

# Design and Multiplierless Implementations of ECG-Based 1st Order Gaussian Derivative Wavelet Filter with Lattice Structures

Jassim M. Abdul-Jabbar\*

Abdulhamed M. Jasim\*\*



\* University of Mosul - College of Engineering.

\*\* University of Mosul - College of Electronics Engineering.

## ARTICLE INFO

Received: 00 / 00 /00  
Accepted: 28 / 5 /2022  
Available online: 19/7/2022  
DOI: 10.37652/juaps.2012.63369

### Keywords:

ECG;  
Gaussian derivatives wavelet function;  
Filter banks;  
Lattice structures;  
Multiplierless implementation.

## ABSTRACT

In this paper, the 1<sup>st</sup> order Gaussian derivative wavelet function utilized in the design of some corresponding filter banks. Since the 1<sup>st</sup> order Gaussian derivative function has a similar shape to QRS complex part of the ECG, it can be used in QRS feature extraction. Using this characteristic of such wavelet function, the designed FIR wavelet filter banks can be realized in highly-efficient lattice structures which are easy to implement in two standard deviation values ( $\sigma = 1$  and  $\sigma = 2$ ). The resulting lattice structures reduce the number of filter banks coefficients and this reduces, in turn the number of multiplications and improves the filter banks efficiencies as it reduces the number of computations performed. The resulting quantized multiplier values can also lead to multiplierless realizations using sum-of-power-of-two (SOPOT) method. FPGA implementations of such structures are achieved with less-complexity.

## Introduction

The ECG is a graphic record of the direction and magnitude of the electrical activity that is generated by depolarization and repolarization of the atria and ventricles. Most of the clinically useful information in the ECG is found in the intervals and amplitudes defined by its features [1]. The normal wave pattern of an electrocardiogram is shown in Figure 1 [2].

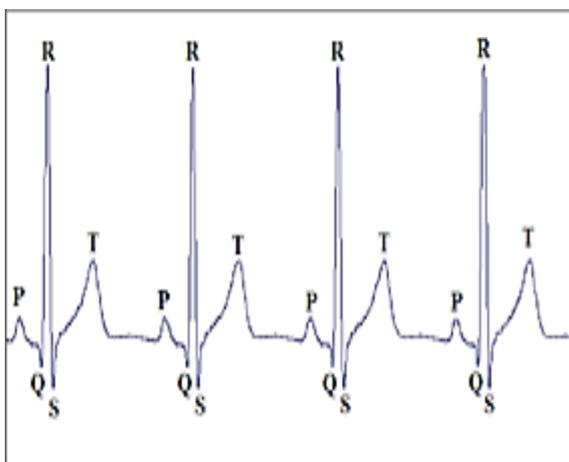


Fig 1: Four cycles of an Electrocardiogram.

Any ECG gives two kinds of information. The first is the duration of the electrical wave crossing the heart which in turn decides whether the electrical activity is normal or slow or irregular and the second is the amount of electrical activity passing through the heart muscle which indicates whether the parts of the heart are too large or overworked [3]. One of the most important ECG components is the QRS complex, which is associated with electrical ventricular activation [4].

The ECG feature extraction system provides fundamental features (amplitudes and intervals) to be used in subsequent automatic analysis. In recent years, many techniques have been proposed to detect these features [1],[5]. The previously proposed methods of ECG signal analysis were based on time domain analysis. But this is not always adequate to study all the features of ECG signals. Therefore, the frequency representation of a signal is required. In recent years, many classifying methods have been proposed including

\* Corresponding author at: University of Mosul - College of Engineering-.E-mail address: [drjssm@yahoo.com](mailto:drjssm@yahoo.com)

digital signal analysis, Fuzzy Logic methods, Artificial Neural Networks [4],[5], Hidden Markov Model, Genetic Algorithm, Support vector Machines, Self-Organizing Map, Bayesian and other hybrid methods. Each of these approaches exhibits its own advantages and disadvantages [5].

The wavelet transform is a recently developed signal processing technique, created to overcome the limitations of both classical FT and STFT to deal with non-stationary signals like biomedical signals. The wavelet transform of a signal is calculated by taking the convolutive product between the biological signal and bases functions, measuring the similarity between them. The result of this product is a set of coefficients. This set of coefficients indicates how similar is the signal relative to the bases functions. In the case of wavelet analysis, the bases functions are scaled (stretched or compressed) and translated versions of the same prototype function; called the mother wavelet  $\varphi(t)$  [6] which is a 1<sup>st</sup> order Gaussian derivative function in this paper.

The resulting filter banks can be realized in highly-efficient lattice structures which are easy to implement. The lattice structure reduces the number of coefficients and this in turn reduces the number of multiplications and improves both; filter bank efficiency and processing speed, as it reduces the number of computations performed [7]. Hardwarely, this leads to less-complex implementations.

Besides this introductory section, Section 2 of this paper contains the design of the FIR filter bank utilizing the 1<sup>st</sup> order Gaussian derivative function as a wavelet function. Section 3 illustrates the lattice structure of such filter bank with a standard deviation  $\sigma = 1$ . A multiplierless realization of such structure is also proposed in this section. The lattice structure of the

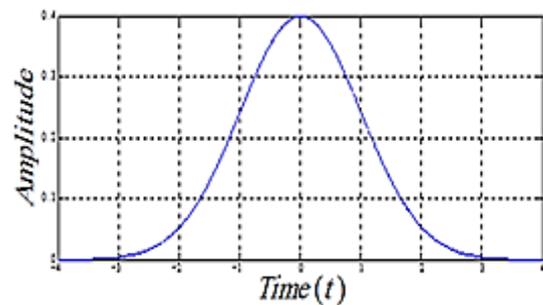
proposed filter with a standard deviation of  $\sigma = 2$  and its multiplierless realization are described in section 4. FPGA implementations and results are given in section 5. Section 6 illustrates the used ECG feature extraction method. Finally, Section 7 concludes this paper.

### Filter bank design

One of the functions in the wavelet techniques is the Gaussian function that is shown in Figure 2 and defined by

$$\psi(t; \sigma) = \frac{1}{\sigma\sqrt{2\pi}} e^{-\left(\frac{(t-m)^2}{2\sigma^2}\right)} \quad (1)$$

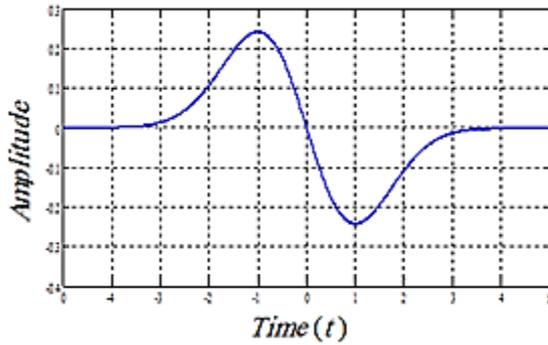
where  $\psi(t)$  is the Gaussian function in terms of the time  $t$ ,  $\sigma$  is the standard deviation and  $m$  is the center of the wave



**Fig 2: The Gaussian function  $\varphi(t)$  with  $m=0$ .**

In this paper, the 1<sup>st</sup> order Gaussian derivative wavelet function is shown in Figure 3 is selected. The 1<sup>st</sup> order Gaussian derivative function has a similar shape to QRS complex part of the ECG and can be used for QRS feature extraction. It is given by

$$\psi'(t; \sigma) = \frac{m-t}{\sigma^3\sqrt{2\pi}} e^{-\left(\frac{(t-m)^2}{2\sigma^2}\right)} \quad (2)$$



**Fig 3: The 1st order Gaussian derivative function  $\psi'(t ; \sigma=1)$  with  $m = 0$ .**

To design a corresponding FIR filter bank, the values of such FIR filter coefficients simulating the 1<sup>st</sup> order Gaussian derivative response must be determined. To do so, the Gaussian derivative function must be truncated in a way that assumes getting coefficients number depends upon standard deviation within the truncated function. To determine the values of these coefficients, we must choose the value of the standard deviation. The 1<sup>st</sup> order Gaussian derivative function is approximately zero for  $|t| > 4\sigma$ . For example,  $\varphi'(t, \sigma) < 0.0004$  for  $|t| > 4\sigma$  [8].

Since the designed FIR filter bank responses have the anti-symmetric property, then the required number of filter coefficients (*i. e.*, multipliers) is  $(1 + 4\sigma)$ . Therefore, the FIR wavelet filter can be designed at various values of standard deviation,  $\sigma$ . In the next section, FIR filter banks will be designed at  $\sigma = 1$  and  $\sigma = 2$ .

**The first order Gaussian derivative FIR filter bank with  $\sigma = 1$**

$$\begin{aligned}
 H_{even}(z^2) &= h_0 + h_2z^{-2} + h_4z^{-4} + \left. \begin{aligned} &h_6z^{-6} + h_8z^{-8} \\ &h_7z^{-6} \end{aligned} \right\} \quad (5) \\
 H_{odd}(z^2) &= h_1 + h_3z^{-2} + h_5z^{-4} +
 \end{aligned}$$

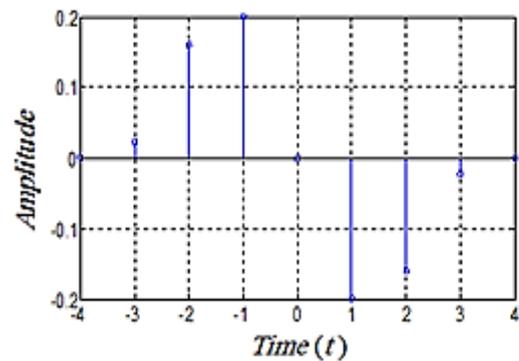
As a function of  $z$ , the forms (5) can be rewritten as

$$\begin{aligned}
 H_{even}(z) &= h_0 + h_2z^{-1} + h_4z^{-2} + \left. \begin{aligned} &h_6z^{-3} + h_8z^{-4} \\ &h_7z^{-3} \end{aligned} \right\} \quad (6) \\
 H_{odd}(z) &= h_1 + h_3z^{-1} + h_5z^{-2} +
 \end{aligned}$$

**a) Lattice Structure**

In this case, the filter response will have 9 coefficients with the following system function that corresponds to the sampled version of the wavelet function in Figure 4:

$$H(z) = h_0 + h_1z^{-1} + h_2z^{-2} + h_3z^{-3} + h_4z^{-4} + h_5z^{-5} + h_6z^{-6} + h_7z^{-7} + h_8z^{-8} \quad (3)$$



**Fig 4: Impulse response of 1st order Gaussianderivative function with  $\sigma = 1$ .**

By the property of quadrature mirror filters (QMFs),  $G(z) = H(-z)$ , the system function  $G(z)$  that corresponds to the scaling function can be written as

$$G(z) = h_0 - h_1z^{-1} + h_2z^{-2} - h_3z^{-3} + h_4z^{-4} - h_5z^{-5} + h_6z^{-6} - h_7z^{-7} + h_8z^{-8} \quad (4)$$

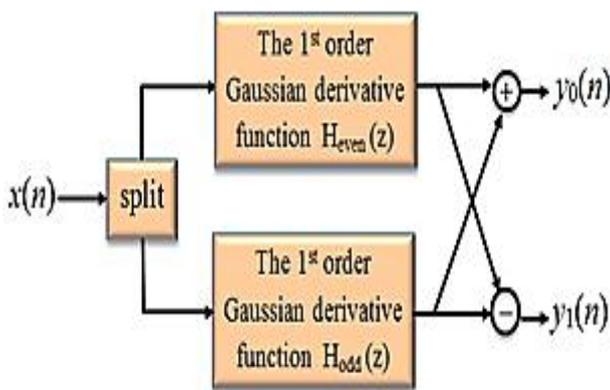
The first design step is to find the polyphase matrix of the specified filter bank and a similar matrix of the lattice structure (shown in Figure 5), where the filters' polyphase representations are expressed as

From (6), the two filters simulating the two outputs in Figure 5 ( $y_0(n)$  and  $y_1(n)$ ) are derived, respectively as

$$\begin{aligned}
 H(z) &= H_{\text{even}}(z) + z^{-1}H_{\text{odd}}(z) \\
 \text{And} \\
 G(z) &= H_{\text{even}}(z) - z^{-1}H_{\text{odd}}(z) \quad (7)
 \end{aligned}$$

with the following coefficients values:

$$h_0 = -h_8 = 0.0008, \quad h_1 = -h_7 = 0.0227, \quad h_2 = -h_6 = 0.1599, \quad h_3 = -h_5 = 0.1999, \quad h_4 = 0.$$



**Fig 5: Lattice structure of 1<sup>st</sup> order Gaussian derivative designed.**

The previous values of coefficients can be scaled (by  $\alpha$ ) to give a maximum frequency response value equals to one, for the case of no-energy level variation during transformation. This value of  $\alpha$  for  $[ |H(e^{j\omega})| \leq 1 ]$  turns to be 1.5538. Therefore, the new scaled coefficients values are as follows:

$$h_0 = -h_8 = 0.0013, \quad h_1 = -h_7 = 0.0353, \quad h_2 = -h_6 = 0.2485, \quad h_3 = -h_5 = 0.3106, \quad h_4 = 0.$$

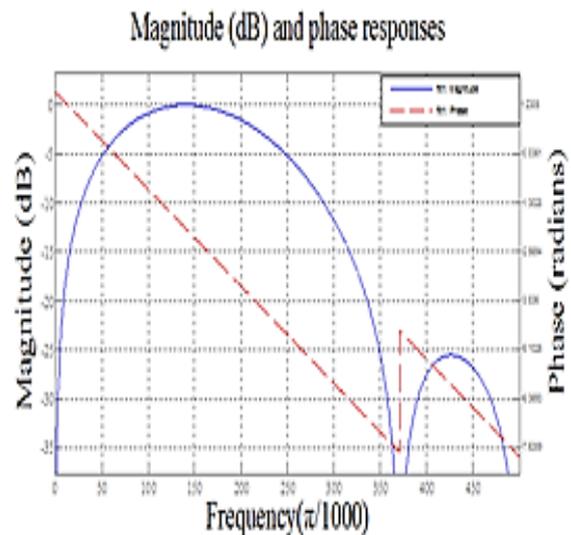
After getting these coefficients, the design of the FIR wavelet filter is accomplished. The magnitude and phase responses are shown in Figure 6. The final structure in Matlab simulation is shown in Figure 7.

**b) A Multiplierless Realization**

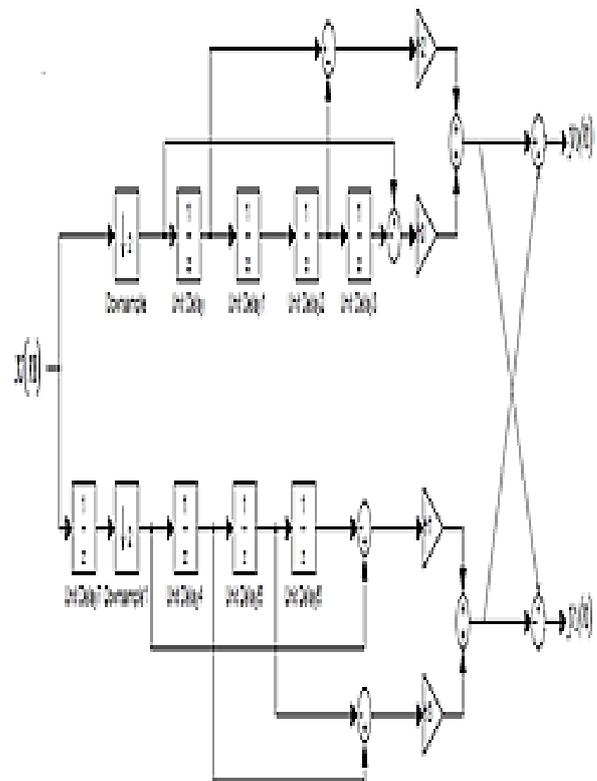
The rounded values of the resulting coefficients for different wordlengths are illustrated in Table 1.

The approximated values of Table 1 are used in a Matlab simulation for best selection of maximum and

average error values in the resulting filter magnitude response and the resulting SNR values



**Fig 6: The magnitude and phase responses of  $H(z)$  filter with  $\sigma = 1$ .**



**Fig 7: The final lattice FIR wavelet (one-level) filter bank structure with  $\sigma = 1$ .**

**TABLE 1:** The rounded coefficient values for different wordlengths with  $\sigma = 1$ .

wordlength (bits)	Coefficients for different wordlengths									
	3	4	5	6	7	8	9	10		
Original Coefficients										
0.0013	0	0	0	0	0	0	0	0	0	9.765 x10 <sup>-4</sup>
0.0353	0	0	0.0313	0.0313	0.0313	0.0352	0.0352	0.0352	0.0352	
0.2485	0.1250	0.1875	0.2188	0.2344	0.2422	0.2461	0.2480	0.2480	0.2480	
0.3106	0.2500	0.2500	0.2813	0.2969	0.3047	0.3086	0.3105	0.3105	0.3105	

Calculations of the values of average error and deviation are carried out by following equations:

$$\Delta_{avg} = (1/L) * \sum_1^L \Delta(e^{j\omega})$$

$$L = \text{length}(H_0(e^{j\omega}))$$

$$\Delta(e^{j\omega}) = |H_{original}(e^{j\omega}) - H_{wordlength}(e^{j\omega})|$$

$$\text{Deviation} = 1 - \max\{H_{wordlength}(e^{j\omega})\} \quad (8)$$

where:  
 $H_{original}(e^{j\omega})$  is the original frequency response.  
 $H_{wordlength}(e^{j\omega})$  is the frequency response at a specified wordlength.

*Deviation* is the amount of error in the frequency response at any wordlength.

From (8), Table 2 is obtained. It will lead us to the correct choice of coefficients wordlengths. It can be seen, in Table 2 that a wordlength of 6 bits can be chosen for acceptable values for average error and deviation. Also, Table 3 returns the suitable number of ECG samples for maximum

SNR value. From Table 3, the 6-bit representation tolerates a suitable number of ECG samples of 40 samples for a maximum SNR value of 27.2824 dB. Since the FIR filter response is anti-symmetric,

therefore the number of multipliers can be reduced to almost half of its original value. Thus, only 4 multipliers are to be used. In addition, these multipliers can be represented in sum-of-power-of-two (SOPOT) resulting in a multiplierless realization (*i.e.*, a limited number of hardware shifters and adders or subtractors) as shown in Table 4.

**TABLE 2:** Maximum and average deviations with  $\sigma=1$ .

6-BIT REPRESENTATION OF COEFFICIENTS	SOPOT REPRESENTATION OF COEFFICIENTS
$h_0 = 0$	None
$h_1 = 0.0313$	$2^{-5} = \text{Five shifts only}$
$h_2 = 0.2344 = 0.2500 - 0.0156$	$2^{-2} - 2^{-6} = \text{Shift and subtract}$
$h_3 = 0.2969 = 0.25 + 0.0313 + 0.0156$	$2^{-2} + 2^{-5} + 2^{-6} = \text{Shift and add}$

**TABLE 3:** The values of SNR with respect to no. of samples for the input signal (ECG) with  $\sigma = 1$ .

Wordlength (bits)	SNR values in (dB) for different wordlengths							
	3	4	5	6	7	8	9	10
No. of ECG samples								
20	9.6264	13.3505	19.6050	26.1716	32.9738	41.0269	49.0253	55.7440
40	13.3079	14.7292	20.3759	27.2824	34.7282	42.8849	56.9035	62.4665
60	8.1282	11.8750	19.3760	25.3649	31.1603	40.2931	48.1736	54.6757
80	6.7212	11.3233	18.7823	24.7021	30.4004	39.2745	46.8832	52.8525

100	7.3632	10.8342	19.0356	24.6652	29.8734	39.6177	47.2408	53.4032
120	7.1317	10.6880	18.8879	24.4196	29.4868	38.6939	45.5305	52.2317
140	7.2455	10.4642	18.9662	24.3674	29.2273	38.7652	45.4758	52.3533
160	7.2556	10.3052	18.9789	24.2862	29.0013	38.6267	45.0707	52.1556
180	7.0549	10.3108	18.8730	24.1898	28.9210	38.2821	44.4774	51.7026
200	6.9960	10.2720	18.8359	24.1256	28.8195	38.0736	44.0804	51.4373

TABLE 4 : Multiplierless representations of coefficients with  $\sigma = 1$ .

Wordlength (bits)	3	4	5	6	7	8	9	10
$\Delta_{avg}$	0.0916	0.0916	0.0388	0.0190	0.0092	0.0026	$1.69 \times 10^{-4}$	$1.691 \times 10^{-4}$
$H_{word\ length}(e^{j\omega})$	0.6495	0.7622	0.8937	0.9484	0.9758	0.9931	0.9997	0.9989
Deviation	0.3505	0.2378	0.1063	0.0516	0.0242	0.0069	$2.702 \times 10^{-4}$	0.0011

The first order Gaussian derivative FIR filter bank with  $\sigma = 2$ .

a) Lattice Structure

$$G(z) = h_0 - h_1z^{-1} + h_2z^{-2} - h_3z^{-3} + h_4z^{-4} - h_5z^{-5} + h_6z^{-6} - h_7z^{-7} + h_8z^{-8} - h_9z^{-9} + h_{10}z^{-10} - h_{11}z^{-11} + h_{12}z^{-12} - h_{13}z^{-13} + h_{14}z^{-14} - h_{15}z^{-15} + h_{16}z^{-16} \quad (10)$$

The same procedure of the previous section can be followed. The filter response will have 17 coefficients with the following system function that corresponds to the sampled version of the wavelet function

in Figure 8 :

$$H(z) = h_0 + h_1z^{-1} + h_2z^{-2} + h_3z^{-3} + h_4z^{-4} + h_5z^{-5} + h_6z^{-6} + h_7z^{-7} + h_8z^{-8} + h_9z^{-9} + h_{10}z^{-10} + h_{11}z^{-11} + h_{12}z^{-12} + h_{13}z^{-13} + h_{14}z^{-14} + h_{15}z^{-15} + h_{16}z^{-16} \quad (9)$$

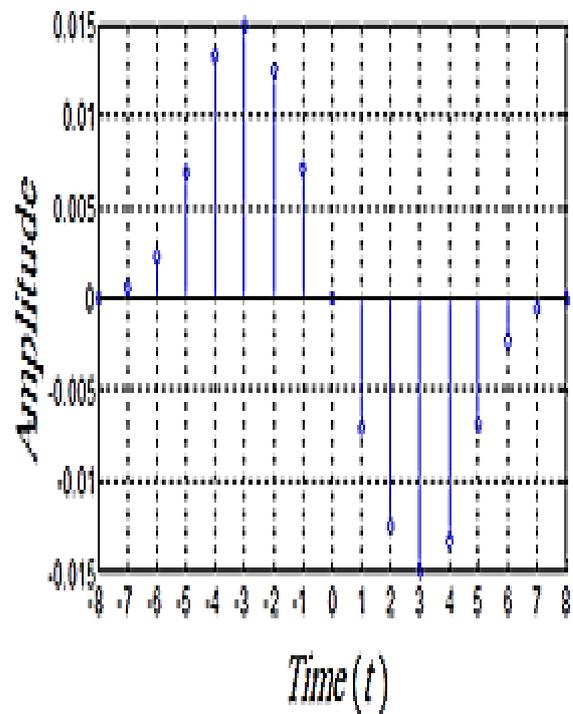


Fig 8: Impulse response of 1<sup>st</sup> order Gaussian derivative function with  $\sigma = 2$ .

By the same property of quadrature mirror filters (QMFs),  $G(z) = H(-z)$ , the system function  $G(z)$  can be written as

The branches in Figure 5 can now be expressed as

$$\left. \begin{aligned} H_{even}(z^2) &= h_0 + h_2z^{-2} + h_4z^{-4} + h_{12}z^{-12} + h_{14}z^{-14} + h_{16}z^{-16} \\ H_{odd}(z^2) &= h_1 + h_3z^{-2} + h_5z^{-4} + h_{13}z^{-12} + h_{15}z^{-14} \end{aligned} \right\} \begin{aligned} &h_6z^{-6} + h_8z^{-8} + h_{10}z^{-10} + \\ &h_7z^{-6} + h_9z^{-8} + h_{11}z^{-10} + \end{aligned} \quad (11)$$

As a function of  $z$ , the forms (11) can be rewritten as

$$\left. \begin{aligned} H_{even}(z) &= h_0 + h_2z^{-1} + h_4z^{-2} + h_{12}z^{-6} + h_{14}z^{-7} + h_{16}z^{-8} \\ H_{odd}(z) &= h_1 + h_3z^{-1} + h_5z^{-2} + h_{13}z^{-6} + h_{15}z^{-7} \end{aligned} \right\} \begin{aligned} &h_6z^{-3} + h_8z^{-4} + h_{10}z^{-5} + \\ &h_7z^{-3} + h_9z^{-4} + h_{11}z^{-5} + \end{aligned} \quad (12)$$

Using (7), the wavelet filters simulating mother and scaling functions can be derived with the following coefficients values:

$$\begin{aligned} h_0 = -h_{16} = 0.0001, & \quad h_1 = -h_{15} = 0.0005, & \quad h_2 = \\ -h_{14} = 0.0023, & \quad h_3 = -h_{13} = 0.0070, & \quad h_4 = -h_{12} = \\ 0.0133, & \quad h_5 = -h_{11} = 0.0150, & \quad h_6 = -h_{10} = 0.0125, \\ h_7 = -h_9 = 0.0072, & \quad h_8 = 0. \end{aligned}$$

Using the same scaling procedure, the value of  $\alpha$  for  $[ |H(e^{j\omega})| \leq 1 ]$  turns to be 10.61392588. Therefore, the new scaled coefficients values are:

$$\begin{aligned} h_0 = -h_{16} = 0.0008, & \quad h_1 = -h_{15} = 0.0057, & \quad h_2 = \\ -h_{14} = 0.0245, & \quad h_3 = -h_{13} = 0.0738, & \quad h_4 = -h_{12} = \\ 0.1409, & \quad h_5 = -h_{11} = 0.1589, & \quad h_6 = -h_{10} = 0.1326, \\ h_7 = -h_9 = 0.0759, & \quad h_8 = 0. \end{aligned}$$

The magnitude and phase responses are shown in Figure 9. The number of coefficients can be reduced to 9 coefficients due to anti-symmetric property. The final structure in Matlab simulation is shown in Figure 10.

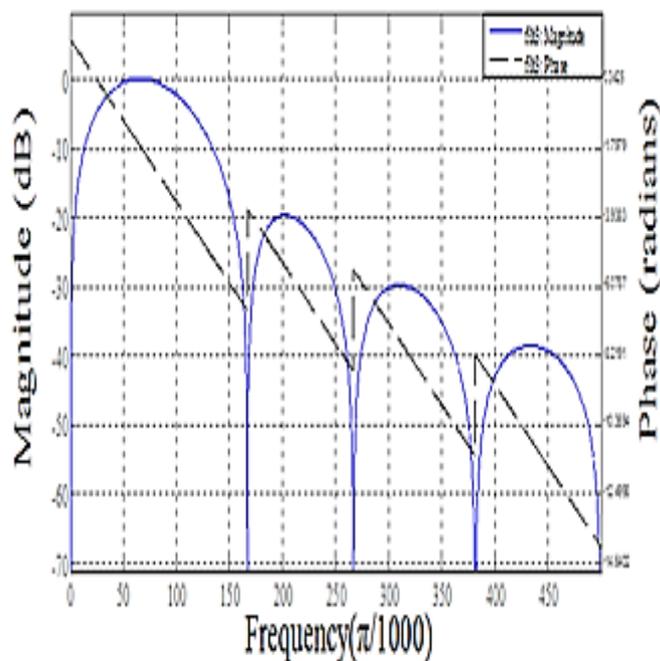
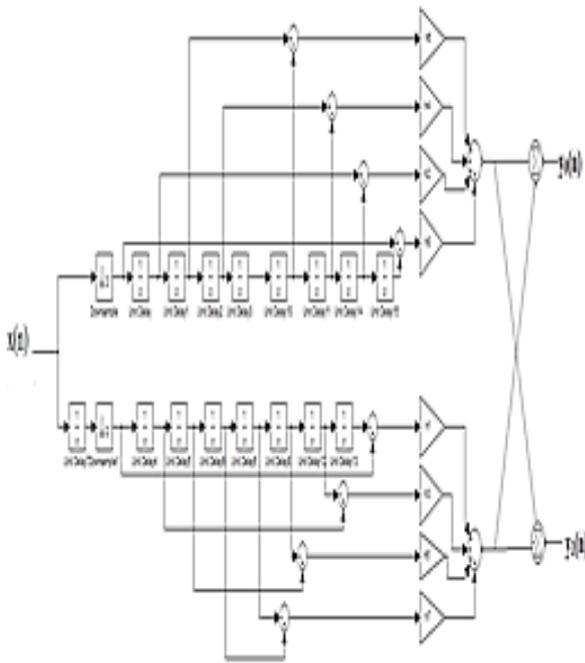


Fig 9: The magnitude and phase responses of H(z) filter with  $\sigma = 2$ .



**Fig 10: The final lattice FIR wavelet (one- level) filter bank structure with  $\sigma = 2$ .**

**b) A Multiplierless Realization**

By same previous procedure, the wordlength of 6 bits also can be chosen where the maximum and average deviations are acceptable (maximum error is 0.0588 and average deviation is 0.0223) and the maximum SNR is 25.3362 dB at an ECG number of samples of 60. The resulting SOPOT multiplierless realization is as illustrated in Table 5.

**TABLE 5: Multiplierless representation of coefficients with  $\sigma = 2$ .**

COEFFICIENTS	SOPOT
$h_0 = 0$	None
$h_1 = 0$	None
$h_2 = 0.0156$	$2^{-6} =$ Six shifts only
$h_3 = 0.0625$	$2^{-4} =$ Four shifts only
$h_4 = 0.1406 = 0.1250 + 0.0156$	$2^{-3} + 2^{-6} =$ Shift and add
$h_5 = 0.1563 = 0.1250 + 0.0313$	$2^{-3} + 2^{-5} =$ Shift and add
$h_6 = 0.1250$	$2^{-3} =$ Three shifts only
$h_7 = 0.0625$	$2^{-4} =$ Four shifts only

**FPGA Implementations and Results**

The filter bank architectures are implemented using one of the Xilinx FPGA devices, the SPARTAN-3E kit. This device has a capacity of (4656) logic slices

where the total number of CLBs is (1164) (each CLB contains four slices). It has 20 dedicated multipliers, 20 BRAMs and can operate at a maximum clock speed of 50MHz. The pipeline technique is used to perform the implementation. Pipeline implementations

have faster operation but suffer from occupying larger areas of the FPGA devices they operate on. Therefore, it is suited very well to applications where high speed is required regardless of the occupied area [9].

A hardware implementation of the 1<sup>st</sup> order Gaussian derivative FIR filter bank structure can be accomplished by the use of two Processing Elements (PEs); each represents an FIR filter (either  $H_{even}$  or  $H_{odd}$ ). The order of the FIR wavelet filter bank depends upon the standard deviation which can be executed by a stream of instructions.

The results of three-level pipeline implementation using SPARTAN-3E device are summarized in Table 6 for the two cases ( $\sigma = 1$  and  $\sigma = 2$ ).

**ECG feature extraction**

This section exhibits the data of ECG signal which is adopted as input signal to the designed systems. These data represent a group of diseases in addition to the normal state.

The extracted feature from the ECG signal plays a vital role in diagnosing the cardiac disease. Therefore it is necessary that the feature extraction system performs accurately. The purpose of feature extraction is to find as few properties as possible within ECG signal that would allow successful abnormality detection and efficient prognosis. To get the feature vector for any ECG signal, the concatenation of the resulting  $y_0(n)$  coefficients of the last decomposition level and some  $y_1(n)$  coefficients is usually obtained [9]. In this paper,

the concatenation of the third level wavelet and scaling coefficients is used to form the feature vector in each tested ECG signal. These vectors are illustrated in Tables 7 and 8 for standard deviation values  $\sigma = 1$  and  $\sigma = 2$ , respectively.

**TABLE 6: Three-level pipelining results.**

Filter bank type	Resource	Used	Available	Utilization ratio
1 <sup>st</sup> order Gaussian derivative filter bank with $\sigma = 1$ .	<i>Slices</i>	236	4656	5%
	<i>Slices flip flops</i>	228	9312	2%
	<i>4-input LUTs</i>	339	9312	3%
	<i>Multipliers</i>	0	20	0%
	<i>Max. Frequency</i>	60.219 MHz		
1 <sup>st</sup> order Gaussian derivative filter bank with $\sigma = 2$ .	<i>Slices</i>	441	4656	9%
	<i>Slices flip flops</i>	380	9312	4%
	<i>4-input LUTs</i>	528	9312	5%
	<i>Multipliers</i>	0	20	0%
	<i>Max. Frequency</i>	60.724MHz		

Conclusions

ECG-based FIR filter banks have been designed. The 1st order Gaussian derivative wavelet function has been utilized. Sampled versions of such wavelet function are used as impulse responses to the designed filter banks. These banks have been realized in a highly-efficient lattice structures which are easy to implement. The numbers of filter banks coefficients have been reduced to less than half of their original numbers, resulting in reducing the number of multiplications and improving the filter banks efficiencies as the final number of the performed computations is reduced. This has lead to less-complex hardware implementations and high operating frequencies when implemented on FPGA kit. SOPOT method has been applied to quantized different multiplier values, leading to multiplierless realizations for such multiplier values (shift and add only). In spite of the need for at least more than one level decompositions for ECG-QRS feature extraction, the proposed lattice structures can also serve for that

purpose because of their less-complex and computational-efficient realizations.

**TABLE 7:  $y_0(n)$  and  $y_1(n)$  coefficients (three-level) of tested ECG signals with  $\sigma = 1$ .**

ECG signal	Normal ECG signal		Bradycardia ECG signal		Tachycardia ECG signal		Hyperkalemia ECG signal		WPW syndrom ECG signal		Pacemaker ECG signal		Established angina ECG signal	
	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	
0	0	1	0	0	0	0	0	0	0	0	0	0	0	0
2	FF	1	0	0	0	1	0	0	3	FD	1	2	0	1
FF	0	0	FF	0	FF	0	FF	FC	3	FD	3	1	FD	3
0	2	0	0	0	0	0	0	0	2	FF	1	1	FD	1
2	1	FE	0	1	FE	1	FE	1	2	FF	1	2	FE	FE
FF	FE	FE	FE	FE	FE	3	FE	4	3	FD	4	4	FC	FC
FF	FF	1	0	FE	2	1	0	3	FC	1	FD	3	FC	0
0	2	3	1	0	2	FE	1	FE	5	2	FE	1	FF	1
2	1	1	0	2	1	0	3	0	4	0	FE	2	FE	2
0	FD	FF	0	1	FF	1	FF	0	0	4	0	0	FF	3
FF	FF	FF	0	0	0	2	0	0	1	0	FF	0	0	FF

**TABLE 8:  $y_0(n)$  and  $y_1(n)$  coefficients (three-level) of tested ECG signals with  $\sigma = 2$ .**

Established angina ECG signal	Pacemaker ECG signal		WPW syndrom ECG signal		Hyperkalemia ECG signal		Tachycardia ECG signal		ECG signal		Bradycardia ECG signal		ECG signal		Normal ECG signal		
	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.	$y_0(n)$ coef.	$y_1(n)$ coef.
1	FF	0	1	0	0	0	0	0	1	1	FF	1	FF	1	FF	1	1
3	1	FE	0	1	3	FC	0	0	FF	FE	FE	2	FE	2	FE	0	0
FF	4	5	FD	FF	3	8	FE	4	FB	FF	FF	6	FB	3	FB	1	1
FF	FE	FE	1	FE	4	FC	FD	FF	6	FA	6	FA	3	FA	3	F9	F9
FD	1	1	2	FA	F4	1	0	0	F9	1	1	6	2	2	2	3	3
FC	5	1	2	F9	3	0	3	3	5	1	1	0	3	3	3	3	3
FD	F7	FD	0	2	1	0	1	6	FE	0	0	0	FD	0	FD	0	0
3	2	5	FD	FF	FE	1	7	5	FF	1	5	1	FB	1	FB	1	1
8	4	FE	1	FE	2	FF	1	3	FE	0	3	0	3	0	3	F9	F9
FF	FF	1	2	FA	FF	FD	FD	F4	FC	FE	FE	0	2	2	2	3	3
1	0	1	1	F8	1	FE	2	0	0	4	FC	4	4	4	4	3	3
4	FF	FF	2	0	4	1	1	1	1	FF	4	0	0	0	0	0	0
FF	4	0	1	0	0	FF	FF	FF	FF	2	0	0	0	0	0	0	0
FE	FF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0
FF	FF	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0	0

References

1. Mahmoodabadi, S. Z., Ahmadian, A. and Abolhasani, M. D. (2005). ECG Feature Extraction Using Daubechies Wavelets. *proceedings of the Fifth IASTED International Conference. Visualization, Imaging, and Image Processing. Benidorm, Spain, September 7-9.*
2. Bhojwani, S. N. (2007). Simulation of Physiological Signals Using Wavelets. *M.Sc. Thesis, University of Akron, On:* <http://etd.ohiolink.edu/sendpdf.cgi/bhojwani%20soniya%20naresh.pdf> *num=akron 1193079604.*
3. Saritha, C., Sukanya V. and Narasimha Murthy Y. (2008). ECG Signal Analysis Using Wavelet Transforms. *Bulg. J. Phys. 35, pp: 68-77.*
4. Engin, M. (2004). ECG Beat Classification Using Neuro-Fuzzy Network. *Pattern Recognition Letters 25 Elsevier, pp: 1715-1722.*
5. Karpagachelvi, S., Arthanari, M. and Sivakumar, M. (2010). ECG Feature Extraction Techniques-A Survey Approach. *International Journal of Computer Science and Information Security, vol. 8, no: 1, pp: 76-80, On:* [http://sites.google.com/site/ijcsis/.](http://sites.google.com/site/ijcsis/)
6. German-Sallo, Z. (2009). Adapted Discrete Wavelet Function Design for ECG Signal Analysis. *Electrical and Mechanical Engineering, Acta Universitatis Sapientiae, pp: 155-161, On:* [http://www.acta.sapientia.ro-acta-emeng-C1-emeng1-13.pdf.](http://www.acta.sapientia.ro-acta-emeng-C1-emeng1-13.pdf)
7. Sripathi, D. (2003). Efficient Implementations of Discrete Wavelet Transforms Using FPGAs. *Master's Thesis, Electrical and Computer Engineering Department, Florida State University.*
8. Hale, D. (2006). Recursive Gaussian Filters. *Center for Wave Phenomena, Colorado School of Mines, Golden CO 80401, USA. pp: 269-278 On:* [http://www.cwp.mines.edu/Meetings/Project06/cwp546.pdf.](http://www.cwp.mines.edu/Meetings/Project06/cwp546.pdf)
9. Al-Gherify, M. F. K. (2007). Image Compression using BinDCT for Dynamic Hardware FPGA's. *Ph.D. Thesis, General Engineering Research Institute (GERI), Liverpool John Moores University.*

## التصميم والبناء بلا مضارب لمرشح موجي نوع مشتقة كاوس الأولى المعتمد على إشارة ECG باستخدام الهيكل المتشابك

جاسم محمد عبد الجبار عبد الحميد محمد جاسم

[drjssm@yahoo.com](mailto:drjssm@yahoo.com)

### الخلاصة

في هذا البحث، تم إختيار المشتقة الأولى لدالة كاوس كدالة أم موجية واستغلالها لتصميم وبناء بعض أجراف مرشحات مقابلة. وبما إن المشتقة الأولى لدالة كاوس لها شكل مماثل لجزء QRS المعقد في إشارة ECG، فإنها يمكن أن تستعمل لإستخلاص ميزات QRS. وإعتماداً على هذه الخاصية في مثل هذه الدالة الموجية الأم، فإن المرشحات المصممة يمكن تحقيقها بهياكل متشابكة كفوءة جداً وسهلة البناء في قيمتي الانحراف المعياري ( $\sigma = 1$ ) و ( $\sigma = 2$ ). الهياكل المتشابكة الناتجة تخفّض من عدد معاملات أجراف المرشحات وهذا بدوره يخفّض عدد المضارب ويحسن كفاءة أجراف المرشحات حيث أنه يخفّض عدد الحسابات المؤداة. وعلى صعيد المكونات المادية فإن هذا سيؤدي إلى بنات أقل تعقيداً. إن قيم المضارب المكتملة الناتجة يمكن أن تقود إلى تحقيق بلا مضارب.