Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
https://doi.org/10.31272/IJES2024.80.S.S4

Bayesian new lasso composite quantile regression with updated Hierarchical

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Keywords: Composite quantile regression, new hierarchical algorithm, new Gibbs sampler, new lasso

Abstract
The quantile regression lines are infinite, but determining an informative quantile regression line is a very hard matter, Composite quantile regression has been used to treat this issue. If the number of independent variables is large, it becomes difficult to interpret the variables accurately to overcome this problem may use the lasso procedure via the Bayesian technique via using the Laplace distribution as the prior distribution. In this paper, we will introduce a new Bayesian hierarchy is a mixture of a new formulation for Laplace prior distribution and composite quantile regression. The proposed method is compared with two other two existing methods in the same field via testing the performance of two other methods via simulation studies examples, and real data scenarios.

1. Introduction
The quantile regression (Qua-Reg) model is one of the important regression models**.** In Qua- reg does not require any statistical assumptions, also it is very robust against outlier data(Cade, B. S., & Noon, B. R. (2003). All these traits, make the Qua- reg of an important regression model. Therefore, it is used in many scientific applications such as Econometrics, finance, biological sciences, agricultural Sciences and medicine. For dependent distributions that are strongly skewed and non-central, the Qua- reg model is adequate. Using different quantile levels, the Qua- Reg model can be used to estimate the relationship between independent variables and a dependent variable in any position of the dependent distribution (Levin, J. (2002)). The quantile regression levels refer to τ_{th}, ere τ_{th} belongs to the interval(0,1). The Qua- reg model is evaluated through the real relationship between the independent variables and a dependent variable conditional function man quanta levels, where, $Q_{y_{i|x_i}}(\tau)$ is the quantile function at the many quantile levels (τ_{th}).
The quantile function is equivalent to the inverse distribution function as follows: $Q_{y_{i|x_i}}(\tau) = F^{-1}_{y_{i|x_i}}(\tau) \quad 0 \leq \tau \leq 1$.There are infinite quantile regression lines to ddescribe the relationship between one response variable and many explanatory variables . But choosing optimal quantile regression line is hard mater . To solve this issue, we will use composite quantile regression (CO- Qua-Reg). Consider Q

Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
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different quantile levels $,0 < \tau_1 < \tau_2 < \cdots < \tau_Q < 1$ Let $y_i = x_i^T \beta_\tau + \varepsilon_i$ for $i = 1, \ldots\ldots, n \ and \ m = 1, \ldots\ldots, M,$., the composite quantile regression (Zou and Yuan,2008) is showm by

$$(\hat{\beta}) = \begin{matrix} Min \\ \beta_\tau \end{matrix} \quad \sum_{q=1}^{Q} \left\{ \sum_{i=1}^{n} \rho_{\tau_m} \left(y_i - x_i^T \beta_\tau \right) \right\}, \qquad [1]$$

Where $\rho_{\tau_q}(.) is\ the\ loss\ function\ , \tau_q = \frac{m}{M+1} for\ m = 1, \ldots.., M.$

It is impossible to differentiable equation (1) at the zero point. Therefore, minimization above equation can be achieved though algorithm which Koenker and D'Orey (1987) recommended. But it's possible that this algorithm is ineffective. at some quantile level. In order to estimate the (CO- Qua-Reg) parameters, a Bayesian technique has been used.

Now, the random error term ε_i being to asymmetric Laplace distribution (ALD). Where ,the probability density function of the asymmetric Laplace distribution with one assigned to scale parameter is

$$f\left(\varepsilon \middle| \tau_q\right) = \tau_q \left(1 - \tau_q\right) exp\left\{ -\rho_{\tau_q}(\varepsilon) \right\}. \qquad (2)$$

Then the joint distribution of $y = (y_1, \ldots, y_n)^T \ given\ X = (x_1, \ldots, x_n)^T , \beta = (\beta_1, \ldots. \beta_k)^T$ for composite quantile regression is

$$(y|X, \alpha, \beta) = \prod_{q=1}^{Q} \tau_q^n \left(1 - \tau_q\right)^n exp\left\{ -\sum_{i=1}^{n} \rho_{\tau_q}\left(y_i - x_i^T \beta_\tau\right) \right\} \qquad [3]$$

Maximizing the probability function of the dependent variable (y) is equivalent to Minimizing the Equation (1). (Benoit et al.,(2013)) and (Yu et al., 2013)) mentioned to estimate the model parameters via link between Maximizing the probability function of the dependent variable (y) with minimization in Equation (1). But there are difficulty to solve Equation number (3) directly way, because of the mixture of Qua elements. The (Huang and Chen (2015)) show the cluster assignment matrix refer to C whose $(i, q)^{th}$element C_{iq} is belong to 1 if the i_{it} subject belongs to the q_{th} cluster, otherwise C_{iq} is belong to 0. The cluster matrix C_{iq} is treated with the missing values issue. Therefore, likelihood belong to equation (3) is take the following form

$$f(y|X, \alpha, \beta) = \prod_{q=1}^{Q} \prod_{i=1}^{n} \left[\tau_q \left(1 - \tau_q\right) exp\left\{ \rho_{\tau_q}\left(y_i - x_i^T \beta_\tau\right) \right\} \right]^{C_{iq}}$$

By following the researchers Li et al. (2010), Kobayashi and Kozumi (2011), The dependent variable $\boldsymbol{y_i}$ takes the following form

$$\boldsymbol{\varepsilon_i = \vartheta_1 \sigma^{-1} v_i + \vartheta_2 \sigma^{-1} \sqrt{v_i}\ z_i} \qquad \textbf{(4)}$$

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Where $\boldsymbol{v}_{\boldsymbol{i}} \sim \sim Exp\left(\tau_q\left(1-\tau_q\right)\right)$ and $\boldsymbol{z}_{\boldsymbol{i}} \sim$ standard normal.

$\boldsymbol{\vartheta}_{\mathbf{1}}=\frac{\mathbf{1}-\mathbf{2}\boldsymbol{\tau}_{\boldsymbol{q}}}{\boldsymbol{\tau}_{\boldsymbol{q}}(\mathbf{1}-\boldsymbol{\tau}_{\boldsymbol{q}})}$ **and** $\boldsymbol{\vartheta}_{\mathbf{2}}=\sqrt{\frac{\mathbf{2}}{\boldsymbol{\tau}_{\boldsymbol{q}}(\mathbf{1}-\boldsymbol{\tau}_{\boldsymbol{q}})}}$.

$$\boldsymbol{E(\varepsilon_i)=\vartheta_1\sigma^{-1}E(v_i)+\vartheta_2\sigma^{-1}\sqrt{v_i}\ E(z_i)}$$
$$\boldsymbol{E(z_i)=0}$$
$$\boldsymbol{E(\varepsilon_i)=\vartheta_1\sigma^{-1}E(v_i)}$$
$$\frac{\mathbf{1}-\mathbf{2}\boldsymbol{\tau}_{\boldsymbol{q}}}{\boldsymbol{\tau}_{\boldsymbol{q}}(\mathbf{1}-\boldsymbol{\tau}_{\boldsymbol{q}})}=\boldsymbol{\vartheta}_{\mathbf{1}}\frac{\boldsymbol{\sigma}^{-\mathbf{1}}}{\tau_q\left(1-\tau_q\right)}\acute{\Rightarrow}\boldsymbol{\vartheta}_{\mathbf{1}}=\frac{\mathbf{1}-\mathbf{2}\boldsymbol{\tau}_{\boldsymbol{q}}}{\boldsymbol{\tau}_{\boldsymbol{q}}(\mathbf{1}-\boldsymbol{\tau}_{\boldsymbol{q}})}*\frac{\tau_q\left(1-\tau_q\right)}{\boldsymbol{\sigma}^{-\mathbf{1}}}\acute{\Rightarrow}\boldsymbol{\vartheta}_{\mathbf{1}}=\frac{1-2\theta_h}{\boldsymbol{\sigma}^{-\mathbf{1}}}$$

$$\boldsymbol{Var(\varepsilon_i)=var(\vartheta_1\sigma^{-1}E(v_i))+\vartheta_2^2(\sigma^{-1})^2\sqrt{Var(v_i)}\ Var(z_i)}$$
$$\boldsymbol{Var(\varepsilon_i)=\vartheta_2^2(\sigma^{-1})^2v_i}$$

Then $\boldsymbol{Var(\varepsilon_i)=\frac{2}{\sigma^{-1}}}$

Let $\boldsymbol{t=\sigma^{-1}v_i}$,therefore($\boldsymbol{t}$) was distributed exponential distribution with the shape parameter $\left(\frac{\tau_q(1-\tau_q)}{\boldsymbol{\sigma}}\right)$. The formula **(4)** takes the following form

$$\boldsymbol{y_i=x_i{}^T\beta_\tau}+\left(1-2\tau_q\right)\boldsymbol{t_i+2\sigma^{-1/2}\sqrt{t_i}\ z_i}$$

the our hierarchical model for the variables $\boldsymbol{t_i,z_i,i=1,2,\dots.n}$ **are**

$$\boldsymbol{y_i=x_i{}^T\beta_\tau}+\left(1-2\tau_q\right)\boldsymbol{t_i+2\sigma^{-1/2}\sqrt{t_i}\ z_i},$$
$$\boldsymbol{t|\sigma\sim}\qquad \boldsymbol{\sigma^n}\ \left(\tau_q\left(1-\tau_q\right)\right)^{\boldsymbol{n}}\ \boldsymbol{e^{-\sigma\sum_{i=1}^n t_i}}$$

$$\boldsymbol{z\sim}\qquad \boldsymbol{\left(\frac{1}{\sqrt{2\pi}}\right)^n\ \ exp\left(-\frac{1}{2}\sum_{i=1}^{n}z_i{}^2\right)}$$

2. The Bayesian Hierarchical of composite quantile regression

By following fadel alhusseini (2017) the lasso composite quantile (La- CO- Qua-Reg) penalized regression solution is

$$\widehat{\boldsymbol{\beta}}=\boldsymbol{argmin}\sum_{q=1}^{Q}\left\{\sum_{i=1}^{n}\rho_{\tau_m}\left(y_i-x_i^T\beta_\tau\right)\right\}+\boldsymbol{\lambda}\sum_{\boldsymbol{j=1}}^{\boldsymbol{k}}|\boldsymbol{\beta_j}| \qquad (\mathbf{5})$$

The Bayesian (La- CO- Qua-Reg) model based on **(5),** for prior distribution must be enforced a Laplace density in a loss quantile regression model, in accordance with Tibshirani (1996).

$$\boldsymbol{\pi(\beta|\sigma,\lambda)=\frac{\sigma\lambda}{2}\ e^{-\sigma\lambda|\beta|}} \qquad (\mathbf{6})$$

Now that we have the prior distribution as in equation (6), we can formulate it as a scale mixture of a uniform distribution and a standard exponential distribution as in the following statement.

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Proposition: in this part ,the formula for the Laplace distribution is scale mixture of standard exponential distribution and uniform distribution .
It can be demonstrated mathematically as follows

$$e^{-\sigma\lambda|x|} = \int_{z>\sigma|x|}^{\cdot} \lambda e^{-\lambda z}\, dz$$

$$\frac{\sigma\lambda}{2} e^{-\sigma\lambda|x|} = \int_{z>\sigma|x|}^{\prime} \frac{\sigma\lambda}{2}\, \lambda e^{-\lambda z} dz$$

$$\lambda z = m$$

$$= \int_{w>\lambda\sigma|x|}^{\prime} \frac{\sigma\lambda}{2}\, \lambda e^{-m} \frac{1}{\lambda}\, dm$$

$$\frac{\sigma\lambda}{2}\, e^{-\sigma\lambda|x|} = \int_{w>\sigma\lambda|x|}^{\prime} \frac{\sigma\lambda}{2}\, e^{-m} \quad dm \qquad (7)$$

Now by letting $x = \beta$ in **(7),** we get

$$\frac{\sigma\lambda}{2}\, e^{-\sigma\lambda|\beta|} = \int_{w>\sigma\lambda|\beta|}^{\prime} \frac{\sigma\lambda}{2}\, e^{-m} \quad dm \qquad (8)$$

More details see (Remah ,2021)
Hence , the Bayesian hierarchical model is

$$y_i = x_i^{T}\beta_\tau + (1-2\tau_q)t_i + 2\,\sigma^{-1/2}\sqrt{t_i}\;z_i,$$

$$f(y|,\beta,t_i) = \prod_{i=1}^{n} \frac{1}{\sqrt{4\pi t_i}}\, e^{\frac{1}{2}\sum_{i=1}^{n}\sum_{q=1}^{Q}\frac{c_{iq}\,(y-x_i^T\beta_\tau-(1-2\theta_q)t_i)^2}{2\,\sigma^{-1/2}\;t_i}}$$

$$t|\sigma \sim \qquad \sigma^n(\,\tau_q(1-\tau_q))^n e^{-\sigma\sum_{i=1}^{n} t_i}$$

$$z \sim \quad \left(\frac{1}{\sqrt{2\pi}}\right)^n \quad exp\left(-\frac{1}{2}\sum_{i=1}^{n} z_i^{\,2}\right)$$

$$\beta|\sigma,\lambda\sim \quad uniform(-\frac{1}{\sigma\lambda},\frac{1}{\sigma\lambda})$$

$$m\sim \qquad standard\ exponential$$

$$\sigma\sim \qquad \sigma^{a-1}e^{-b\sigma}$$

$$\lambda\sim \qquad \lambda^{c-1}e^{-d\lambda}$$

(a, b, c and d) are hyper parameter

The Conditional Posterior distribution of parameters

1- The Posterior distribution of parameter (C_{iq})

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The posterior distribution of parameter $C_i = (C_{i1}, C_{i2}, \dots\dots, C_{iq})^T$ is the multinomial distribution ;

$$p(C_i|y, X, \alpha, \beta, z) \propto \text{multinomial}(1, \hat{p}_1, \dots., \hat{p}_q), \quad (9)$$

where $\hat{p}_H = \frac{\exp[-(y_i - x_i^T\beta - (1-2\tau_q)m_i/2z_i]}{\sum_{q=1}^{Q}\exp[-(y_i - x_i^T\beta - (1-2\tau_h)z_i/2z_i]}$

2- The Posterior distribution of parameter (Z_i)

Given q , the posterior distribution of parameter $\boldsymbol{z_i}$ for $i = 1, \dots, n$, is the Inverse Normal ,$invG(\delta_i^T, \gamma^T), where\ \delta_i^T = \sqrt{\sum_{q=1}^{q} C_{iq}/(y_i - x_i^T\beta)^2)}$

And $\gamma^T = \sum_{h=1}^{H} C_{ih}/2$

3- The Posterior distribution of parameter (t_i)

the posterior distribution of parameter t_i is the inverse Normal distribution with shape parameter $(\frac{\sigma\tau^2}{w^2} + 2\sigma)$ and scale parameter $\sqrt{\frac{w_j}{\beta_j^2}}$. $s_i^{-1} \sim InvG(\mu_i^T, \varphi^T)$

4- The Posterior distribution of parameter (β)

the posterior distribution of parameter $\boldsymbol{\beta}$ is multivariate distribution $\boldsymbol{N(x_i{}^T\beta} + (1 - 2\tau_q)\boldsymbol{t_i}, \boldsymbol{2\ \sigma^{-1/2}}\sqrt{\boldsymbol{t_i}})$**.**

5-The Posterior distribution of parameter (w)

the posterior distribution of parameter $\boldsymbol{w}$ is standard exponential

6-The Posterior distribution of parameter(σ)

The posterior distribution of parameter $\boldsymbol{\sigma}$ is a gamma distribution with shape parameter $(\boldsymbol{a} + \left(\frac{\boldsymbol{3n}}{\boldsymbol{2}}\right))$ **and scale parameter** $(\sum_{i=1}^{n}(\frac{(y_i - x_i{}^T\beta - \xi_1 t_i)^2}{2\xi_2^2 t_i} + \boldsymbol{t_i}) + \boldsymbol{b})$

7- The Posterior distribution of parameter(λ)

The posterior distribution of parameter $\boldsymbol{\lambda_j}$ is a gamma distribution

with shape parameter $(\boldsymbol{q} + \boldsymbol{c})$ **and scale parameter** $(\prod_{i=1}^{q} \boldsymbol{I}\left\{\boldsymbol{\lambda_j} < \frac{w_j}{\sigma_j|\beta_j|}\right\})$

We build a good Gibbs sampling algorithm for the above parameters, with giving some initial values for $\boldsymbol{y}, \boldsymbol{z}, \boldsymbol{t}, \boldsymbol{\beta}, \boldsymbol{\sigma}$, and $\boldsymbol{\lambda}$ .Also give initial values for **hyper parameter (a, b, c, d)**

The our MCMC algorithm is run 13000 iterations , but burn-in 3000 first samples in order to stationary of samples MCMC. The our proposed algorithm was an fast and very efficient.

3. Simulation approach

4. In this paper, set of scenarios simulation approach ,for testing the efficiency of our method (BCQRegN lasso) has been used. The new our proposed method was compared with two other methods , like (BAYESIAN COMPOSITE QUANTILE REGRESSION WITH NEW LASSO refer to (BCQRegU)) that introduced by alhuseini ,fadel hamid hadi (2017), and (Bayesian Lasso Quan-

Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
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Reg that introduced by (Li et al. (2010)) .We run the MCMC algorithm in with three quantile levels ($\tau = 0.25, \tau = 0.50, \text{and } \tau = 0.95$) under various types of error distributions term, first error distribution is standard normal with mean zero and variance one, second error distribution is normal mature distribution $e_i \sim 0.5N(2,2) + 0.5N(-2,2)$, and third error distribution is t- distribution with 3 degree freedom $e_i \sim t_{(3)}$. For choosing best method , we will used (MMAD) where $MMAD = median(mean(|x^T\hat{\beta} - x^T\beta|))$

3.1 Sparse Case: In this case, we give the true vector $\beta = (1,0,2,0,1,0,0)^t$, then the regression model is

$$y_i^* = x_{i1} + 2x_{i3} + x_{i5} + e_i \qquad i=1,2,\ldots,100,$$

We generate the explanatory variables from multivariate $N_p(0, \Sigma_x)$ (p is number of independent variables)with $(\Sigma_x)_{kl} = 0.5^{|k-l|}$. The error distribution generated with the three various types . Three sample size used in this study . Below table shows MMAD values in sparse case.

Table 1.The (MMADs) for the simulation in sparse case.

	Comparison Methods		$e_i \sim N(0,1)$	$e_i \sim 0.5N(2,2) + 0.5N(-2,2)$	$e_i \sim t_{(3)}$
Sim1	n=50	$LassoN_{\tau=0.25}$	1.979 (0.872)	1.892 (0.865)	1.319 (0.737)
		$LassoN_{\tau=0.50}$	1.635(0.883)	1.562(0.873)	1.182(0.815)
		$LassoN_{\tau=0.95}$	1.041(0.793)	1.172(0.623)	1.003(0.568)
		BCQRegU	0.852(0.551)	0.973(0.461)	0.819(0.451)
		BCQRegN lasso	0.719(0.467)	0.636(0.511)	0.783(0.413)
	n=100	$LassoN_{\tau=0.25}$	1.878(0.948)	1.969(0.915)	1.573(0.892)
		$LassoN_{\tau=0.50}$	1.583(0.932)	1.461(0.896)	1.348(0.941)
		$LassoN_{\tau=0.95}$	1.263(0.852)	1.143(0.867)	1.045(0.897)
		BCQRegU	0.829(0.581)	0.750(0.474)	0.987(0.584)
		BCQRegN lasso	0.622(0.368)	0.704(0.484)	0.761(0.587)
	n=150	$LassoN_{\tau=0.25}$	1.963(0.986)	1.983(0.978)	1.813(0.968)
		$LassoN_{\tau=0.50}$	1.952(0.896)	1.872(0.902)	1.614(0.856)
		$LassoN_{\tau=0.95}$	1.452(0.768)	1.572(0.751)	1.314(0.652)
		BCQRegU	0.732(0.464)	0.642(0.419)	0.686(0.429)
		BCQRegN lasso	0.511(0.314)	0.484(0.289)	0.541(0.352)

Note: In the parentheses are SDs.

Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
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Table 1 presents the results for MMAD and SD for the three existing methods in the comparison under various types of error distributions and three various of sample size . It is Cleary from Table 1 that the performance of our method (BCQRegN lasso) appears, compared to the other two approaches, it is well. In general, the (MMAD) generated by our method (BCQRegN lasso) is smaller than the (MMAD) generated by other two methods (BCQRegU and $LassoN$) for all sample size and error distribution under consideration

3.2 dense Case: In this case, we give the true vector

$\beta = (0.85,0.85,0.85,0.85,0.85,0.85,0.85)^t$, then the regression model is

$$y_i^* = 0.85x_{i1} + 0.85x_{i2} + 0.85x_{i3} + 0.85x_{i4} + 0.85x_{i5} + 0.85x_{i6} + 0.85x_{i7} + e_i \quad i=1,2,\ldots\ldots,100,$$

We generate the explanatory variables from multivariate $N_p(0, \Sigma_x)$ (p is number of independent variables)with $(\Sigma_x)_{kl} = 0.5^{|k-l|}$. The error distribution generated with the three various types . Three sample size used in this study . Below table shows MMAD values in sparse case.

Table 2. The (MMADs) for the simulation in dense case.

	Comparison Methods		$e_i \sim N(0,1)$	$e_i \sim 0.5N(2,2) + 0.5N(-2,2)$	$e_i \sim t_{(3)}$
Sim2	n=50	$LassoN_{\tau=0.25}$	**0.969 (0. 607)**	**0. 914 (0.575)**	**0.921 (0.782)**
		$LassoN_{\tau=0.50}$	**0.838 (0.653)**	**0.855 (0.659)**	**0.749 (0. 580)**
		$LassoN_{\tau=0.95}$	**0.812 (0.549)**	**0. 756 (0.573)**	**0.729 (0.509)**
		BCQRegU	**0.682 (0.489)**	**0.597 (0.419)**	**0. 599 (0.453)**
		BCQRegN lasso	**0.424 (0.268)**	**0.480 (0.307)**	**0.413 (0.240)**
	n=100	$LassoN_{\tau=0.25}$	**0. 914 (0.736)**	**0.826 (0.715)**	**0.832 (0.665)**
		$LassoN_{\tau=0.50}$	**0.818 (0.616)**	**0. 813 (0.507)**	**0.802 (0.627)**
		$LassoN_{\tau=0.95}$	**0.745 (0.569)**	**0.849 (0.680)**	**0.739 (0.596)**
		BCQRegU	**0.581 (0.407)**	**0.599 (0.509)**	**0.521 (0.382)**
		BCQRegN lasso	**0.417 (0.295)**	**0.476 (0.294)**	**0.450 (0.323)**
	n=150	$LassoN_{\tau=0.25}$	**0.949 (0.781)**	**0.908 (0.749)**	**0.856 (0.715)**
		$LassoN_{\tau=0.50}$	**0.852 (0.679)**	**0.890 (0. 670)**	**0.8691 (0.662)**
		$LassoN_{\tau=0.95}$	**0.749 (0.547)**	**0.802 (0.527)**	**0.789 (0.519)**
		BCQRegU	**0.589 (0.408)**	**0.572 (0.381)**	**0.494 (0.306)**
		BCQRegN lasso	**0.411 (0.253)**	**0.352 (0.179)**	**0.465 (0.217)**

Note: In the parentheses are SDs.

Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
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Table 2 presents the results of the MMAD and SD for the three existing methods in the comparison under various types of error distributions and three various of sample size . It is Cleary from Table 2 that the performance of our method (BCQRegN lasso), compared to the other two approaches, it is well.

In general, the MMAD generated by our method (BCQRegN lasso)is smaller than MMAD generated by other two methods (BCQRegU and $LassoN$) for all sample size and error distribution under consideration.

figure-1- histogram of our proposed method (BCQRegN lasso) parameter

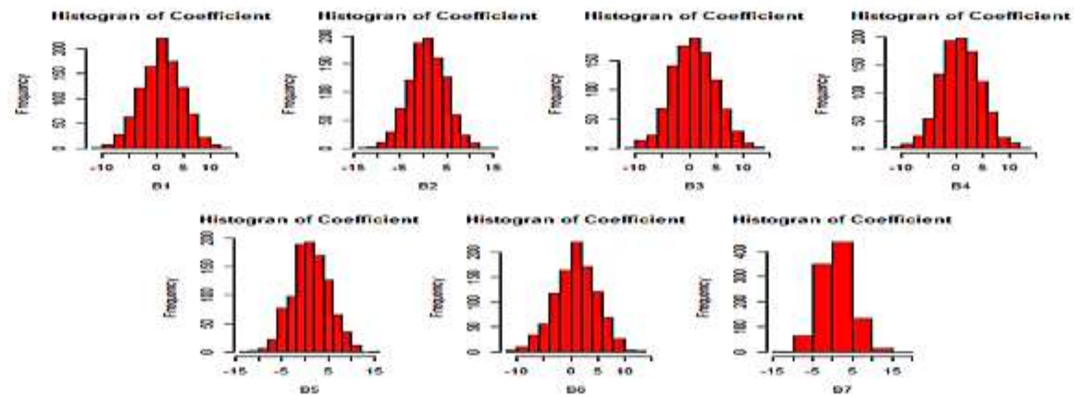

figure-1- histogram of our proposed method (BCQRegN lasso) parameter estimate for simulation in sparse case

Figure -2- Trace plots of our proposed method (BCQRegN lasso) for simulation in sparse case

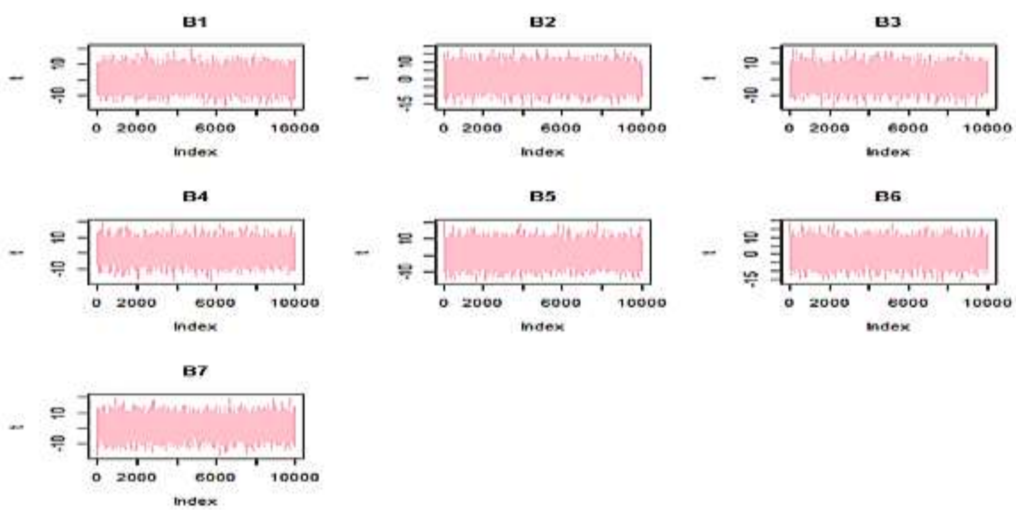

figure-3- Histogram of our proposed method (BCQRegN lasso) parameter estimate for simulation in dense case

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https://doi.org/10.31272/IJES2024.80.S.S4

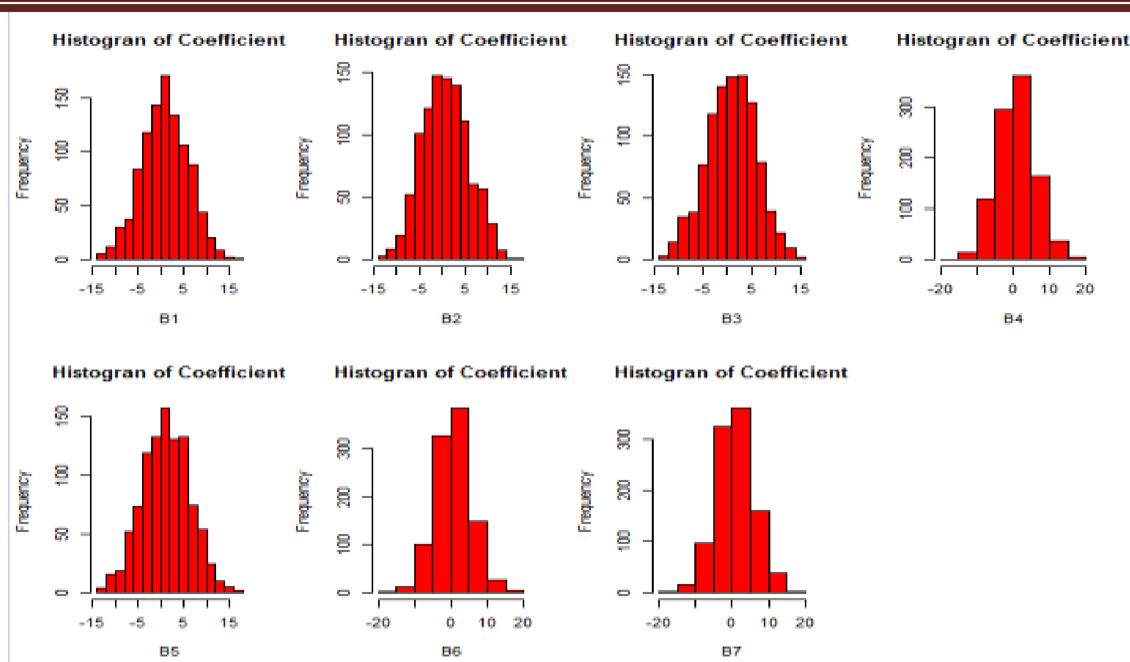
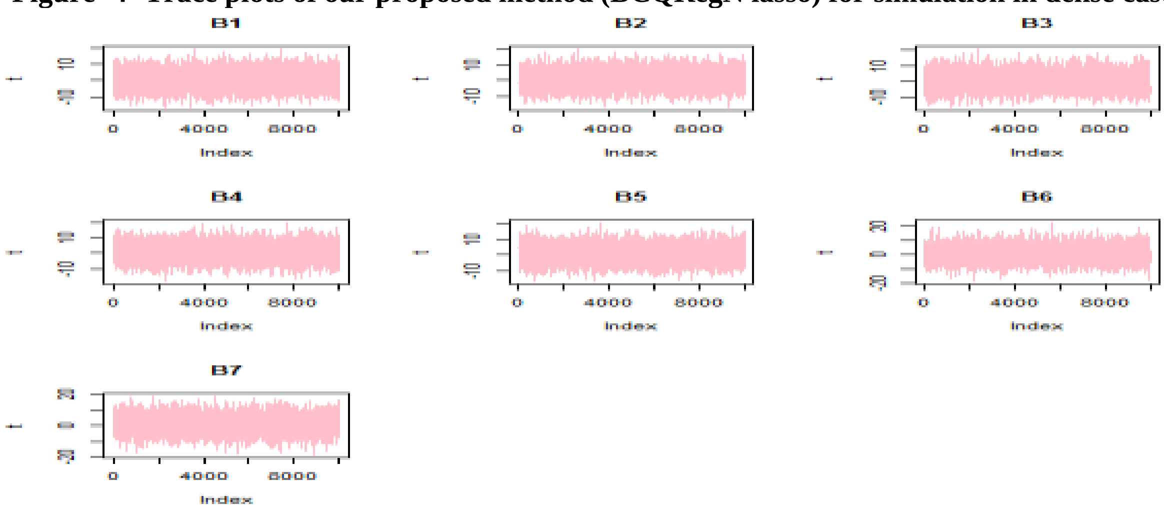


Figure -4- Trace plots of our proposed method (BCQRegN lasso) for simulation in dense case



Figures 1-4 shows the histogram graphs and Trace plots through two simulation examples. From the histogram graphs to parameters estimates that belong to our method (BCQRegN lasso) is very closed from histogram of normal distribution. from trace plot ,we see our MCMC samples of the posterior distribution was convergence to very stationary.

4. Air Pollution Dataset

three methods are being used for the analysis of the air pollution data using the quantile regression model. within the R program's (bayesQR) package contains the

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air pollution data (Lindmark and Karlsson, 2011). The Norwegian Administration of the Public Roads measured this dataset. The air pollution data consists of one independent variable and a set of independent variables as shows in plow table

table -3- Show the dependent variable and independent variables

Name variables	Symbol variables	Type variables
the log (concentration of NO2 per hour)	y	Dependent variable
log (number of cars per hour)	x_1	Independent variable
temperature	x_2	Independent variable
wind speed in meters per second	x_3	Independent variable
temperature difference	x_4	Independent variable
wind direction	x_5	Independent variable
time of day in hours	x_6	Independent variable
day number	x_7	Independent variable

In this section , we analyzed the above data by three methods shows in simulation section . To evaluating these methods under study mean absolute error (MAE) has been used, under three quantile levels $\tau \in \{0.25, 0.50\ and\ 0.95\}$. From results listed in table 4 , we see our method (BCQRegN lasso) have a good performance until with real data compared with other methods , because of the MAE is computed by our method (BCQRegN lasso)is smaller than MAE is computed by other two methods.

Table. 4 Mean absolute errors (MAE)for the our proposed method and other two methods.

Methods	$LassoN_{\tau=0.25}$	$LassoN_{\tau=0.50}$	$LassoN_{\tau=0.95}$	BCQRegU	BCQRegN lasso
MAE	34.432	31.732	28.971	12.452	9.591

From results listed in table -5- , we see the independent variables have positive and negative effective as showing the below tables

Table 5. Parameter estimates for our method (BCQRegN lasso) and other two methods.

	$assoN$			BCQRegU	BCQRegN lasso
Variables	$\tau_1 = 0.25$	$\tau_2 = 0.50$	$\tau_3 = 0.95$	Q=4	Q=4
Intercept	142.775	321.826	203.687	262.832	183.981
x_1	−2.613	−1.025	−3.762	-1.624	-2.526
x_2	0.178	0.142	0.062	0.002	0.000
x_3	0.000	0.000	0.000	0.000	0.000
x_4	−2.167	−1.831	3.762	0.523	1.230
x_5	0.054	0.071	0.542	0.000	0.000
x_6	0.032	0.089	0.215	0.672	0.452
x_7	0.039	0.167	0.362	0.341	0.521

Iraq Journal For Economic Sciences / ISSN:1812-8742 / ISSN ONLIN:2791-092X
https://doi.org/10.31272/IJES2024.80.S.S4

From results listed in table 5, in $LassoN$ method the variable (x_3) is unimportant in response variable ,but rest independent variables have importance in response variable under all quantile levels. But in the BCQRegU method there are two variables (x_3 and x_5) are unimportant in response variable .But in the BCQRegN lasso method there are three variables (x_2, x_3 and x_5) are unimportant in response variable.

5. Conclusion and Recommendation

5.1 Conclusion: This paper introduced a new contribution for Bayesian new lasso composite quantile regression with a new hierarchical . The our Gibbs sampler algorithm was simple to implement and effective compared with other algorithm . The our method (BCQRegN lasso) has a well performance compared with other methods in the same filed, this clear from simulation approach and real dataset.

5.2 Recommendation

The our method , Bayesian new lasso composite quantile regression with a new hierarchical (BCQRegN lasso)will encourage the researchers for developing other Bayesian composite quantile regression model through using a new formulation of Laplace distribution to presents a new formula provide as a good algorithm.

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