On the Support Sets of Acyclic and Transitive Digraphs

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Abstract

For any "acyclic digraph σ there is defined a connected component" $G_\sigma(X)$ of the graph $G(X)$ containing relation σ, there are defined non-empty "support sets":

$$S(\sigma) \square \{y \in X : \sigma(x,y) = 0 \ for \ all \ x \in X\}, \ \ S^{'}(\sigma) \square \{x \in X : \sigma(x,y) = 0 \ for \ all \ y \in X\},$$

there are defined the families:

$$S(G_\sigma) \square \{S(\tau) \subseteq X : \tau \in G_\sigma(X)\}, \ S^{'}(G_\sigma) \square \{S^{'}(\tau) \subseteq X : \tau \in G_\sigma(X)\}$$

"consisting of all support sets of acyclic digraphs" τ included in the component $G_\sigma(X)$. In this work we proved that the equality $S(G_\sigma) = S^{'}(G_\sigma)$ is valid and we investigated some features of the concepts of "support sets of acyclic and transitive digraphs".

The family $S(G_\sigma) = (S^{'}(G_\sigma))$ is a specific partially ordered set with respect to the natural relation of inclusion of sets. Specificity is that, together with each element, the family $S(G_\sigma)$ contains all non-empty subsets of this element, and, in addition, $S(G_\sigma)$ contains all singlton subsets of the set X. Moreover, if σ is a "transitive digraph", then the family $S(G_\sigma)$ contains all two-element subsets of the set X. The latter circumstance can play an important role in the process of separating transitive digraphs from acyclic digraphs. In connection with this fact, we consider the central problem of an independent description of families $S(G_\sigma)$ (or their maximal elements).

Key words: support sets, partial order graphs , acyclic and transitive digraphs.

الخلاصة

لأي بيان موجه دائري مباشر σ عرفنا في هذا البحث المركبات المتصلة $G_\sigma(X)$ للبيان $G(X)$ والحاوية على المجموعات الداعمة التالية

$$S(\sigma) \square \{y \in X : \sigma(x,y) = 0 \ for \ all \ x \in X\}, \ \ S^{'}(\sigma) \square \{x \in X : \sigma(x,y) = 0 \ for \ all \ y \in X\},$$

حيث عرفنا العوائل التالية

$$S(G_\sigma) \square \{S(\tau) \subseteq X : \tau \in G_\sigma(X)\}, \ S^{'}(G_\sigma) \square \{S^{'}(\tau) \subseteq X : \tau \in G_\sigma(X)\}$$

والتي تحوي على كل المجموعات الداعمة من البيانات الموجه الدائرية والمتعدية والتي تتضمن المركبات $G_\sigma(X)$ في هذا البحث برهنا ان $S(G_\sigma) = S^{'}(G_\sigma)$ وكذلك برهنا العديد من الخصائص التي تتعلق بمفهوم المجموعات الداعمة للبيانات الموجه المباشرة الدائرية والمتعدية. العائلة $S(G_\sigma) = (S^{'}(G_\sigma))$ التي تحوي كل علاقات الترتيب الجزئي برهنا بأنها تحوي على عنصر واحد من المجموعات الجزئية الداعمة وإذا كان σ بيان موجه مباشر متعدي اذن العائلة $S(G_\sigma)$ برهنا بأنها تحوي على عنصرين فقط من المجموعات الجزئية الداعمة.

الكلمات المفتاحية: المجموعة الداعمة ، بيان علاقات الترتيب الجزئي، البيان الموجه الدائري والمتعدي.

1. Definitions and auxiliary propositions

Definition 1.1 Any "binary relation $\sigma \subseteq X^2$ (X –arabitrary set), generates a characteristic function" $\sigma' : X^2 \to \{0,1\}$,(if $(x,y) \in \sigma$, then $\sigma'(x,y) = 1$, otherwise $\sigma'(x,y) = 0)$, and this mapping is bijectve.

Remarks 1.2 1) from the definition above we called the subset $\sigma \subseteq X^2$ as the relationships and functions (sometimes digraphs).
2) If X finite set then the characeteristic function can be interepreted as a binary matrix (the matrix consisting of 0 and 1).

Definition 1.3 the relations $\sigma, \tau \subseteq X^2$ called adjacent if there exists a disjoint of union of two subsets $X = Y \cup Z$, such that:

1) $\sigma(x,y) = 0$ for all $(x,y) \in Y \times Z$;
2) $\tau(x,y) = 0$ for all $(x,y) \in Z \times Y$;
3) $\tau(x,y) + \sigma(y,x) = 1$ for all $(x,y) \in Y \times Z;$
4) $\sigma(x,y) = \tau(x,y).$ for all $(x,y) \in Y^2 \cup Z^2$.

Remark 1.4 1) From the definiition above, that if the relation τ adjaceent with a relation σ , then σ adjaceent with a relation τ , and this fact we write in the form of a diagram $\sigma \xleftrightarrow{Y\times Z} \tau$.

$$\begin{array}{c|cc} & Y & Z \\ \hline Y & & 0 \\ Z & \sigma(x,y) & \end{array} = \sigma \xleftrightarrow{Y\times Z} \tau = \begin{array}{c|cc} & Y & Z \\ \hline Y & & 1-\sigma(x,y) \\ Z & 0 & \end{array}$$

Example 1.5: For $X = \{1,2,3,4\}$ we have the following adjaceent relations:

$$\begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 1 & 1 & 1 & 1 \\ 1 & 1 & 0 & 1 \end{matrix} \xleftrightarrow{\{1\}\times\{2,3,4\}} \begin{matrix} 1 & 1 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 1 & 1 & 1 \\ 0 & 1 & 0 & 1 \end{matrix} \xleftrightarrow{\{2,4\}\times\{1,3\}} \begin{matrix} 1 & 0 & 0 & 0 \\ 0 & 1 & 0 & 0 \\ 0 & 0 & 1 & 0 \\ 1 & 1 & 0 & 1 \end{matrix} \xleftrightarrow{\{2,3\}\times\{1,4\}} \begin{matrix} 1 & 0 & 0 & 0 \\ 1 & 1 & 0 & 0 \\ 1 & 0 & 1 & 1 \\ 1 & 0 & 0 & 1 \end{matrix}$$

Example 1.6: $X = \{1,...,6\}$, Y={1,2}, Z={3,4,5,6}, Then the adjacent relation is:

$$\left[\begin{array}{c|c} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & 0 \\ \hline \begin{matrix} 0 & 0 \\ 1 & 0 \\ 0 & 0 \\ 1 & 1 \end{matrix} & \begin{matrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \end{array}\right] = \sigma \xleftrightarrow{Y\times Z} \tau = \left[\begin{array}{c|c} \begin{matrix} 1 & 0 \\ 0 & 1 \end{matrix} & \begin{matrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \end{matrix} \\ \hline 0 & \begin{matrix} 1 & 0 & 1 & 0 \\ 1 & 1 & 1 & 0 \\ 0 & 0 & 1 & 0 \\ 0 & 0 & 0 & 1 \end{matrix} \end{array}\right]$$

"Thus, the set X generates a pair $\langle 2^{X^2}, E(X)\rangle$, where 2^{X^2} is the set of vertices, consist of the set of all binary relations of the set X, and $E(X)-$ is a set of edges, consist of all unordered distinct pairs of adjaceent of binary relations of the set X. The pair $G(X) \doteq \langle 2^{X^2}, E(X)\rangle$ will be called "undirected graph of binary relations of the set X".

The following theorem proved that in [Al' Dzhabri, and Rodionov 2015].

Theorem 1.7: If $card\ X \neq 1$, then $diam(G(X)) = 2$.

Remark 1.8: we denoted the "connected component of the graph" $G(X)$ by $G_\sigma(X)$, which contains the given relation $\sigma \in 2^{X^2}$.

2. Certain types of the subgraphs of the graph of binary relations.

"We denoted that the collection of all partial orders defined on the set X by $V_0(X)$. And the collection of all reflexive –transitive relations defined on the set X by $V(X)$ and where X finite sets the collection of all acyclic relations by $A(X)$."

In [Al' Dzhabri, and Rodionov 2013; Al' Dzhabri, and Rodionov 2015; Al' Dzhabri, and Rodionov 2015] we proved that if $\sigma, \tau \in 2^{X^2}$ are adjacent then:

1. $\sigma \in V_0(X)$ if and only if $\tau \in V_0(X)$;
2. $\sigma \in V(X)$ if and only if $\tau \in V(X)$;
3. $\sigma \in A(X)$ if and only if $\tau \in A(X)$.

Therefore, in the graph $\langle 2^{X^2}, E(X)\rangle$ define the following subgrphs:

$$\langle V_0(X), E(X)\rangle, \quad \langle V(X), E(X)\rangle, \quad \langle A(X), E(X)\rangle. \qquad \ldots\ldots\ldots\ldots\ldots\ldots\ldots\ldots(1)$$

Continue to suggest that $card\ X < \infty$ (i.e $X = \{1,\ldots,n\}$). Then we get the following remakes:

Remakes 2.1: 1) If replacing the unit elements $\sigma(x,x)$, zeros, then we get a one-to-one correspondence between the set $V_0(X)$ and the set of all labeled transitive digraphs denoted by $V_0^0(X)$.

2) There exist a one-to-one correspondence between the set $V_0(X)$ and the seet of all labeeled T_0-topoology denoted by $\mathrm{T}_0(X)$.

3) Let $T_0(n) \square\ card\ \mathrm{T}_0(X) \square\ card\ V_0(X) \square\ card\ V_0^0(X)$. Additional suggest that $T_0(0) \square 1$.

In [Al' Dzhabri, and Rodionov 2013] we proved that the number of "connected component of the graph "$\langle V_0(X), E(X)\rangle$ equal to $T_0(n-1)$. we note that for any nataural number n the following equaliities are hold:

$$T_0(n) = \sum_{p_1+\ldots+p_k=n} \frac{n!}{p_1!+\ldots+p_k!} V(p_1,\ldots,p_k), \qquad \ldots\ldots\ldots(2)$$

$$T_0(n)=\sum_{p_1+...+\mathrm{p}_k=n}(-1)^{n-k}\frac{n!}{p_1!+...+\mathrm{p}_k!}w(p_1,...,\mathrm{p}_k),$$

where the summation is over all ordered sets $(p_1,...,p_k)$ of positive integers such that $p_1+...+p_2=n$. The first formula see [Comtet 1966; Erne 1974; Borevich 1982] and second in [Rodionov, 2016].The number of $V(p_1,...,p_k)$ and $W(p_1,...,p_k)$ denote the number of partial orders of a special form, which depends on a set of $(p_1,...,p_k)$.

If $X=\{1,...,n\}$. Then $cardV(X)=\sum_{m=1}^{n}S(n,m)T_0(m)$ see [Comtet 1966; Evans, *et al.*, 1967; Gupta 1968] where $S(n,m)-$ This Stiriling numbers of the 2nd kind in our work we proved that the number of coonnected component of the graph $\langle V(X),E(X)\rangle$ equal to $\sum_{m=1}^{n}S(n,m)T_0(m-1)$.

Remark 2.2 From above there exists a one to one corresponded between the set of all transitive- reflexive relations $V(X)$ and the set of all labeled topologies $T(X)$ defined on the set X.

If $X=\{1,...,n\}$ according to [Rodionov 1992] the following equality holds:

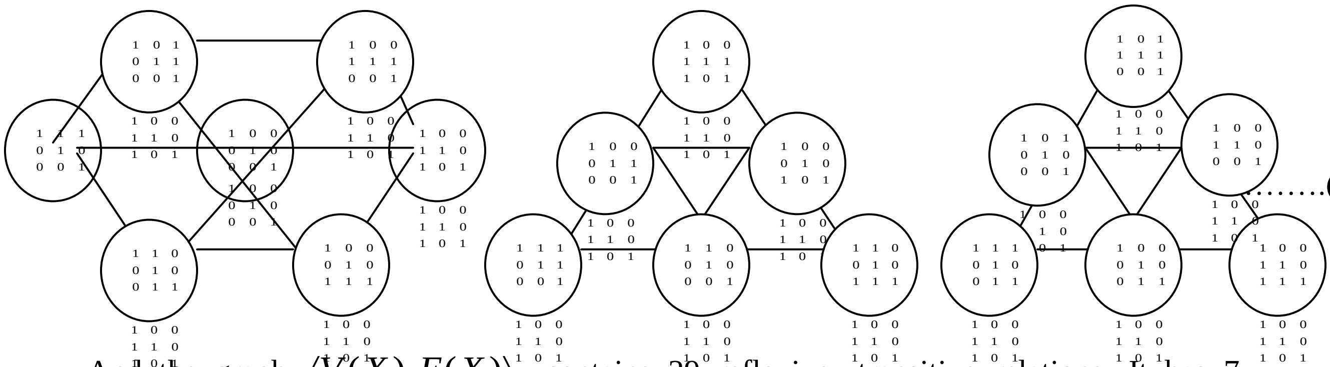
$$cardA(X)=\sum_{p_1+...+p_k=n}(-1)^{n-k}\frac{n!}{p_1!...p_k!}2^{(n^2-p_1^2-...-p_k^2)/2}, \qquad (3)$$

In our work [Al' Dzhabri, V.I. Rodionov] we proved that the number of connected component of the graph $\langle A(X),E(X)\rangle$ equal to $\sum_{p_1+...+p_k=n}(-1)^{n-k}\frac{(n-1)!}{(p_1-1)!p_2!...p_k!}2^{(n^2-p_1^2-...-p_k^2)/2}$.

Remark 2.3 We note that the formulas (2) and (3) have the same struccture, and In the second case if the formula has a finished appearance, then in the first formula remains a problem of calculation of numbers $W(p_1,...,p_k)$.

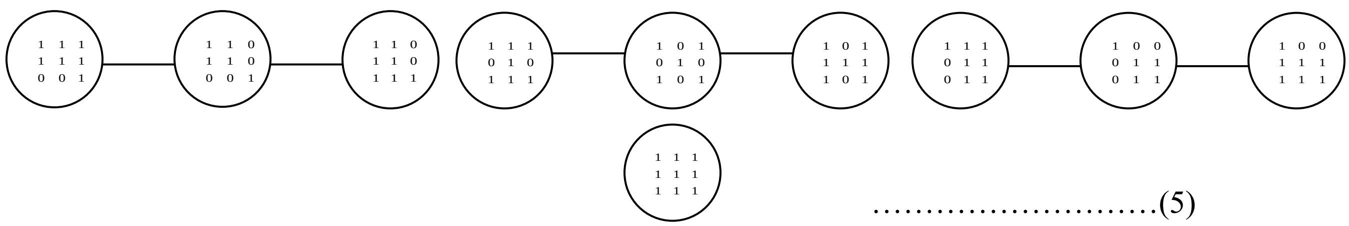
Now from the formulas above we give the following example:

Example 2.4: Let $X=\{1,2,3\}$.Then we get 3 "connected components of the graph" $\langle V_0(X),E(X)\rangle$, contains 19 partial orders:



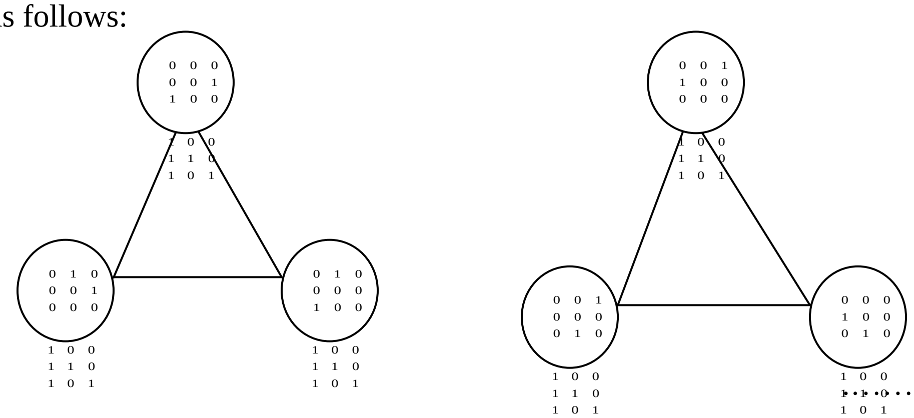
……..(4)

And the graph $\langle V(X),E(X)\rangle$, contains 29 reflexive- transitive relations. It has 7 conneected componeents: 3 components of the graph (4) above and 4 components as:



………………………(5)

And the graph $\langle A(X),E(X)\rangle$ contains 25 acyclice relations . It has 5 connected components :
3 components of the grapha (4) above (in them should be replaced the elements of the set $V_0(X)$ by the elements of the set $V_0^0(X)$)
2 components as follows:



……..(6)

“Where $X=\{1,2,3,4\}$ in subgraphs of the form (1) there is a 219, 355 and 543 vertices respectively and the number of connected components of these subgrags in (1) equal to 19, 45, and 79 respectively.”

3. Supporot sets of acyclic and transitive digraphs

In this section we introduced a new concept "support sets $S(\sigma)$" of acyclic and transitive digraphs of adjacency and determined the algebraic system consisting of all binaray relations of a set and of all unordered pairs of various adjacent binary relations.

Definition 4.1: For any acyclic digraph $\sigma \in A(X)$, $cardX < \infty$ define a non-empty support sets:

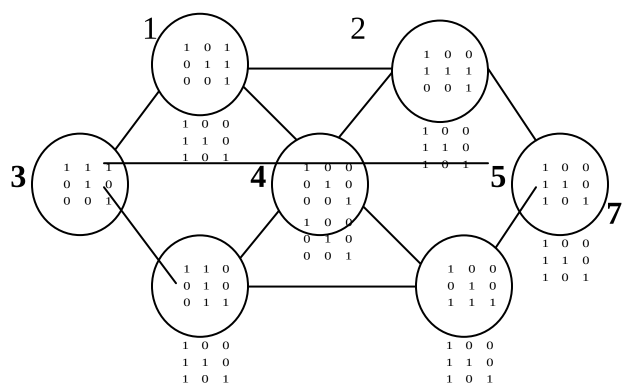
$$S(\sigma) \triangleq \{y \in X : \sigma(x,y) = 0 \text{ for all } x \in X\},$$
$$S^{'}(\sigma) \triangleq \{x \in X : \sigma(x,y) = 0 \text{ for all } y \in X\}.$$

And defined families of support sets:

$$S(G_\sigma) \triangleq \{S(\tau) \subseteq X : \tau \in G_\sigma(X)\},\ S^{'}(G_\sigma) \triangleq \{S^{'}(\tau) \subseteq X : \tau \in G_\sigma(X)\}$$

Where $G_\sigma(X)$ is the "connected of componeent of the graph" $G(X)$, contains σ.

Example 4.2: In example (2.4) the first "connected components of the graph" $\langle V_0(X), E(X)\rangle$,



We can compute: $S(\sigma)$ and $S^{'}(\sigma)$ in all vertices such as:

In the vertex number (1) $S(\sigma) = \{1,2\}$ and $S^{'}(\sigma) = \{3\}$.

In vertex number (2) $S(\sigma) = \{2\}$ and $S^{'}(\sigma) = \{1,3\}$.

In vertex number (3) $S(\sigma) = \{1\}$ and $S^{'}(\sigma) = \{2,3\}$.

In vertex number (4) $S(\sigma) = \{1,2,3\}$ and $S^{'}(\sigma) = \{1,2,3\}$.

In vertex number (5) $S(\sigma) = \{2,3\}$ and $S^{'}(\sigma) = \{1\}$.

In vertex number (6) $S(\sigma) = \{1,3\}$ and $S^{'}(\sigma) = \{2\}$.

In vertex number (7) $S(\sigma) = \{3\}$ and $S^{'}(\sigma) = \{1,2\}$. And we show that:

$$S(G_\sigma) = \{\{1\},\{2\},\{3\},\{1,2\},\{1,3\},\{2,3\},\{1,2,3\}\} = S^{'}(G_\sigma).$$

Remark 4.2 If X finite be set then $S(\sigma) \neq \varnothing$ for any $\sigma \in A(X)$.

Now we prove that the following theorem:

Theorem 4.3: For any $\sigma \in A(X)$, then $S(G_\sigma) = S^{'}(G_\sigma)$.

Proof: Let $\rho \in G_\sigma$ defines the following sets $Z \square S(\rho) \in S(G_\sigma)$ and $Y \square X \setminus Z$, and $\rho(x,y)=0$ for all $(x,y) \in X \times Z = Z^2 \bigcup (Y \times Z)$. Define the relaation $\tau \in G_\sigma$ such that $\rho \xleftrightarrow{Y\times Z} \tau$, and hence $\tau(x,y) = \rho(x,y) = 0$ for all $(x,y) \in Z^2$ and $\tau(x,y)=0$ for all $(x,y) \in Z \times Y$. Thereforre $\tau(x,y)=0$ for all $(x,y) \in Z \times X$, and hencee $Z \subseteq S^{'}(\tau) \in S^{'}(G_\sigma)$, then there exist $\pi \in G_\sigma$ such that $Z = S^{'}(\pi) \in S^{'}(G_\sigma)$. And hence $S(G_\sigma) \subseteq S^{'}(G_\sigma)$. The inverse inclusion is proved by the symmetricc.

Proposition 4.4: Let X finite set . For any non-trivial $\sigma \in A(X)$ and for any $y \in X \setminus S(\sigma)$ there exist $x \in S(\sigma)$ such that $\sigma(x,y)=1$.

Proof: Let $V \square S(\sigma)$, $W = X \setminus V$ since σ non- trivial acyclic then the set W is non empty such that for any $y \in Z$ there exist $x \neq y$ such that $\sigma(x,y)=1$ therefore for σ we have the following representation:

$\sigma =$

	V	D	F
V	1 ⋱ 1	0	
D	0		
F	0	$*$	

$$D = \{\alpha \in W : \sigma(\psi,\alpha) = 0 \textit{ for all } \psi \in V\},$$
$$F = W \setminus D = \{\alpha \in W : \sigma(\psi,\alpha) = 1 \textit{ for some } \psi \in V\}.$$

For proof these proposition its enough to show that $D = \varnothing$. Now we assume that $D \neq \varnothing$.

1) If $F = \varnothing$, then $\sigma(x,y)=0$ for any $(x,y) \in V \times D = (V \bigcup F) \times D$.
2) If $F \neq \varnothing$, then $F \times D \neq \varnothing$. Fix $(x,y) \in F \times D$. since $x \in F$, then there exist $z \in V$

such that $\sigma(z,x)=1$; since $(z,y) \in V \times D$, then $\sigma(z,y)=0$ and from defintion (1.3) we get $\sigma(x,y)=0$ (in the other word all elements in the block $*$ equal to zero) . Thus, in both cases $\sigma(x,y)=0$ for any $(x,y) \in (Y \bigcup J) \times I$. and hence for $y \in I$ there exist $x \in I$, such that $x \neq y$ and $\sigma(x,y)=1$ and this contradiction with remark (4.2) . Really in the block I^2 is located acyclic $\tau \square \sigma|_I$ which is the restriction σ on I and therefore by remark (4.2) we get that $S(\tau) \neq \varnothing$.

Corollary 4.5 Let X finite set and $\sigma \in A(X)$ such that $S(\sigma) = \{x\}$ singleton set then $\sigma(x,y)=1$ for any $y \in X$.

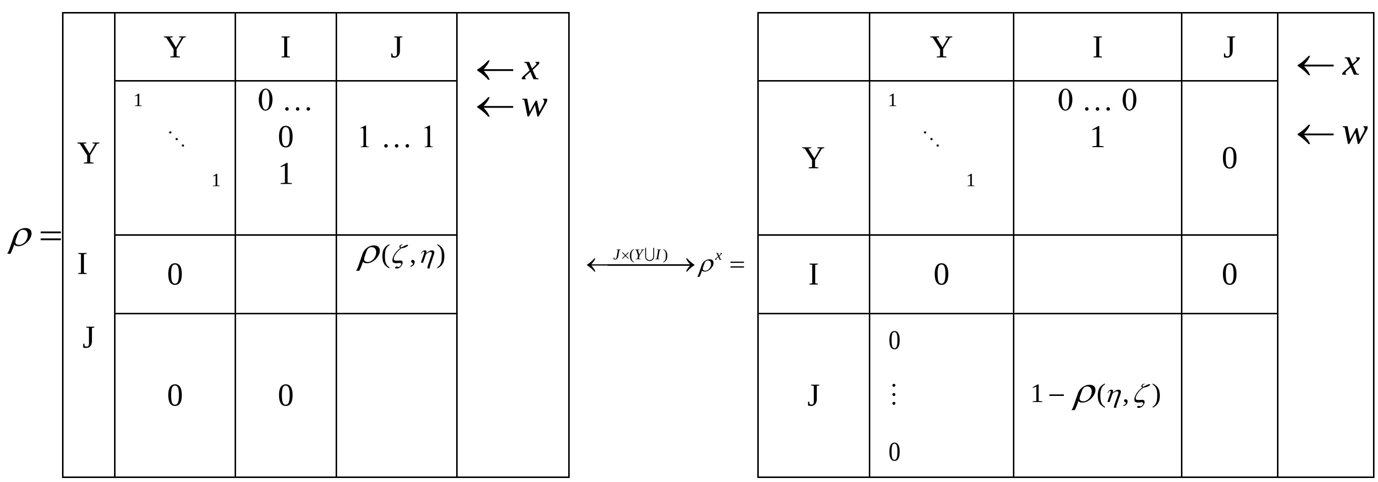
Proposition 4.6: Let $\sigma \in A(X)$ and $\rho \in G_{\sigma}(X)$. Then $S(\sigma) = S(\rho)$ if and only if $\sigma = \rho$.

Proof : Where $card\ X \leq 2$ The proposition is triivial. Now suppose that $card\ X \geq 3$. Let $Y := S(\sigma) = S(\rho)$ and $Z := X \setminus Y$. Assume that $\sigma \neq \rho$ in another word $\sigma(x,z) \neq \rho(x,z)$ for some pair $(x,z) \in X \times Z$.

1) We assume first $x \in Y$. Without lose of generality, we can assume that $\sigma(x,z) = 1$, and $\rho(x,z) = 0$. If $card\ Y = 1$, then $Y = \{x\}$, then by Corollary (4.4) $\rho(x,z) = 1$ and this a contradiction and hence $card\ Y \geq 1$, therefore by proposition (4.3) there exist $w \in Y$ such that $w \neq x$, and $\rho(w,z) = 1$.

Let $I := \{\eta \in Z : \rho(x,\eta) = 0\}$ and $J := \{\eta \in Z : \rho(x,\eta) = 1\}$. It is cleare that $z \in I$, therefor $I \neq \varnothing$ and $Y \times I \neq \varnothing$. If $(\zeta,\eta) \in J \times I$, then $\rho(x,\zeta) = 1$, $\rho(x,\eta) = 0$, therefore :

$\rho(\zeta,\eta) = \rho(x,\zeta)\rho(\zeta,\eta) \leq \rho(x,\eta) = 0$. And henc $\rho(\zeta,\eta) = 0$ and for ρ we have the diagiram:



In particular, $\rho^{x}(w,z) = 1$.

On the other hand, Let $K := \{\eta \in Z : \sigma(x,\eta) = 0\}$ and $L := \{\eta \in Z : \sigma(x,\eta) = 1\}$. It is clear that $z \in L$, therefore $L \neq \varnothing$ and $Y \times L \neq \varnothing$, And if $(\zeta,\eta) \in L \times K$, then $\sigma(x,\zeta) = 1, \sigma(x,\eta) = 0$, and therefore by definition (1.3) $\sigma(\zeta,\eta) = 0$, and for σ we have the diagiram:

$\sigma =$

	Y	K	L	$\leftarrow x$ $\leftarrow w$
Y	$1 \ddots 1$	$0 \dots 0$	$1 \dots 1$	
K	0		$\sigma(\zeta,\eta)$	
L	0	0		
		$\uparrow z$		

$\xleftrightarrow{L\times(Y\cup K)} \sigma^x =$

	Y	K	L	$\leftarrow x$ $\leftarrow w$
Y	$1 \ddots 1$	$0 \dots 0$	0	
K	0		0	
L	$0 \vdots 0$	$1-\sigma(\eta,\zeta)$		
	$\uparrow x$		$\uparrow z$	

Hence $\sigma^x(\zeta,\eta)=0$ for all $\zeta \in Y;$ in particular $\sigma^x(w,z)=0 \neq \rho^x(w,z);$ therefore $\sigma^x \neq \rho^x$ and this contrdiction with lemma (2) in [1] then $x \notin Y.$

2) So, $(x,z)\in Z^2$ and $\sigma(\zeta,\eta)=\rho(\zeta,\eta)$ for all $(\zeta,\eta)\in Y\times Z.$ Fix any $y\in Y,$ and let :

$$I := \{\eta \in Z : \sigma(y,\eta)=\rho(y,\eta)=0\},\ J=\{\eta\in Z:\sigma(y,\eta)=\rho(y,\eta)=1\}.$$

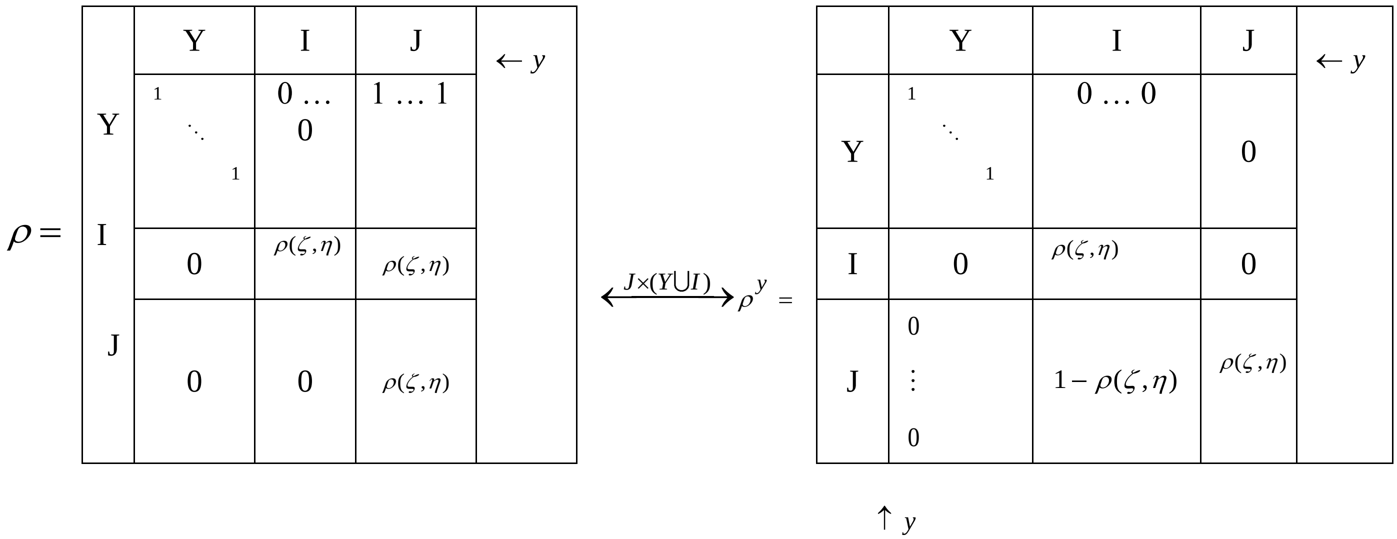
Repeating the calculations of the previous subsection, we get that $\sigma(\zeta,\eta)=\rho(\zeta,\eta)=0$ for all $(\zeta,\eta)\in J\times I.$ Then we get the following diagram:

$\sigma =$

	Y	I	J	$\leftarrow y$
Y	$1 \ddots 1$	$0 \dots 0$	$1 \dots 1$	
I	0	$\sigma(\zeta,\eta)$	$\sigma(\zeta,\eta)$	
J	0	0	$\sigma(\zeta,\eta)$	

$\xleftrightarrow{J\times(Y\cup I)} \sigma^y =$

	Y	I	J	$\leftarrow y$
Y	$1 \ddots 1$	$0 \dots 0$	0	
I	0	$\sigma(\zeta,\eta)$	0	
J	$0 \vdots 0$	$1-\sigma(\eta,\zeta)$	$\sigma(\zeta,\eta)$	



Since from above diagram, we show that $\sigma^{y} = \rho^{y}$, then from the representations for σ^{y} and ρ^{y} it follows that $\sigma(\zeta,\eta) = \rho(\zeta,\eta)$ for all $(\zeta,\eta) \in Z^{2}$, and this contradiction and hence $\sigma = \rho$. The converse is trivial.

Proposition 4.7: Let X finite set and $\sigma \in A(X)$, for any a non-empety subset $S \subseteq S(\sigma)$ there exist a unique $\rho \in G_{\sigma}(X)$ such that $S(\tau) = S$ where ρ is adjacent to σ.

Proof :

Suppose that $U \square S(\sigma),\ V \square U \setminus S,\ W \square X \setminus U,\ I \square \{\eta \in W : \sigma(\zeta,\eta) = 0 \ for\ all\ \zeta \in S\}$ in the other word in the block $**$ (see the diagram below) for any $\eta \in J$ there exist $\zeta \in S$ such that $\sigma(\zeta,\eta) = 1$. Fix $(x, y) \in J \times I$. Since $x \in J$ then there exist $z \in S$, such that $\sigma(z, x) = 1$ and since $(z, y) \in S \times I$ then $\sigma(z, y) = 0$ and from definition (1.3) we get that $\sigma(x, y) = 0$ (in the othere word in the block $*$ all elements equal to zero) :

$\sigma =$

	S	V	I	J
S	$1 \ddots 1$	0	0	$**$
V	0	$1 \ddots 1$	$\sigma(\zeta,\eta)$	
I	0	0		
J	0	0	$*$	

$\xleftrightarrow{(S \cup J) \times (V \cup I)}$

$\rho =$

	S	V	I	J
S	$1 \ddots 1$	1	1	$**$
V	0	$1 \ddots 1$	$\sigma(\zeta,\eta)$	0
I	0	0	$1 \ddots 1$	0
J	0	$1-\sigma(\eta,\zeta)$	$1-\sigma(\eta,\zeta)$	$1 \ddots 1$

From the diagram above, we can see that $S(\tau) = S$.

From the propostions above we get the following remarks:

Remarks 4.8: 1) Where $\sigma \in A(X)$.The set $S(G_\sigma)$ It is a specific partially ordered set with respecet to the natural inclusion relation of sets. “The specificity is that, together with each element, the family $S(G_\sigma)$ contains all non-empty subsets of this element, and in addition, $S(G_\sigma)$ contains all singleton subsets of X .”

2) f $\sigma \in V_0^0(X) \subset A(X)$, then the family $S(G_\sigma)$ necessarily contains all two-element subsets of the set X . The latter circumstance can play an important role in the process of separating transitive graphs from acyclic.
3) Analysis of the structure of a partially ordered set $S(G_\sigma)$ shows that for its description it is sufficient to indicate all its maximal elements.
4) Transpose procedure for matirices $\sigma(x, y) \to \sigma^{T}(x, y)$ generates for the sets $S^{'}(\sigma)$ and $S^{'}(G_\sigma)$ similar propostions as above.

Example 4.9: In example (2.4) we note that in the graph (4) the support sets $S(\sigma)$ contains two elements, but in the graph (6) the support sets $S(\sigma)$ contains only one element.

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