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Polynomials Over Splitting Fields

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ABSTRACT

In this paper we study some results concerning the existence of splitting fields which are generated by roots of polynomials. Also we study the roots of cubic polynomials.

Introduction and preliminaries

These results are basic to Galois theory consider the polynomial ring $K[X]$ over field K .Let f(x) belong to K[X] in the quotient ring K[X]/f(x). We let g(x) denotes the coset (g(x)+f(x)). Thus if $g(x)=\sum_{i=0}^{n} K_i x^i$, then by the definition of addition and multiplication of cosets we have that $\overline{g(x)}=\sum_{i=0}^{n} \bar{K}_i x^{\bar{i}}$, we considered a field K contains in a complex numbers ℂ and a cubic polynomial $f(x)=x^3+px+q\in K[X]$. Also, we obtained explicit expression involving extraction of square and cubic roots for the three roots α_1,α_2 and α_3 of f(x) in C and we were beginning to study the splitting field extension $E=K(\alpha_1,\alpha_2.\alpha_3)$ .If f(x) factors in $K[X]$ either all the roots are in K or exactly one of them (say α_3) is in K and the other two roots of irreducible quadratic polynomial in $K[X]$ In this case $E=K(\alpha_1)$ is a field extension of dimension 2 over K. Therefore if α_1 denotes one of the roots, we know that $K(\alpha_1)\cong K(X)/(f(X))$ is a field extension of dimension 3 =deg(f)over K .also we have $K\in K(\alpha_1)\subseteq E$, it follows from the multiplicatively of dimension that 3 divides the dimension of E over K.

Definition. [2]

A polynomial f(x) belong to K[X] is said to split over a field S contains K,if f(x) can be write it factor as product of linear a factors in S[X],such that K is a field.

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Remark .[1]

$\delta=(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)\in E$,since $\delta^2=-4p^3-27q^2\in K$,either K or K(δ) is an extension field of dimension 2 over K, since $K\subseteq K(\delta)\subseteq E$ it follows that 2 also divides $\dim_k$ (E).

$\delta\in K$ and $\dim_k$ (E) =3 or

$\delta\notin K$. and $\dim_k$ (E) =6.

Proposition. [4]

Let K be a field .If f(x) is a non-constant polynomial in $K[X]$, then there exists a field extension F/K such that F contains a root of f(x) .

Now by the following we can show that C is the field of complex numbers [x^2+1 is irreducible in $R[X]$. Now, $R[X]=\{a+b\bar{x}|\ a,b\in R\}$ is a field where $\bar{x}=x+(x^2+1)$ .Since $x^2=-1$, we may call C the field of the complex numbers.]

Definition . [5]

Let K be a field .A polynomial $f(x)\in K[X]$ is said to split over a field S⊇K if f(x) can be factored as a product of line a factors in S[x].

A field S containing K is said to be a splitting field for f(x) over K if f(x) splits over S but over no proper intermediate field of S/K.For example The field of complex numbers C is s splitting field for the polynomial x^2+1 over R .this follows, since $x^2+1=(x+i)(x-i)$ in C[x], and C/R has no proper intermediate field because [C:R]=2 .Now if $C\supseteq L\supseteq R$ where L is an intermediate field of C/R, then 2=[C:R]] [L:R] and so either [C:L][=1 or [L:R]=1 .Then either C=L or C=R and note that C is the splitting field of x^2+1 over Q since x^2+1 splits over Q (L).

Proposition . [5]

Let K be a field and f(x) be a polynomial in $K[X]$ of degree n . Let F/K be a field extension .If f(x)=c(x-c1)(x-c2)….(x-cn) in F(x). then `is a splitting field for f(x) over K.

Also,if we have K a finite field.Then cardinality of K is pn for some prime p and some positive integer n.Every k belong to K is a root of the polynomial XPn –X and K is the splitting field of this polynomial over prime subfield Zp.

Therefore,if the roots are known as α1 and α2 then The field $Q(\lambda,\lambda_3)$ for the last example is a splitting field for x^4-3 over Q.

Now we can say that if K be field and f(x) be constant polynomial over K. Then there is a splitting field for f(x) over K. and if E /K is a field extension and f(x) be an irreducible polynomial in $K[X]$. If $a,b\in E$ are roots of f(x) then $K(a)\cong K(b)$.

Also, we can use other concept to obtain splitting field by normal extension such that ((if a finite extension E / K is normal ,then it is a splitting field over K and f(x) bolong to K[X].)).

Therefore , if E / L and L / K be a finite extensions and if E / K is normal then E / L is normal(E /L is splitting).Now we can give the following fact about two splitting fields[Let $f(x)\in K[x]$. Any two splitting fields for f(x) over K are isomorphic],

also, let F/K be a field extension and $a,b\in F$. Then a and b are called conjugates, if a and b are roots of the same irreducible polynomial over K.

Examples

1-The field $Q(\sqrt{2})=\{a+b\sqrt{2}:a,b\in Q\}$ is a splitting field of $x^2-2\in Q[x]$ over Q

2- A splitting field of $x^2+1\in R[x]$ over R is the field C.

Proposition[2]

If K is field and $f\in K[x]$ then:

There exists splitting field of polynomial; f on K.

Any two splitting fields of f on K are two isomorphism fields on K.

Splitting fields are unique up to isomorphism over K.

Proposition .[3]

Let K be subfield of C let $f(x)=x^3+px+q\in K[X]$ an irreducible cubic polynomial and let E denotes the splitting field of f(x) in C. Let $\delta=(\alpha_1-\alpha_2)(\alpha_2-\alpha_3)$ where α_i are the roots of f(x). If $\delta\in K$, then $\dim_k$ (E)=6

Proposition. [1]

Suppose $K\subseteq L$ is any field extension f(x) $\in K[X]$ and β is the root of f(x) in L. If δ is an automorphism of L leaves F fixed pointwise, then $\delta(\beta)$ is also a root of f(x).

Proof

If $f(x)=\sum f_i x^i$,and since β is one of the roots that is mean f(β)=0 then $\sum f_i\delta(\beta)^i=\delta\sum f_i(\beta^i)=\delta(0)=0$

Example

Let $f(x)=x^3-2$,which is irreducible over Q. The three roots of f in C are $\sqrt[3]{2}$, $\omega\sqrt[3]{2}$ and $\omega^2\sqrt[3]{2}$, where $\omega=\frac{1}{2}+\frac{\sqrt{-3}}{2}$ is a primitive cube root of 1.

Finally,to show that the splitting fields always exist[for if g is any irreducible factor of f,then K[X]/(g)=K(α) is an extension of K for which g(α)=0,where α denotes the image of X.Then g and f are splits off a linear factor,induction implies that exists a splitting field L for f.

Conclusions

We gote that a polynomial f(x) $\in K[X]$ always has a splitting field, namely the field generated by its roots in a given algabric closure $\overline{K}$ of K. Also we can apply these roots of any non-constant polynomials by Galois theory.We obtained a new result (every normal extension is splitting field, and splitting fields are unique. let K be a field by a root of polynomials f(x) $\in K[X]$ we mean an element α in an over field of K such that $f(\alpha)=0$. It is easy to see that a non-zero polynomial in $K[X]$ of degree n has most n roots.

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متعددات الحدود على الحقل المنفصل

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الخلاصة

قمنا في هذا البحث بدراسة بعض النتائج المتعلقة بوجود الحقل المنفصل الذي يتولد عن طريق جذور متعددات الحدود. كذلك قمنا بدراسة نوع واحد من هذه الجذور وهي الجذور التكعيبية .