magnetic fields is the cause of the apparent shift in phase; however, more work is necessary to verify this but will not be carried out here since this is not the focus of our efforts. Returning to our discussion of the electric field, we note that the electric field also shows a decaying behavior as it penetrates into the superconductor. Looking at our solution to the superconducting transmission line problem, we see that we have successfully calculated the electromagnetic fields of this structure using the magnetic vector potential as a mathematical tool to eliminate the need for direct integration.

**Appendix**:

One of the main programs in MATLAB to obtain figures in this paper:

```
%%% Theoretical study of the electromagnetic wave behavior in %%%Josephson transmission line using MATLAB model
%Aziz A. Aziz Al-Barzinjy
%clear all variables
warning off;
clear all,
%Initialize constants
w = 100e9, %rad/s
kx = 100, %1/m
kz = 348, %1/m
c=3e8, %m/s
%Set x and z dimensions for movie axes
xdim=-40e-5:.25e-5:40e-5,
zdim=-15e-3:.25e-3:15e-3,
%initialize movie image counter
l=1,
%begin time loop
for i=1:1:10
%initialize row counter
m=0,
%begin x loop
for j=-40e-5:.25e-5:40e-5
%increment row counter
m=m+1,
%initialize column counter
n=0,
%begin z loop
for k=-15e-3:.25e-3:15e-3
%increment column counter
n=n+1,
%calculate x-component of Electric field
Efieldx=c^2*(kz*kx*(exp(kx*j)-exp(-
kx*j))*exp(sqrt(-1)*(kz*k-w*i))),
%calculate z-component of Electric field
Efieldz=c^2*(kx^2*(exp(kx*j)+exp(-
kx*j))*exp(sqrt(-1)*(kz*k-w*i))),
%calculate magnitude of electric field
Efield(m,n) = sqrt(Efieldx^2+Efieldz^2),
%calculate Magnetic field
Bfield(m,n)=-kx*(exp(kx*j)-exp(-kx*j))*exp(sqrt(-
1)*(kz*k-w*i)),
%end z loop
end
%end x loop
end
%String Concactinate time counter
istr = strcat(num2str(i),' (s)'),
%Start a new figure for Efield movie image
figure(2)
%Clear previous figure
Clf,
%Plot 3-d countours of Electric field using 100 contours
contour3(zdim,xdim,Efield,100)
%Hold the current figure to plot over
hold on
%Set axis dimensions and ticklength
axis([-15e-3,15e-3,-40e-5,40e-5])
set(gca,'TickLength',.025,.025])
%Label x and y axes and enter current time as title
xlabel('Distance (m)')
ylabel('Distance (m)')
title(istr)
%Add frame to movie
M(:,l)=getframe,
%Turn off figure hold
hold off
%Start new figure for Bfield movie image
figure(3)
%Similar setup as for Efield
Clf,
contour3(zdim,xdim,Bfield,100)
hold on
axis([-15e-3,15e-3,-40e-5,40e-5])
set(gca,'TickLength',[.025,.025])
xlabel('Distance (m)')
ylabel('Distance (m)')
title(istr)
%Add frame to Bfield movie
D(:,l)=getframe,
hold off
%Increment movie frame
l=l+1,
%Use if statements to save files at time = 1,2,3,and 4 seconds
if i==1
%save Efield1.dat Efield -ascii -tabs
save Bfield1.dat Bfield -ascii -tabs
elseif i==2
save Efield2.dat Efield -ascii -tabs
save Bfield2.dat Bfield -ascii -tabs
elseif i==3
save Efield3.dat Efield -ascii -tabs
save Bfield3.dat Bfield -ascii -tabs
elseif i==4
save Efield4.dat Efield -ascii -tabs
save Bfield4.dat Bfield -ascii -tabs
end
%end time loop
end
%Initialize file and store Electric field movie
```

```
aviobj = avifile('EfieldVid','compression','indeo5', 'FPS', 10),
%Add frames to Electric field movie
aviobj = addframe(aviobj,M),
%Initialize file and store Magnetic field movie
aviobj = avifile('BfieldVid','compression','indeo5', 'FPS', 10),
aviobj = addframe(aviobj,D),
%Close AVI files so they are no longer writeable and can be viewed.
aviobj = close(aviobj),
end
```

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# دراسة نظرية لكيفية تصرف الموجات الكهرومغناطيسية داخل وصلات خطوط ارسال جوزيفسون باستخدام برنامج ماتلاب

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**الملخص**

هذا البحث قدم وصفا تفصيليا لكيفية حساب متجه الجهد المغناطيسي لوصلة لانهائية و رقيقة من وصلات جوزيفسون ذات الابعاد الهندسية المستوية. وباستخدام متجه الجهد المغناطيسي تم حساب كل من المجالين الكهربائي و المغناطيسي ، وباستخدام موديل ماتلاب *MATLAB* (*Model*) تم الحصول على نتائج التفريق نفسها كما في الدراسات المنشورة . مبتدئين بوصلة جوزيفسون و تقنيات صياغة المعادلات لدراسة المجال المغناطيسي داخل وصلة جوزيفسون المصنوعة من مواد فائقة التوصيل. اضافة الى انه تم اشتقاق معادلات المجال لمجال مغناطيسي مستعرض لخطوط الارسال بشكل مفصل.