P- ISSN 1991-8941 E-ISSN 2706-6703 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access
2007,(1), (3 ) :96-102

# THE $\delta(E2/M1)$ MIXING RATIO AND $X(E0/E2)$ RATIO OF TRANSITIONS IN SOME OF SAMARIUM ISOTOPES

**A.M ASHIJ**

DEPT. OF PHYSICS -COLLEGE OF SCIENCE- UNIVERSITY OF AL-ANBAR

**ARTICLE INFO**

Received: 15 / 4 /2006
Accepted: 20 / 8 /2007
Available online: 14/6/2012
DOI: 10.37652/juaps.2007.15650

**Keywords:**
$\delta(E2/M1)$,
$X(E0/E2)$,
Transitions ,
Samarium Isotopes.

**ABSTRACT**

Samarium isotopes (Z=62) lies in the traditional rotational to transitional-spherical region that occurs at the range of deformed nuclei. Gamma ray $\delta(E2/M1)$ mixing ratio and $X(E0/E2)$ ratio for selected transitions in Sm148-154 are calculated in the frame work of proton-neutron interacting boson model (IBM-2). The results obtained for Samarium isotopes are reasonably in a good agreement with the previous experimental results.

## Introduction

Naturally occurring samarium is composed of 4 stable isotopes,Sm144, Sm150, Sm152, Sm154 and three radioisotopes Sm147, Sm148 and Sm149, with Sm152 being the most abundant (26.75%) [1] .These isotopes Z=62 and N=82 to 92 are well abrupt changes in nuclear properties between almost spherical in N=82 to well deformed in N=92 [2]. So Sm isotopes have provided a useful testing ground for nuclear structure calculations.

The Interacting Boson Approximation (IBA) model [3-4] has been remarkably successful in the description of the low-lying collective states in many medium to heavy even-even nuclei. The neutron-proton version of the interacting boson model (IBA-2), later suggested by Iachillo and Arima [5] which distinguishes between neutron $(\nu)$ and proton $(\pi)$, is used in the present work, a full description of the IBA-2 is found in reference [6].

* Corresponding author at: DEPT. OF PHYSICS -COLLEGE OF SCIENCE- UNIVERSITY OF AL-ANBAR, Iraq.E-mail address: Monean_m_957@yahoo.com

The Hamiltonian operator in IBA-2, which been used to calculated the energy level and hence the gamma transitions matrix elements, has three parts, one for proton bosons, one for neutron bosons and one that describes the interaction between unlike bosons:

$$H = H_\pi + H_\nu + H_{\pi\nu} \quad (1)$$

The Hamiltonian generally used in phenomenological calculations can be written as

$$H = \varepsilon_d(n_{d\nu} + n_{d\pi}) + \kappa(Q_\nu . Q_\pi) + V_{\nu\nu} + V_{\pi\pi} + M_{\nu\pi} \quad (2)$$

where the dot denoted the scalar product. The first term represents the single-boson energies for neutron and protons, $\varepsilon_d$ is the energy difference between s- and d- boson and $n_{d\rho}$ is the number of d-bosons, where $\rho$ correspond to $\pi$ (proton) or $\nu$ (neutron) bosons. The second term denotes the main part of the boson-boson interaction, i.e. the quadrupole-quadrupole interaction between neutron and proton bosons with the strength $\kappa$. The quadrupole operator is

$$Q_\rho = [d_\rho^{+} s_\rho + s_\rho^{+} d_\rho]^{(2)} + \chi_\rho [d_\rho^{+} d_\rho]^{(2)} \quad (3)$$

where $\chi_\rho$ determines the structure of the quadrupole operator and is determined empirically. The square bracket in Eq. (3) denotes angular momentum coupling.

The terms $V_{\pi\pi}$ *and* $V_{\nu\nu}$,in equation (2) which correspond to interaction between like-boson, are sometimes included in order to improve the fit to experimental energy spectra. They are of the form

$$V_{\rho\rho} = \frac{1}{2} \sum_{L=0,2,4} C_L^{\rho} ([d^+{}_\rho d^+{}_\rho]^{(L)}.[d_\rho d_\rho]^{(L)}). \tag{4}$$

However, their effects are usually considered minor and often neglected [7].The Majorana term, $M_{\nu\rho}$, which contains three parameters $\xi_1$ , $\xi_2$ *and* $\xi_3$ may be written as

$$M_{\nu\pi} = \frac{1}{2}\xi_2([s_\nu^+ d_\pi^+ - d_\nu^+ s_\pi^+]^{(2)}.[s_\nu d_\pi - d_\nu s_\pi]^{(2)}) - \sum_{k=1,3} \xi_k([d_\nu^+ d_\pi^+]^{(k)}.[d_\nu d_\pi]^{(k)}). \tag{5}$$

This work aimed at two thinks: first, to give the Hamiltonian of IBA-2 in terms of the formalism; second is to study the transitions probability and mixing ratio of Sm isotopes by use of this Hamiltonian.

## The Calculation and Results

The isotopes chosen in this work are A=148,150, 152, 154 due to the presents of experimental data for the mixing ratios and X(E0/E2) values. We have $N_\pi = 6$, (12 protons outside the closed shell 50), and $N_\nu$ varies from 2 for Sm148 to 5 for Sm154, measured from the closed shell at 82. While the parameters $\kappa, \chi_\rho$, *and* $\varepsilon_\rho$ ,as well as the Majorana parameters $\xi_k$, with k=1,2,3, were treated as free parameters and their values were estimated by fitting with the experimental values. The procedure was made by selecting the traditional value of the parameters and allowing one parameter to vary while keeping the others constant until the best fit with the experimental obtained. This was carried out until one overall fit was obtained. The best values for the Hamiltonian parameters are given in table-1.

Concentration was made on the 21+ to make a reasonable fit to experimental data. A sample of experimental and theoretical decay scheme is presented in figure-1. As one can see an over all a good agreement was obtained for the gamma and beta bands for Sm152. The results in figure-2 show a comparison between experimental and theoretical energy levels of the ground band in Sm isotopes, the agreement is very good for the 21 and 41, but the model does not able to predict the 61 and this may be due to the high spin of this state. Actually this has slim effects on calculations of transitions probability.

Ichillo and Arima in their original interacting boson approximation give the M1 operator in the restricted case of U(5) dynamic symmetry [3] and as well as the general case used by Sholton [9]. How ever, even when starting with general operator, they derived the E2 / M1 mixing ratio by neglecting the term which break the SU(3) symmetries. It follows that the reduced mixing ratio is given by the same simple formula which contains only one parameter and the initial and final spin, which can be derived as fallows:

In IBA-2, the E2, transition operator is given by,

$$T^{(E2)} = e_\pi Q_\pi + e_\nu Q_\nu \tag{6}$$

where $Q_\rho$ is the same as in equation (3) and $e_\pi$ *and* $e_\nu$ are boson effective charges depending on the boson number $N_\rho$ and they can take any value to fit the experimental results. The method explained in reference [10]. The effective charges calculated by this method for Sm isotopes were $e_\nu = 10.0 e\ fm^2\ and\ e_\pi = 13.0 e\ fm^2$ .

After calculated the E2 matrix elements we lock after the M1 matrix elements as fallows:

The M1 operator obtained by letting $l = 1$ in the single boson operator of the IBM-1 and can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2}(g_\pi L_\pi^{(1)} + g_\nu L_\nu^{(1)}) \qquad (7)$$

where $g_\pi, g_\nu$ are the boson g-factors in units of $\mu N$ and $L^{(1)} = \sqrt{10}(d^+ x\tilde{d})^{(1)}$ . This operator can be written as

$$T^{(M1)} = \left[\frac{3}{4\pi}\right]^{1/2}\left[\tfrac{1}{2}(g_\pi + g_\nu)(L_\pi^{(1)} + L_\nu^{(1)}) + \tfrac{1}{2}(g_\pi - g_\nu)(L_\pi^{(1)} - L_\nu^{(1)})\right] \qquad (8)$$

The first term on the right hand side ,in the above equation, is diagonal and therefore for M1 transitions the previous equation may be written as

$$T^{(M1)} = 0.77\left[(d^+\tilde{d})_\pi^{(1)} - (d^+\tilde{d})_\nu^{(1)}\right](g_\pi - g_\nu) \qquad (9)$$

The direct measurement of B(M1) matrix elements is difficult normally, so the M1 strength of gamma transition may be expressed in terms of the multipole mixing ratio which can be written as [11]

$$\delta(E2/M1) = 0.835 E_\gamma (MeV).\Delta \qquad (10)$$

where $$\Delta = \frac{\langle I_f \| T^{E2} \| I_i \rangle}{\langle I_f \| T^{M1} \| I_i \rangle} \quad in\ \ eb/\mu N$$

Having fitted E2 matrix elements, one can then use them with to obtain M1 matrix elements and then the mixing ratio $\delta(E2/M1)$ ,compare them with the prediction of the model using the operator ( 9 ). The $g_\pi\ and\ g_\nu$ have to be estimated, if they not been measured in the case of Sm isotopes. The g factors my be estimated from experimental magnetic $(\mu)$ moment of the $2^+_1$ state (μ=2g). In phenomenological studies $g_\pi\ and\ g_\nu$ are treated as parameter and kept constant for a whole isotope chain [1]. The total g factor defined by Sambataro et.al. [13] as

$$g = g_\pi \frac{N_\pi}{N_\pi + N_\nu} + g_\nu \frac{N_\nu}{N_\pi + N_\nu} \qquad (11)$$

Many relations could be obtained for a certain mass region and then the average $g_\pi\ and\ g_\pi$ values for this region could be calculated. One of the experimental B(M1) and the relation above been used to find that $g_\pi - g_\nu = 0.53 \mu N$ .The estimated values of the parameter are $g_\pi = 0.84 \mu N\ and\ g_\nu = 0.31 \mu N$ ,these were used to calculate the ratio $\Delta(E2\backslash M1)$ and then the mixing ratio $\delta(E2\backslash M1)$. The ratios were calculated for some selected transitions and listed with the available experimental data in table-2.

**<u>The X( E0/ E2) Ratio</u>**

Monopole transitions (E0) are known to be pure penetration effect, where the transition is caused by an electromagnetic interaction between the nuclear charge and the atomic electron penetrating the nucleus. An E0 transition occurs between two states of the same spin and parity by transferring the energy and zero unit of angular momentum. Thus E0 has no competing gamma ray. These transitions are different from zero only in the case where the transition is accompanied by the nucleus surface change. For example in the nuclear models where the surface is assumed to be fixed E0 transitions are strictly forbidden. Electric monopole transitions can occur not only in 0+--- 0+ transition but also, in competition with gamma multipole transition and depending on transition selection rules may compete in any $\Delta I = 0$ decay such as a 2+ --- 2+. At transitions energies greater than $2m_o c^2$, monopole pair production is also possible.

The E0 reduced transitions probability written as [14]

$$B(E0; I_i - I_f) = e^2 R^4 \rho^2(E0) \qquad I_i = I_f \tag{12}$$

where e in the electronic effective charge, R is the nuclear radius and $\rho(E0)$ is the transition matrix element. However, there are only limited cases where $\rho(E0)$ can be measured directly. In most cases we have to determine the intensity ratio of E0 to the competing E2 transition calling this as $X(\frac{E0}{E2})$ value [15] which can be written as

$$X(E0/E2) = \frac{B(E0; I_i - I_f)}{B(E2; I_i - I_{f'})} \tag{13}$$

where $I_f = I_{f'}$ for $I_i \neq 0, and\ I_f = 0, I_{f'} = 2\ for\ I_i = 0.$

The $T^{(E0)}$ operator my be found by setting $l = 0$ on the IBM-2 operator [16]

$$\rho_{if}(E0) = \frac{Z}{R_0^2} \sum \tilde{\beta}_{0\rho} \left\langle f \middle| d^+{}_\rho x d_\rho \middle| i \right\rangle \tag{14}$$

where R0=1.2A1/3 fm and $\rho(E0)$ is a dimensionless quantity. The two parameter $\tilde{\beta}_{0\pi}, \tilde{\beta}_{0\nu}$ in equation(14) may be estimated by fitting in isotope shift, which is the difference in the square radius $\delta\langle r^2 \rangle$ between neighboring isotopes in their ground state [17].i.e.

$$\delta\langle r^2 \rangle = \langle 2_1^+ | T_0 | 2_1^+ \rangle - \langle 0_1^+ | T_0 | 0_1^+ \rangle. \tag{15}$$

In the case of Samarium isotopes the measured isotopes shift [18] were used to fined the parameters used in the IBM-2 calculations and they are $\tilde{\beta}_\pi = 0.045\ fm^2$ and $\tilde{\beta}_\nu = 0.031\ fm^2$ produced the monopole matrix elements. The results of the calculation are listed in table-3.

Discussion and Conclusions

Good agreements with the experimental energy for the low laying levels was obtained and according to that the mixing ratios were calculated after calculate E2 and M1 matrix elements. All the experimental and theoretical mixing ratios for Sm isotopes indicate a small M1 components which means that in the band-mixing transitions, M1 components is almost forbidden. In the calculation of $\delta(E2/M1)$ it is found that there is a great effect of the Majorano parameter $\xi_2$ on the value and sign of E2 and M1 matrix elements. However, an acceptable overall agreement between the experiment and theory were obtained as shown in table-2.The lack of agreement indicates that the perturbation expansion of angular momentum works well for strongly deformed nuclei and does not describe nuclei in the transitional region. The model was not able to produce the sing of the mixing ratio of the transition from gamma to ground band.

The present study provides further support for the idea of the co-existence of spherical vibrator in Sm148 to deformed- rotor characteristics in the N=88 isotones of Sm, Gd and Nd as been shown by Gupta and Kumar [19]

The E0 quantities predicted by the two theories for 03+ states were on the whole in poor agreement with experiment. In general we note that the E0 transition matrix elements provide a highly critical test of nuclear model. Furthermore, the microscopic characters of the ground states and the excited 0+ states in Sm150-152 would appear to be sufficiently complex to eliminate any description of them in the term of simple model. We notice that most of the experimental and theoretical values for the $X(E0/E2)$ ratio are small, which means that there is a small contribution of E0 transition on the life time of the $0^+$ states. There are two high values of X in transitions from $0_{\beta\beta}$ to $0^+{}_g$ in Sm148 and in Sm152 means that this state decay mostly by the E0 and according to this one could say that the study of ths state give information about the shape of the nucleus, because the E0 transitions matrix elements connected strongly with the penetration of the atomic electron to the nucleus. So combination of the wave function of atomic electron, which is well known, and the nuclear surface give good information of the nuclear shape.

**References**


1. N.N. Greenwood, A. Earnshaw, "Chemistry of elements", Program Press, Oxford, UK.(1984).
2. Passoja, J. Kantele, P. Toivonen and P. lipas, J. phys. G: Nucl. Phys.12, 1047, (1986)
3. F. Iachello and A. Arima, Ann. Phys, NY, 111, 253, (1976).
4. F. Iachello and A. Arima, Ann. Phys, NY, 111, 201, (1978).
5. F. Iachillo and A. Arima, " The interacting Boson model", Camb. Un. Press., (1987)
6. W.Pfeifer "An Interacting Boson IBM), vdf hochschulverlag- Zurich. (1998)
7. D.S. Mosbah, J.A. Evans and W.D. Hamilton, J.Phys. G: Nucl. Phys.,20, 787, (1994)
8. M. Lederer and V. Shirley. " Table of Isotopes", 7ed, Wiley and sons, New York, (1978)
9. O. Sholton, PVI Report, 63, (1990)
10. A.R.H. Subber, P. Park and W.D. Hamilton, J. Phys. G:Nucl. Phys.,13, 161, (1987)
11. J. Lange, K. Kumar and J.H. Hamilton, Rev. of mod. Phys., 54, 119, (1982)
12. M.Barlan and M.A. Atlihan, Turk. J. Phys., 26, 305, (2002)
13. M. Sambatora, O. Scholton, A.Dieperink and G. Piccitto, Nucl. Phys., A423, 333, (1984)
14. E.L. Church and J. Weneser, Phys. Rev., 103, 1035, (1956)
15. J.O. Rasmissen, Nucl. Phys. 19, 85, (1960)
16. P.O.val and D.R. Barret, Phys. Rev. C23, 492, (1981)
17. R.bije R, A. Dieperink and O.Scholton , Nucl. Phys., A344, 207, (1980)
18. J.G. England, I.S.Grant, J.A.R. Griffith, D.E.evans and P.M. Walker, j.Phys. G: Nucl. Part.phys., 16, 105, (1990)
19. J.B. Gupta and K. kumar, J.Phys. G: Nucl. Phys., 7, 673, (1981).


**Table-1. The IBM-2 parameters used in this work**

| *Isotope* | $\varepsilon(MeV)$ | $X_\nu$ | $\kappa$ |
|---|---|---|---|
| **$Sm^{148}$** | **0.80** | **-0.8** | **-0.13** |
| **$Sm^{150}$** | **0.74** | **-0.8** | **-0.06** |
| **$Sm^{152}$** | **0.43** | **-0.8** | **-0.023** |
| **$Sm^{154}$** | **0.34** | **-0.8** | **-0.039** |
| $X_\pi = -1.2$ | $\xi_1 = \xi_3 = 0.12 \, and \, \xi_2 = 0.1 MeV$ | $C_\pi^L = C_\nu^L = -0.5, 0.4, -0.8 MeV$ | |

**Table-2. Experimental and theoretical $\delta(E2 \backslash M1)$ multipole mixing ratios of Samarium isotopes.**

| **Isotope** | **Spin** $I_i \rightarrow I_f$ | **Transition** $E_\gamma$ **(MeV)** | **Mixing Ratio** $\delta(E2/M1)$ This work | Experimental |
|---|---|---|---|---|

| | | | | |
|---|---|---|---|---|
| $Sm^{148}$ | $2_\beta \rightarrow 2_g$ | 904 | +5.22 | - |
| | $2_\gamma \rightarrow 2_g$ | 1114 | -1.33 | - |
| | $3_\gamma \rightarrow 2_g$ | 1182 | -3.66 | - |
| $Sm^{150}$ | $2_\beta \rightarrow 2_g$ | 712 | -9.33 | -4.8 (+0.5;-0.4) |
| | $2_\gamma \rightarrow 2_g$ | 860 | -2.11 | +3.6 (+1.7;-1.0) |
| | $3_\gamma \rightarrow 2_g$ | 1171 | 16.45 | 13 (+ ∞ ;-7) |
| $Sm^{152}$ | $2_\beta \rightarrow 2_g$ | 689 | -10.34 | -9.6(3) |
| | $2_\gamma \rightarrow 2_g$ | 965 | +0.48 | - |
| | $3_\gamma \rightarrow 2_g$ | 1113 | -2.34 | -6.5(3) |
| $Sm^{154}$ | $2_\beta \rightarrow 2_g$ | 1069 | +3.55 | - |
| | $2_\gamma \rightarrow 2_g$ | 1194 | +0.34 | |
| | $3_\gamma \rightarrow 2_g$ | 1457 | -7.35 | - |

**Table-3. Calculated X(E0/E2) ratios compare with experimental data in even Samarium isotopes.**

| Isotope | Initial level (KeV) | $I_i$ | $I_f$ | X(E0/E2) Experimental | X(E0/E2) Theoretical |
|---|---|---|---|---|---|
| $Sm^{148}$ | 1427 | $0^+_\beta$ | $0^+_g$ | | 0.013 |
| | 1454 | $2^+_\gamma$ | $2^+_g$ | | 0.009 |
| | 1664 | $2^+_\beta$ | $2^+_g$ | | 0.0005 |
| | 1923 | $0^+_{\beta\beta}$ | $0^+_g$ | | 0.13 |
| $Sm^{150}$ | 740 | $0^+_\beta$ | $0^+_g$ | $1.23 (14)x10^{-2}$ | 0.0078 |
| | 1046 | $2^+_\beta$ | $2^+_g$ | | 0.0006 |
| | 1193 | $2^+_\gamma$ | $2^+_g$ | | 0.0067 |
| | 1255 | $0^+_{\beta\beta}$ | $0^+_g$ | $8.3 (8)x10^{-2}$ | 0.09 |
| $Sm^{152}$ | 684 | $0^+_\beta$ | $0^+_g$ | $9.4 (17)x10^{-3}$ | 0.0056 |
| | 810 | $2^+_\beta$ | $2^+_g$ | | 0.0008 |
| | 1082 | $0^+_{\beta\beta}$ | $0^+_g$ | 14(4) | 1.2 |
| | 1085 | $2^+_\gamma$ | $2^+_g$ | | |

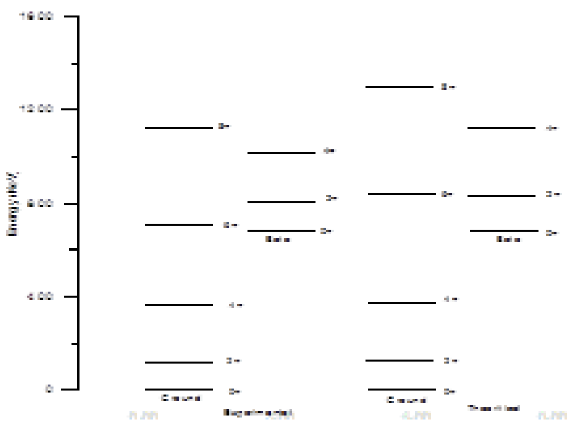

**Fig-1. The experimental and theoretical energy levels of $Sm^{152}$. Experimental values are from ref. [8]**

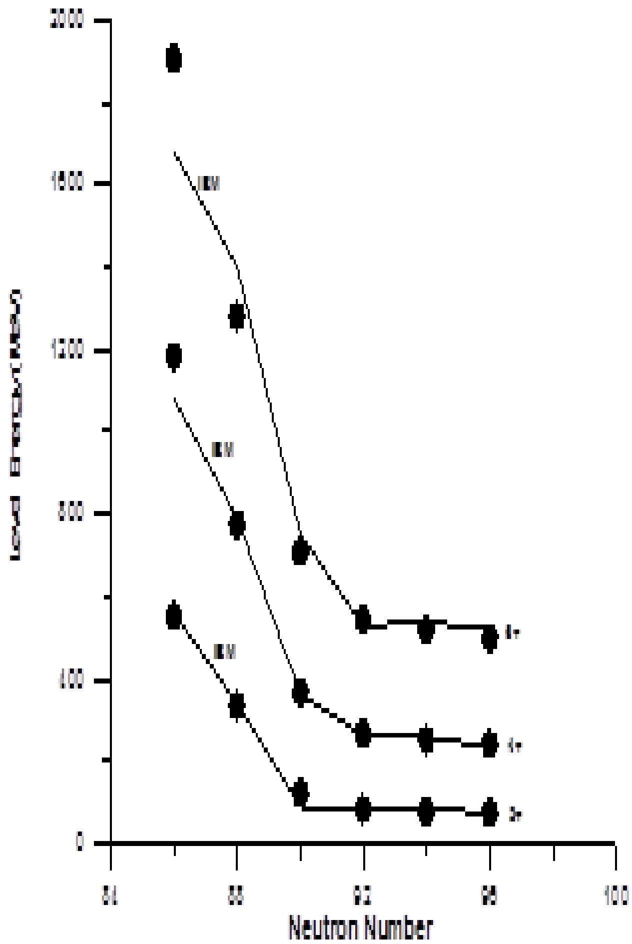


**Fig-2. A comparison between experimental [8] and IBM-2 calculated values of the ground band levels in even- even Sm isotopes**

P- ISSN 1991-8941 E-ISSN 2706-6703 2007,(1), (3 ) :96-102 *Journal of University of Anbar for Pure Science (JUAPS)* Open Access

# نسبة الخلط في $\delta(E2/M1)$ و $X(E0/E2)$ لبعض الانتقالات في نظائر السماريوم.

**عبد المنعم مصلح عاشج**

قسم الفيزياء- كلية العلوم– جامعة الانبار
*E-mail*: Monean_m_957@yahoo.com

**الخلاصة**

نظائر السماريوم (Z=62) تقع في المنطقة المعروفة ذات الأنتقال من ألأنوية دائمة التشوه الى الأنوية الانتقالية وثم الى الأنوية الكروية في المدى المعروف للأنوية الثقيلة دائمة التشوه. نسبة الخلط $\delta(E2/M1)$ لانحلال كاما وكذلك نسبة $X(E0/E2)$ لمجوعة مختارة من النظائر قد حسبت باستخدام نموذج البوزونات المتفاعلة الثاني (IBM-2) وتم الحصول على نتائج نظرية و مقارنتها مع النتائج العملية السابقة وكانت نسبة التوافق مقبولة.