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Design a Fuzzy PID Controller for Trajectory Tracking of Mobile Robot

***Abstract**- In this paper, a trajectory tracking control for a non-holonomic differential wheeled mobile robot (WMR) system is presented. A big number of investigations have been used the kinematic model of mobile robot which is a nonlinear model in nature, thus a hard task to control it. This work focuses on the design of fuzzy PID controller tuned with a firefly optimization algorithm for the kinematic model of mobile robot. The firefly optimization algorithm has been used to find the best values of controller's parameters. The aim of this controller is trying to force the mobile robot tracking a pre-defined continuous path with the least possible value of error. Matlab Simulation results show that a good performance and robustness of the controller. This is confirmed by the value of minimized tracking error and the smooth velocity especially concerning presence of external disturbance or change in initial position of mobile robot.*

***Keywords**- Fuzzy PID controller, Firefly algorithm, Mobile robot, Trajectory tracking.*

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1. Introduction

The investigation of differential wheeled mobile robots has increased recently. The mobile robot is mechanical device that able to move in an environment with a specific degree of autonomy, which is used in various service and industrial applications, such as transportation, factory automation, etc. The mobile robot with non-holonomic constraints means the motion of the robot is not completely free, where the robot can be moved in some directions (forward and backward) but not in others (sideways) [1,2].

The control problem of the mobile robot caused by the wheels motion has freedom in three degrees (x, y, θ), whereas for the mobile robot control, it needs only two control signals (U_L, U_R) under the kinematic and non-holonomic constraints. There are several published studies to solve the control problems of mobile robot (navigation problems) which can be divided as three types: The first type is the position estimate control method for problems of navigation. The second type is path planning and implementation. While the third type is trajectory tracking which is the goal of this work [3]. Three prime causes increase tracking error for mobile robot. The first one is the discontinuity of the radius of path rotation of the differential driving mobile robot. The second is due to the small radius of rotation overlap with the exact drive of the mobile robot. The third is due to the radius of rotation which is not uniform either randomly curvature or complicated curve [3].

There are various controllers have been suggested for trajectory tracking of mobile robots, Some of these proposed controllers are conventional controllers that applied linear or nonlinear feedback control, while the rest used other techniques such as predictive control technique [3]. The adaptive trajectory-tracking controller applied on the robot dynamics was discussed in [4]. Moreover, an intelligent control architecture using neural network and fuzzy logic for two-wheeled mobile robot was developed in [5], and fuzzy controller is developed as following:

'Lacvice' and 'velagic' [6] developed a fuzzy logic controller for mobile robot position where it membership functions tuned by using the genetic algorithm. The aim of this work is to ensure smooth velocity and exact trajectories tracking. The suggested controller has two inputs and two outputs. The input includes the angel and the distance between the reference cart and mobile robot. The outputs represent the linear and angular velocity commands, respectively.

'Turki' et al. [7] proposed a fuzzy logic controllers of Takagi Sugeno type that optimized by using (PSO) method. The goal of this work to control a trajectory tracking of mobile robot by using two optimized fuzzy controllers for speed control and azimuth control, results show a very good performance for the suggested controller.

'Mikova' and 'Gmiterko' [8] proposed the modelling mobile robot. Focus has been on the kinematic model characteristics, taking into computation the mobility constraints that caused by different links. The conventional "tracking

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controllers" are not favorable for this type of operation; because they do not guarantee that the robot remains on the reference path. The goal of this work is to suggest and to achieve the control method, which guarantee that the "output" of the mobile robot will move along the specified path., by using the computer simulation method.

Fuzzy logic control is selected in this work because it is very convenient to apply on a highly nonlinear model. Generally, FLC operates with a combination of control rules that derived from experts' learning. There are several structures of fuzzy logic controller, which are similar to the classical PID controllers. The fuzzy PID controller that used in this work is combined between the PI fuzzy and PD fuzzy controllers with input and output gains as explained in [9]. It is known that fuzzy PI control type is more feasible than fuzzy PD control type, and then it is hard for the fuzzy PD to eliminate the steady state error. While the fuzzy PI control type is famous that it gives bad performance in the transient response for higher order operation because of the inner integration process. To combine the performance of the fuzzy PI control and fuzzy PD control at the same time, a fuzzy (PI+PD) controller is chosen such that to keep the accurate characteristics of the PID controller by employ the error and the rate of change of error as its inputs [10]. The gains of this fuzzy (PI+PD) controller are tuned using the firefly algorithm as optimization method to find the best values of controller gains, as in [11,12].

The remainder of the paper is arranged as follows. In section two, kinematic model of the mobile robot is explained. Section 3, the suggested fuzzy PID controller and firefly optimization algorithm are illustrated. The results of simulation and the conclusion are given in sections four and five respectively.

2. The Kinematic Model of Differential Wheeled Mobile Robot

The wheeled mobile robot scheme is shown in Figure 1. The structure of mobile robot contains a Wagon with two wheels for driving installed on the same shaft and have an omni-directional wheel in the front of cart. The front wheel carries the mechanical construction and make the platform of the mobile robot steadier. Two autonomous similar DC motors represent the actuators of right and left wheels to move and orient the mobile robot [13,14]. The two driving wheels with same radius denoted by R (m), and 2L (m) is the space between the wheels. The center of gravity of the mobile robot is situated at point c, and there is distance from center of mass to center of axis of wheels is denoted by a.

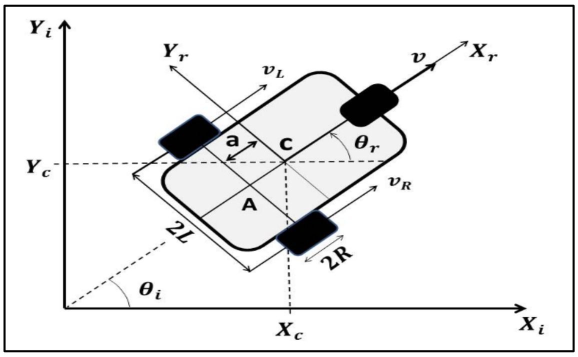


Figure 1: The mobile robot structure and coordinates [14]

Where $\{X_i, Y_i, \theta_i\}$ is inertial frame (global frame) that represent the static coordinate system in the robot plane, and $\{X_r, Y_r, \theta_r\}$ is robot frame (local frame) coordinate system that attached to robot [2].

θ: Angel of rotation for mobile robot (rad).
v: The linear or (average) velocity of the platform in the robot frame (m/sec).
ω: The rotational velocity of the cart in the global and local frames (rad/sec).

The aim of kinematic modeling of the robot is to manage the velocity of robot in the global frame as the wheels velocities function and the robot geometrical parameters. Where should be established the robot speed [14]:

$$\dot{q} = [\dot{x} \quad \dot{y} \quad \dot{\theta}]^T \tag{1}$$

As a function of the wheels rotational velocities $\dot{\varphi}_R$ and $\dot{\varphi}_L$ (rad/sec): which means the rotational velocity of the right and left wheels respectively. The velocity of any wheel in the local frame is $R\dot{\varphi}$, where R (m) is radius of wheels. Therefore, the average velocity in the local (robot) frame is the translational velocity or linear velocity $v(t)$ (m/sec) that combine the linear velocity of left wheel $v_L(t)$ (m/sec), and linear velocity of right wheel $v_R(t)$ (m/sec) [14]:

$$v_L(t) = R\,\dot{\varphi}_L(t) \tag{2}$$

$$v_R(t) = R\,\dot{\varphi}_R(t) \tag{3}$$

$$v(t) = R\,\frac{\dot{\varphi}_L(t) + \dot{\varphi}_R(t)}{2} \tag{4}$$

$$v(t) = \frac{v_L(t) + v_R(t)}{2} \tag{5}$$

In addition, the rotational velocity or angular velocity is:

$$\omega(t) = \frac{d\theta(t)}{dt} = \frac{R}{2L}(\dot{\varphi}_L(t) - \dot{\varphi}_R(t)) \quad (6)$$

The velocities also can be defined by immediate curvature radius of the robot trajectory that is denoted by $r(t)$ (m) is (distance from the instantaneous center of rotation ICC or ICR (m) to midpoint (A) the middle of two wheels) [2], and L (m) is the distance between each driving wheel and point (A), as shown in Figure 2.

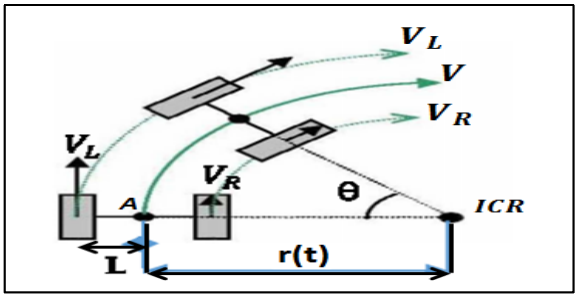


Figure 2: Instantaneous center of rotation [11].

$$v(t) = \omega(t).r(t) \quad (7)$$

$$v_L(t) = (r(t) + L)\,\omega(t) \quad (8)$$

$$v_R(t) = (r(t) - L)\,\omega(t) \quad (9)$$

After solving the above equations, the rotational velocity and the instantaneous curvature radius of the robot trajectory relative to the mid-point axis (c) are found as given in following equations:

$$\omega(t) = \frac{v_L(t) - v_R(t)}{2L} \quad (10)$$

$$r(t) = \frac{L(v_L(t) + v_R(t))}{v_L(t) - v_R(t)} \quad (11)$$

It is simulated the non-holonomic constraint of wheeled mobile robot, which represents the ideal rolling no lateral slip [2]:

$$\dot{y}\cos\theta(t) - \dot{x}\sin\theta(t) = 0 \quad (12)$$

The kinematic equations of mobile robot that represent the speed in the inertial or global frame is [2]:

$$\dot{x}(t) = v(t)\cos\theta(t) \quad (13)$$
$$\dot{y}(t) = v(t)\sin\theta(t) \quad (14)$$
$$\dot{\theta}(t) = \omega(t) \quad (15)$$

To determine the wheeled mobile robot position and orientation, that is need to integrate above equations as showed below:

$$x(t) = x_o + \int_0^t v(t)\cos\theta(t)\,dt \quad (16)$$
$$y(t) = y_o + \int_0^t v(t)\sin\theta(t)\,dt \quad (17)$$
$$\theta(t) = \theta_o + \int_0^t \omega(t)\,dt \quad (18)$$

In the program simulation, it expressed as the currently format of the equations:

$$x(k) = \frac{1}{2}[v_L(k) + v_R(k)]\cos\theta(k)\Delta t + x(k-1) \quad (19)$$

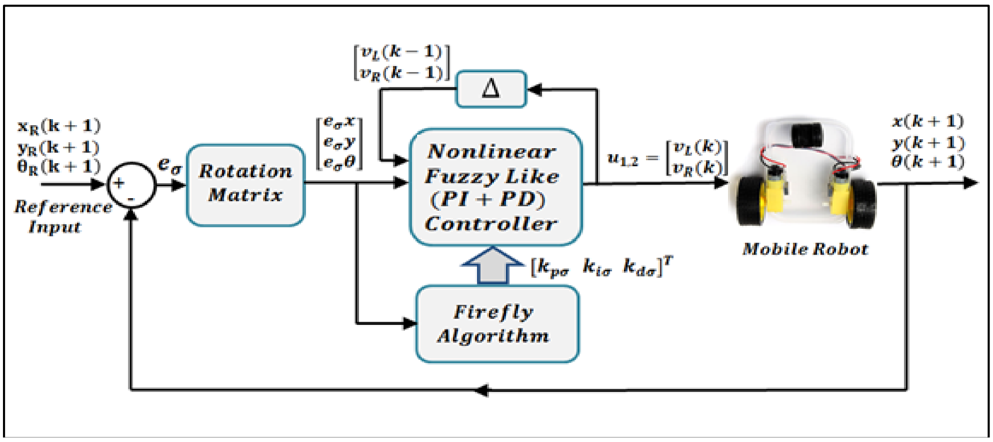
$$y(k) = \frac{1}{2}[v_L(k) + v_R(k)]\sin\theta(k)\,\Delta t + y(k-1) \quad (20)$$

$$\theta(k) = \frac{1}{2L}[v_L(k) - v_R(k)]\Delta t + \theta(k-1) \quad (21)$$

Where Δt is the sampling interval between two sampling time. The above equations applied to construct the mobile robot model and applied for simulation of mobile robot in Matlab program.

3. Fuzzy (PI+PD) Trajectory Tracking Controller Design

Commonly, fuzzy logic control principle is used to control systems or process which are complicated or difficult to model and/or nonlinear and complex. The complete modified fuzzy control system is consist of single-loop feedback control system. The requirements to design fuzzy logic control in order to track a desired path of mobile robot are the data of desired trajectory, the structure of fuzzy logic controller, model of mobile robot, and optimization algorithm used to tune the gains of controller. In the final stage of the controller design, a network should be constructed to solve the control problem of the mobile robot. This problem belongs to the motion of the wheels that has freedom in three degrees (x, y, θ), where it will transformed the three-control signal of (x, y, θ) into two control signals(U_L, U_R). The feedback of fuzzy PID controller is very essential to stabilize the error of path tracking system when the mobile robot output is deviated from the desirable trajectory. The fuzzy PID controller for mobile robot MIMO system is shown in Figure 3.



The proposed nonlinear fuzzy PID controller state cannot be equivalent to conventional PID controller state because of twice proportional gains involved in structure of PI+PD controller as shown in Figure 4. The physical joining remains the same between derivative and integral time constants. Fuzzy logic controller can be modified using the same method as for conventional controllers. Where the decrease of integral time constant or increase of proportional gain leads the system to higher oscillations. The oscillations can be compensated to some range by increment of derivative time constant. In a slow time response case, it can be decreased the integral time constant or increased the proportional gain to make the response faster [15].

The output of PI fuzzy controller in its standard form is the integral of output of PD fuzzy controller as shown:

$$u_{PI}(t) = \int_0^t \left[K_p e(t) + K_d \frac{d}{dt} e(t)\right] \quad (22)$$

$$u_{PI}(t) = K_i \int_0^t e(\tau)d\tau + K_p e(t) \quad (23)$$

Where $u(t)$ the control output, K_p, K_d, K_i the proportional, derivative and integral gains respectively, The PI fuzzy logic control which is called the incremental control output and denoted by $\Delta u_{PI}(t)$ can be obtained the control signal $u_{PI}(t)$ as represented by the following equation [15]:

$$u_{PI}(t) = u_{PI}(t-1) + \Delta u_{PI}(t) \quad (24)$$

The typical PID control law in its standard form is:

$$u_{PID}(t) = K_p \left[e(t) + T_d \frac{d}{dt} e(t) + \frac{1}{T_i} \int_0^t e(\tau)d\tau\right] \quad (25)$$

Where T_d the derivative time constant and T_i the integral time constant. The equation can also be written as [15]:

$$u_{PID}(t) = K_p e(t) + K_d \frac{d}{dt} e(t) + K_i \int_0^t e(\tau)d\tau \quad (26)$$

Where: $K_d = K_p T_d$ and $K_i = \frac{K_p}{T_i}$, so the fuzzy controller which is constructed from individual PI and PD controllers has a combined output to perform as a fuzzy PID controller :

$$u_{PID}(t) = u_{PI}(t) + u_{PD}(t) \quad (27)$$

The designing of Fuzzy Logic System (FLS) includes the design of input scale factors, a rule base, a proper choice of the membership functions (MF), and output scale factors. Where the input scale factors convert the real inputs of fuzzy controller into normalized values to be in the range (-1,1), and the scale factors of output convert the normalized outputs of fuzzy controller into actual values [2]. Several researches have indicated that the performance of FLS depends on the design of membership function more than rule base design [9].

The general construction of the fuzzy logic controller is included four major sections. The fuzzification, which transforms the crisp values into fuzzy sets. The fuzzy sets come in the inference mechanism as inputs. Fuzzy conclusions are determined depending on the fuzzy rule base, which are built upon prior experience on the system and its characteristics. The results are passed through a defuzzification process as final stage. Defuzzification is an inverse operation of fuzzification where it converts the fuzzy value to crisp value [2].

The fuzzy set has a many of membership functions that represent the experiential variables in a arithmetical method. The MF represents a curve that specify how the input range points are transformed to a membership degree or membership value in this range [0,1]. The input range is commonly called universe of discourse. The membership functions that applied in input and output fuzzy sets are in the shape of triangular or Gaussian or trapezoid function etc.

Gaussian membership function is common technique for designing fuzzy sets where it used in this work because it have the advantage of being smooth and nonzero at all points [2]. Where, a gaussian MF is specified by two parameters $\{c, \sigma\}$:

$$gaussian(x: c, \sigma) = e^{-\frac{1}{2}(\frac{x-c}{\sigma})^2} \quad (28)$$

A gaussian MF is specified completely by c and σ; which represents the MFs center and MFs width respectively. The standard gaussian MFs that is used in fuzzy controller for error(e), derivative of error ($\dot{e}$) and FLC output (u_{FLC}) are shown in Figure 5. The inputs and output relationship is depicted by a two dimensional linear rule base as shown in Table 1, and Mamdani type inference engine.

There are seven fuzzy linguistic variables where used NB, NM, NS, Z, PS, PM, PB denote to (Negative Big, Negative Medium, Negative Small, Zero, Positive Small, Positive Medium and Positive Big) respectively. So it has become 49 rules in rule base. The FLC output (u_{FLC}) is method of

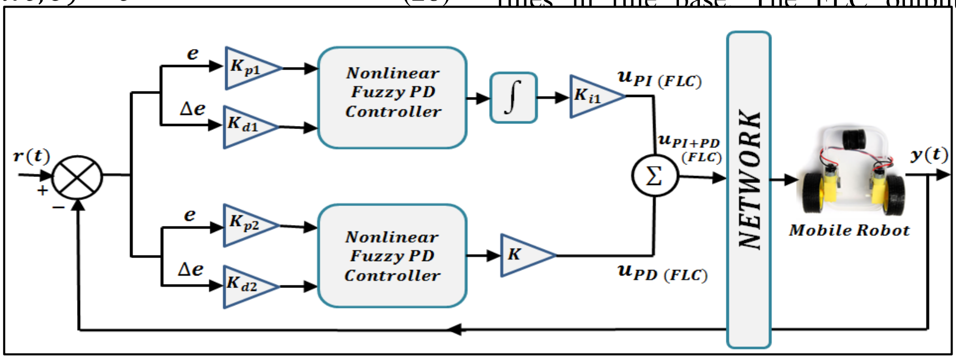


Figure 4: The internal structure of PI+PD FLC [13]

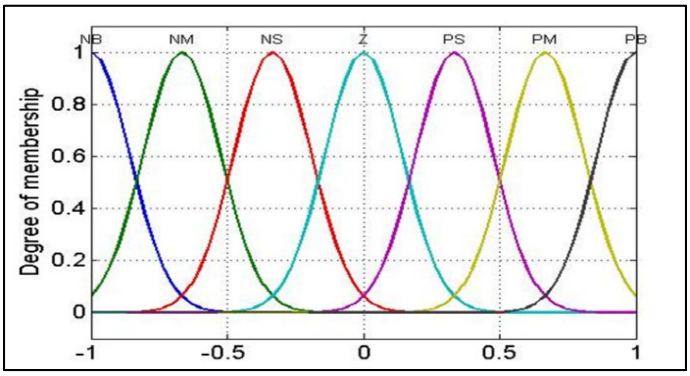


Figure 5: Membership functions for error, error derivative and FLC output.

Table 1: Rule base for error, error derivative and FLC output.

$\dot{e}$ \ e	NB	NM	NS	Z	PS	PM	PB
NB	PB	PB	PB	PB	PM	PS	Z
NM	PB	PB	PB	PM	PS	Z	NS
NS	PB	PM	PM	PS	Z	NS	NS
Z	PM	PM	PS	Z	NS	NM	NM
PS	PS	PS	Z	NS	NM	NM	NB
PM	PS	Z	NS	NM	NB	NB	NB
PB	Z	NS	NM	NB	NB	NB	NB

There are many different optimization algorithms available to use for tuning the parameters of fuzzy PID controller. Some of these algorithms often employed are, firefly algorithm, PSO algorithm and genetic algorithms, etc…
In this paper firefly, optimization algorithm is applied to find the optimal value of the fuzzy PID controller parameters. The objective of optimization for controller's parameters is to get best speed control signal that makes the error of the mobile robot trajectory tracking is least as possible.

Firefly Algorithm

The major work of a firefly's flash is to act as a signal system to attract other fireflies. The firefly algorithm work with some assumptions as shown below: [16,17].
1. The fireflies are unisexual, where any single firefly can be tempted to each other fireflies.
2. The attraction is proportional to their lightening, where any two fireflies, the one that have less light will be tempted by (and consequently move towards) the brighter one, but, the (visible lightening) intensity is reduced when mutual distance (r) increases.
3. If not found the fireflies which brighter than a specified firefly, it will move at random. The lightening must be connected with the objective function.
The procedure of firefly algorithm is illustrated by the following pseudo code:

Begin
1. Objective function: $f(x)$, $x = (x_1 , x_2 , \dots , x_d)$
2. Generate an initial population of fireflies: x_i , $(1 , 2 , 3 , \dots , n)$
3. Formulate light intensity I so that it is associated with $f(x)$, $I \propto f(x)$ or simplified by $I = f(x)$
4. Specify coefficient of absorption γ
While ($t < max\ generation$) { $max\ generation = 1000$}
    for $i = 1 : n$ (n is a number of fireflies where taken n=20)
        for $j = 1 : n$
          if $(I_j > I_i)$
            Vary attractiveness with distance r using $[exp(-\gamma r)]$ where move firefly i towards j;
            Determine new solutions and update lighting strength;
          end if
          end for j
       end for i
       Grade of fireflies and find the best one;
    end while
After processing the results and conception;
End

The light intensity can be defined by:

$$I = I_0 e^{-\gamma r} \tag{29}$$

Where, I_0 is the original light intensity, (r) is distance between firefly i and firefly j. The value of (γ) is depend on the media type, where light is partially absorbed by the media that through in it.

The Cartesian distance between two fireflies is given by:

$$r_{ij} = \left\| x_i - x_j \right\|_2 \tag{30}$$

$$r_{ij} = \sqrt{\sum_{k=1}^{d} (x_{i,k} - x_{j,k})^2} \tag{31}$$

Where $x_{i,k}$ is the k-th component of the i-th firefly.

The attractiveness β of a firefly is defined by:

$$\beta = \beta_{min} + (\beta_0 - \beta_{min}) e^{-\gamma r^2} \tag{32}$$

Where, β_0 is the attractiveness at $r = 0$ when no distance between two fireflies.

The standard update formulation for each pair of fireflies x_i *and* x_j is:

$$x_i^{t+1} = x_i^t + \beta \left(x_j^t - x_i^t \right) + \alpha_t (rand - 1/2) \tag{31}$$

Where; α_t is a step size controlling parameter, $\beta = 0.2$, $\alpha = 0.5$, $\gamma = 1$, and number of parameters (controller gains) that will be optimized =24. The values of firefly parameters $(n, \beta, \alpha, \gamma)$ are chosen with trial and error depending on a several of unrestricted real-valued standard functions was applied to examine the effect of the firefly parameters as explained in [16].

4. Simulation and Result

The modified fuzzy PID controller is applied on the differential mobile robot's kinematic model and also is verified with the computer simulation using Matlab 2014. In simulation, study an infinity path and star path are used as references trajectories to measure the difference between the mobile robot path and reference path. The two wheeled mobile robot physical description was explained, where the wheeled mobile robot has two motors for two wheels and a third wheel for stability and balance. Firefly algorithm was applied to regulate the fuzzy PID controller's parameters depending on the difference between mobile robot actual path and reference path. The result was obtained in the last iteration of firefly algorithm. The dimensions of the mobile robot model are taken as follow: Length=0.4 m, width = 0.2 m, Tire width =0.1m. Tire length=0.2. The system is tested for two cases; the first one is infinity trajectory which is defined in Eq. (34-36) [2].

$$x_R(n) = 0.75 + 0.75 * \sin\left(2 * \pi * \frac{t}{100}\right) \quad (34)$$

$$y_R(n) = \sin\left(4 * \pi * \frac{t}{100}\right) \quad (35)$$

$$\theta_R(n) = atan2\left[\frac{\left(\frac{y_R(n)-y_R(n-1)}{t+\epsilon}\right)}{\left(\frac{x_R(n)-x_R(n-1)}{t+\epsilon}\right)}\right] \quad (36)$$

The second case is star trajectory, which is defined in Eq. (37- 39) [2].

$$x_R(n) = -2.5 * \sin\left(2 * \pi * \frac{t}{30}\right) \quad (37)$$

$$y_R(n) = 2.5 * \sin\left(2 * \pi * \frac{t}{20}\right) \quad (38)$$

$$\theta_R(n) = atan2\left[\frac{\left(\frac{y_R(n)-y_R(n-1)}{t+\epsilon}\right)}{\left(\frac{x_R(n)-x_R(n-1)}{t+\epsilon}\right)}\right] \quad (39)$$

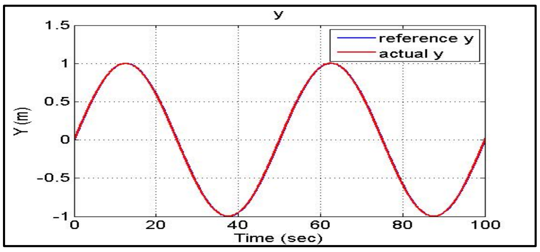


Figure 7.b: The response of y variable.

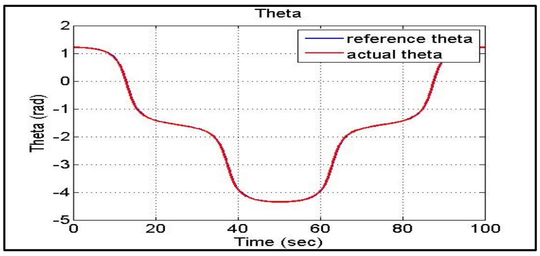


Figure 7.c: The response of θ variable.

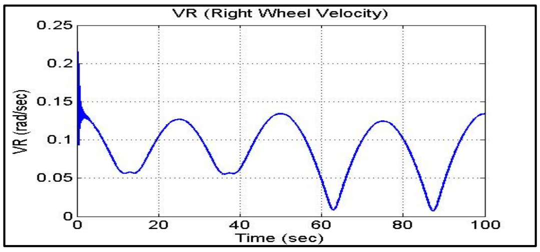


Figure 8.a: The right wheel velocity.

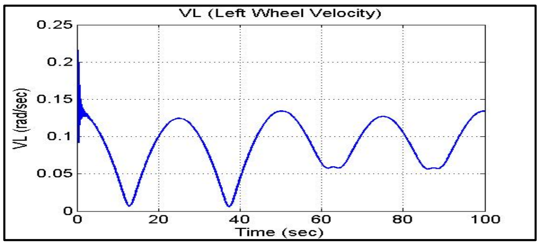


Figure 8.b: The left wheel velocity.

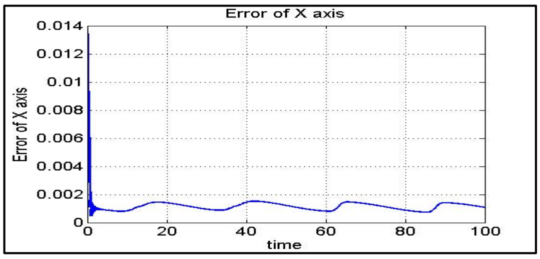


Figure 9.a: The error of x variable.

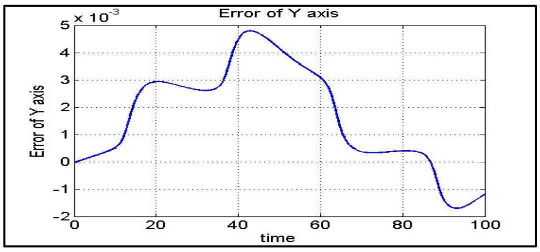


Figure 9.b: The error of y variable.

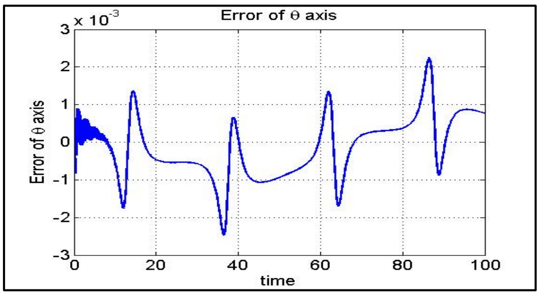


Figure 9.c: The error of θ variable.

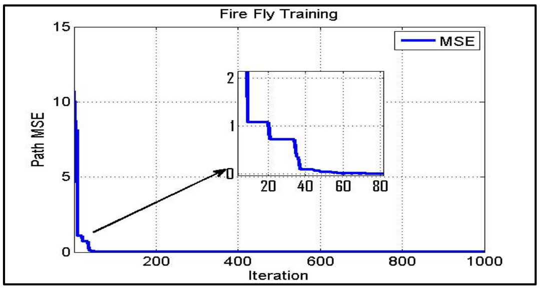


Figure 10: The MSE of path tracking.

The reference and actual trajectories of the infinity path is shown in Figure 6, where they represented by blue and red lines respectively. Figures 7.a-7.c display the desired and actual paths for x, y and θ respectively. The velocities of left and right wheels (v_L, v_R) are shown in Figures 8.a, 8.b respectively. Figures 9.a-9.c, shows error for x, y, and θ respectively. Finally, Figure 10 represented the mean square error (MSE) of path.

The reference and actual trajectory of the star path is shown in Figure 11, where they represented by blue and red lines respectively. Figures 12.a-12.c display the desired and actual path for x, y and θ respectively. The velocities of

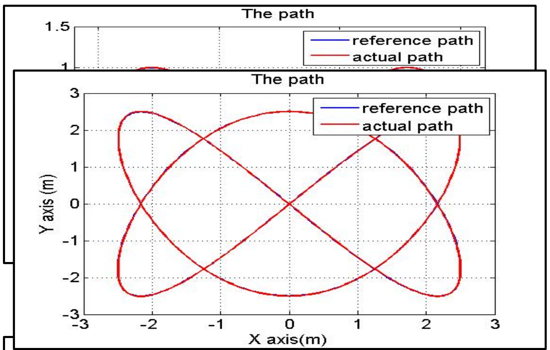


Figure 11: Reference and actual paths.

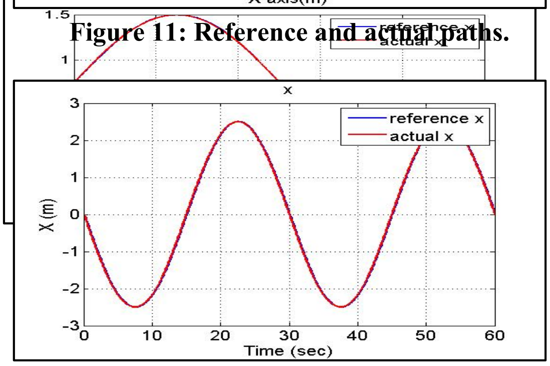


Figure 12.a: The response of x variable.

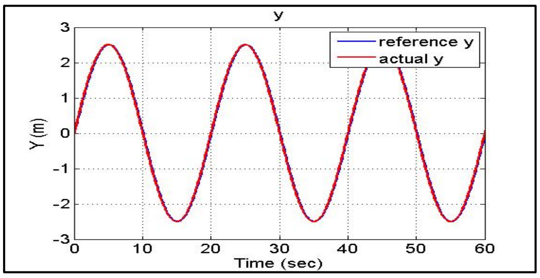


Figure 12.b: The response of y variable.

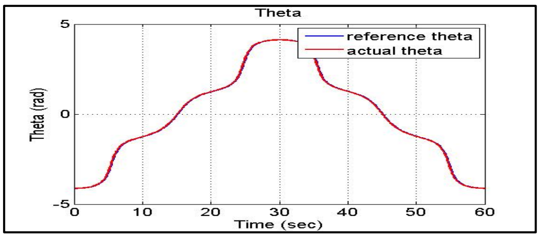


Figure 12.c: The response of θ variable.

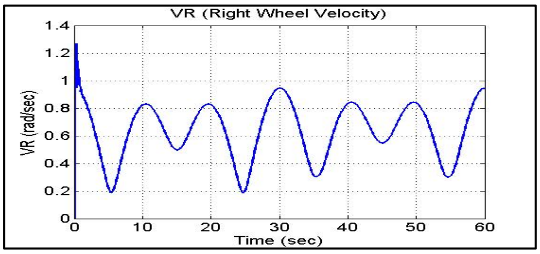


Figure 13.a: The right wheel velocity.

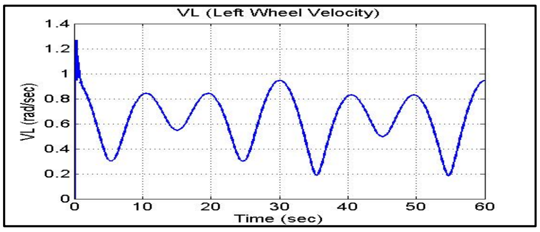


Figure 13.b: The left wheel velocity.

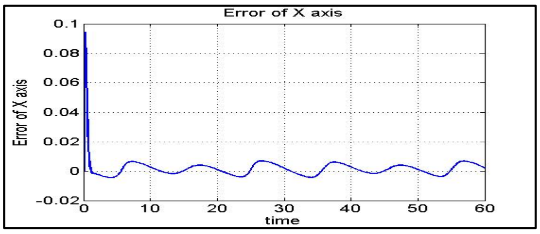


Figure 14.a: The error of x variable.

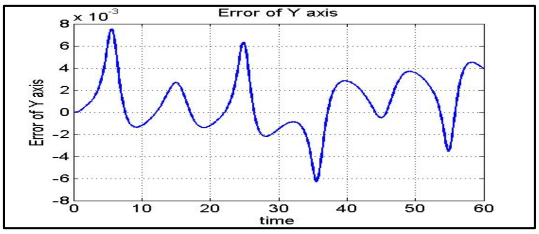


Figure 14.b: The error of y variable.

left and right wheels (v_R , v_L) are presented in Figure 13.a and 13.b respectively. Figure 14.a-14.c shows error for x, y, and θ. Finally, Figure 15 represented the mean square error (MSE) of path.

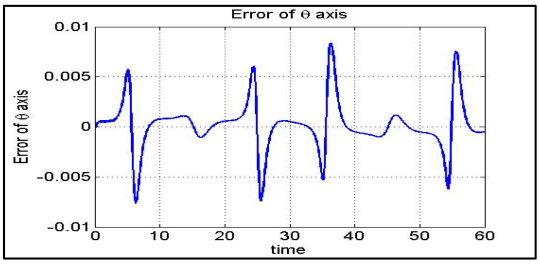


Figure 14.c: The error of θ variable.

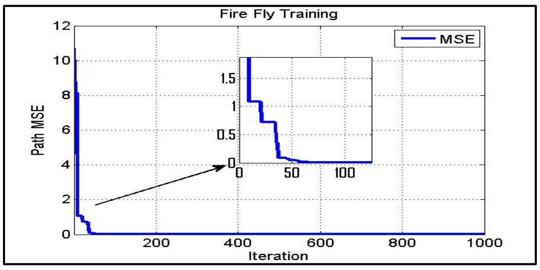


Figure 15: The MSE of path tracking

The obtained results from applying the modified fuzzy PID controller to the kinematic model of differential wheeled mobile robot show that the method is efficient to track the desired trajectory with a very insignificant error. Table 2 indicate to the MSE values for all variables of the infinity and star paths, respectively.

Figure 16, (a, b): Reference and actual paths with presence of dis
(infinity, star) trajectory respe

In order to check the robustness of the modified fuzzy PID controller, another initial posture is taken to test the ability of proposed controller to track the mobile robot on the desired path. if we change the initial position [0.75, 0, 0] of mobile robot for infinity path to $[0.69, -0.02,\ pi/2]$ for $[x, y, \theta]$ respectively, and add unmodeled disturbance term $[0.1sin(2t)\,, 0.1sin(2t)]$ for proving the robustness and ability of fuzzy PID controller. Therefore, without re-training the parameters (gains) of controller, the obtained result as shown in Figure 16.a. The values of MSEs of $(ex, ey, e\theta)$ are $(1.7 * 10^{-5}, 1.1 * 10^{-5}, 5.5 * 10^{-4})$ respectively. This result is much better than the result of using neural controller based on position and orientation Predictor, which has not added disturbance to model [17].

Another test for star path is considered, if we change the initial position of $[x, y, \theta]$ from [0, 0, -4.1243] to [0.2, 0.1, -4.1243] and add unmodeled disturbance term$[0.1sin(2t)\,, 0.1sin(2t)]$. Then without re-training the parameters (gains) of controller so that to prove the ability and robustness of controller. The obtained result is shown in Figure 16.b which is much better than the result of nonlinear PID neural trajectory tracking Controller used in [3]. Table 3 depicts the values of MSEs of both controllers. The results show that the proposed controller has a good a ability to reduce the difference between actual and desired paths quickly and then follow the desired path with a good accuracy.

Table 2: The values of MSE of two cases.

Trajectory	MSE of x-axis	MSE of y-axis	MSE of θ-axis	MSE of Path
Path 1	$1.6529 * 10^{-6}$	$6.22 * 10^{-6}$	$6.7944 * 10^{-7}$	$8.55 * 10^{-6}$
Path 2	$4.9951 * 10^{-5}$	$6.86 * 10^{-6}$	$5.9123 * 10^{-6}$	$6.27 * 10^{-5}$

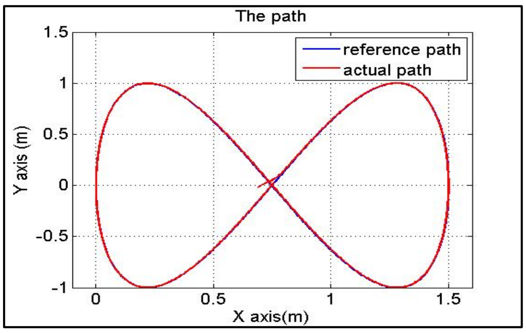


Figure 16.a

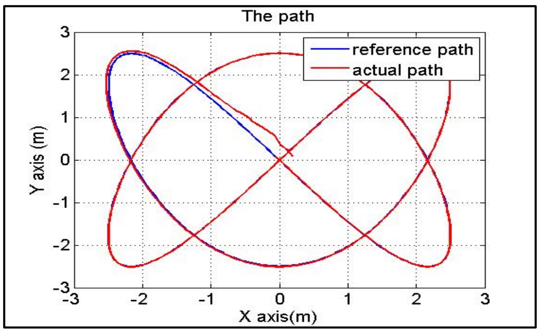


Figure 16.b

Table 3: The comparison of MSE values with Ref. [3] for star path

MSE of Nonlinear Fuzzy PID Controller with Firefly Algorithm			MSE of Nonlinear PID Neural Controller with PSO Algorithm		
$(e(x))$	$(e(y))$	$(e(\theta))$	$(e(x))$	$(e(y))$	$(e(\theta))$
0.000214	**0.00187**	**0.000404**	**0.0013**	**0.0038**	**1.18**

From the figures and values of MSEs in Table 3, it is clear that the fuzzy PID controller give better performance and robustness than nonlinear PID neural controller used in [3]. The results confirms that the fuzzy PID controller is feasible, robust and able to follow the desired trajectory without re-training its parameters.

5. Conclusions

The nonlinear fuzzy PID controller with firefly optimization algorithm technique for MIMO nonholonomic wheeled mobile robot motion model has been presented in this paper. The control algorithm is based upon the mean square tracking error between the actual position and desired position throughout mobile robot trip. This error is feed to fuzzy PID controller to generate correction to controller's parameters in order to minimize tracking error and find the optimal velocity for each driven wheel of the mobile robot. In fact, firefly algorithm is used off-line to tune 24 parameters (gains) of fuzzy PID controller. Simulation results show clearly that the suggested controller model has the ability of produce suitable and smooth velocity (v_L and v_R) without a sharp heels. Moreover, the suggested controller has explained the ability of tracking continuous desirable trajectory especially when the robot has been started form arbitrary initial position and a disturbance term is added to its model.

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