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H-C and H*-C Semi compactness in bitopological- space

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A R T I C L E I N F O

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A B S T R A C T

In this paper we introduce two new concepts, namely H-C-Semi compact and H*- C-Semi compact in bitopological space several propositions and examples about these concepts are introduced.

Introduction

Let X be a non empty set. Let T1 and T2 be two topolgies on X then the triple (X,T1,T2) is called a bitopological space, this concept was first introduced by Kelly [1]
In this work, introduce new concepts namely H-C- Semi compact and H* -C semi compact in bitopological space.

Preliminaries

2-1 Remarks
i) If T1 is a topology on X and T2 is also a topology on X then T1 $\cup$ T2 is not necessarily a topology on X
ii) (T1 $\cup$ T2) means the topology on X generated by T1 $\cup$ T2
Definition : [2]
Let (X, T1, T2) be a bitopological- space let A $\subseteq$ X , we say that A is N-open if and only if is open in the space (X , T3) where T3 = (T1 $\cup$ T2) is the supremum topology on X containing T1 and T2 . A $\subseteq$ X is called S- open if and only if it is T1–open or T2 - open
2-3 Remarks and Example [2]
i) The complement of N- open (S- open) is called N - closed (S- closed)
ii) Each S-open in (X, T1 ,T2) is N- open but the converse is not necessarily true
iii) X= {a, b , c}
T1={Ø, X , {a}}
T2={ Ø , X , {b}}
(T1 $\cup$ T2) = {Ø , X , {a} , {b} , {a,b}}.
{a,b} is N- open but not S - open
{a} is S- open , hence it's also N- open
{ a , b}c = {c} N-closed
{ a }c = {b, c} S-closed
Definition [2] , [3]
A bitopological - space (X, T1, T2) is called N –compact (S- compact) if every N- open cover (S- open cover) of X has a finite subcover.

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Definition [2] , [3]
Let (X ,T) be a toplogical - space we say that X is C- compact if : for each closed set A $\subseteq$ X
And each open cover
F = {W$^{\alpha}$ | $\alpha \in \Omega$ } of A, there exists
α_1 , α_2, …., α_n such that A $\subseteq \overline{W}_{\alpha_1} \cup \overline{W}_{\alpha_2} \cup \ldots \overline{W}_{\alpha_n}$
(that is, there exist a finite sub family whose closures covers A)
3-H-C-Semi- compact- space
In this section, we introduce the concept of H-C- Semi-compact space several properties of this concept are proved
First, we introduce the following definition
Definition
Let (X , T1 , T2) be a bitopological–space let A $\subseteq$ X , we say that A is H- semi open in X iff it is semi - open in the space (X , (T1 $\cup$ T2))
Remarks and examples
i) The complement of H-semi- open is called H- semi-closed.
ii) Every N- open is H- semi- open but the converse is not necessarily true.
iii) Let X= {a ,b,c}
T1= { Ø, X, {a} }
T2={ Ø, X }
(T1 $\cup$ T2) = { Ø ,X ,{a}}=T1
{ a } is N – open , Hence it will be H-semi – open
{a}c= { b, c} is H- semi – closed
Consider A={ a , c } , A is
H-semi – open but A is not N-open
Definition
Let (X ,T1 , T2) be abitopological – space
Let A $\subseteq$ X
F = { W$^{\alpha}$| $\alpha \in \Omega$ } is called
H-semi – open cover of A if
1- W$^{\alpha}$ is H- semi-open in X for each $\alpha \in \Omega$

2- A $\subseteq \bigcup_{\alpha\in\Omega} W\alpha$

Definition

i) A bitopological space (X , T1 , T2) is called H-semi compact iff every
H-semi open cover of X has a finite sub cover

ii) let A $\subseteq X$, We say that A is H- semi compact iff every
H-semi open cover of A has a finite sub cover

Definition

Let (X , T) be a topological-space, X is called C-semi compact if :

Given a semi–closed subset A $\subseteq$ X and given a semi –open cover
F = {Wα | $\alpha \in \Omega$ } of A
Then there exist

α1 , α2, ……., αn Such that A $\subseteq \overline{W}\alpha_1 \cup \overline{W}\alpha_2 \cup \ldots \overline{W}\alpha_n$

Definition

let (X , T1 , T2) be abitopological space we say that X is H-C-semi compac if given H – semi closed set A $\subseteq$ X and given
F = { Wα| $\alpha \in \Omega$ } where F is an H – semi open cover of A
then there exit α1 , α2, …, αn such that
A $\subseteq$ (H – scl Wα1) $\cup$(H – scl Wα2) $\cup \ldots \cup$ (H – scl Wαn) (where H- scl Wα = the smallest H-semi closed Set containing Wα1)

Proposition

Every H- semi closed sub set of H-semi compact space is H-semi compact

Proof

Let (X ,T1 ,T2) be H-semi compact and let A $\subseteq$ X be H-semi closed subset of X let F = {Wα | $\alpha \in \Omega$ } be an H-semi open cover of A.
Now A is H-semi closed, so AC =X-A is H - semi open
Now F* = F $\cup$ { Ac} is an H-semi open cover of X , but X is H-semi compact so $\exists$ α1 , α2, …, αn
such that
X= Wα1$\cup$Wα2$\cup \ldots \cup$Wαn $\cup$ Ac Hence A$\subseteq$ Wα1 $\cup$ Wα2 $\cup \ldots \cup$ Wαn , which means that A is H- semi compact.
The proof of the following proposition is clear .

3.8 Proposition :

i) Every compact space is C-compact
ii) Every semi compact space is C-semi compact
iii) Every H-semi compact space is H-C- semi compact .
iv) the converse of (iii) is not necessarily true .
as show by the following example.
Let (N ,T1 , T2) be a bitopological space where N= the set of natural numbers
T1= the indiscrete topology on N T2= F $\cup$ { N ,Ø }
where F= { Wn | Wn = {1, 2,…, n} , n $\in$ N }
Now (N , T1 , T2) is H- C- semi compact but not H- semi compact

3.9 Proposition

Let (X , T1 , T2) be H-C-semi compact then (X , T1) and (X ,T2) are C- semi compact space.

Proof

Let A $\subseteq$ (X , T1) be semi closed and let F={ Wα|$\alpha \in \Omega$ } be a T1 semi open cover of A .Now A is H-semi closed subset of (X , T1 , T2) and F is an H-semi open cover of A
But (X, T1, T2) is H – C- semi compact so $\exists$ α1 , α2, …, αn such that
A$\subseteq$(H – scl Wα1) $\cup$(H – scl W α2) $\cup \ldots \cup$ (H – scl Wαn) . Now H – scl Wα1 $\subseteq$ T1- scl Wα1

H – scl Wα1 $\subseteq$ T1- scl Wαn

So A $\subseteq$ (T1 – scl Wα1)$\cup \ldots \cup$
(T1 – scl Wαn)
So (X ,T1) is C-semi compact
Similarly we prove that (X , T2) is C- semi compact .

3-10 Remark

The converse of proposition (3-9) is not necessarily true as shown in the following

Example

Let (N, T1,T2) be a bitoplogical space, let T1=P(O+) $\cup$ { N }
and T2 = P(E +) $\cup$ { N}
where P (O +) is the power set of O+ = set of all odd natural numbers and P(E+) is the power set of
E+ = set of all even natural numbers then (N , T1) and (N , T2) are C- semi compact but (N , T1 , T2) is not H-semi compact space

3-11 Definition

Let f: (X, T1,T2) → (Y, T'1,T'2)
Be a function , we say that if is H- semi continuous if the inverse image of H- semi open set in Y is H-semi open in X

3-12 Proposition:

The H- semi continuous image of an H-C-semi compact space is also H-C- semi compact

Proof :

Let (X, T1, T2) be H- C- semi compact we have to prove that (Y, T'1, T'2) is also H-C- semi compact
Let A $\subseteq$ Y be H- semi closed Now B= f-1(A) is H-semi closed in X
Let F = { Wα| $\alpha \in \Omega$ } be an H- semi open cover of A
Hence F*={f-1 (Wα | $\alpha \in \Omega$ } is an H-semi open cover of
B= f-1 (A)
But X is H – C- semi of compact
So $\exists$ α1 , α2, ……., αn ∍

B$\subseteq$ (H – scl f-1 Wα1)
$\cup \ldots \cup$ (H – scl f-1 Wαn)
Hence

A= f (B) $\subseteq$ (H – scl Wα 1)
$\cup$ … $\cup$ (H – scl Wα n)
Hance Y is H-C-semi compact

4- H*- C- compact space

In this section we introduce the concept of H*- C- compact space

Definition

A sub set A of bitopological space (X, T1 , T2) is said to be H*- sime open if it is T1 – semi open or T2 semi open

The complement of H*- sime open set is called H*- semi closed

Definition

Let (X , T1 , T2) be abitopological space , let A $\subseteq$ X. A sub collection of the family T1 $\cup$ T2 is called H* - semi open cover of A if the union of members of this sub collection contains A.

Definition

A bitopological space(X, T1, T2) is said to be H*- semi compact if every H*- semi open cover of X has finite sub cover .

4-4 Definition

A bitopological space (X , T1,T2) is said to be H*-C- semi compact if give H*- semi closed set A $\subseteq$ X and given F = { Wα | $\alpha \in \Omega$ } Which is H*- semi open cover of A then $\exists$ α 1 , α 2, …, α n such that

A $\subseteq$ (H*-sclWα 1) $\cup \cdots \cup$ (H*-scl Wα n)

The proof of the following propositions is similar to the previous one.

Proposition

Every H*- semi closed subset of H*-semi compact space is H*-semi compact.

Proposition

Every H*- semi compact space is H*- C - semi compact.

Proposition

Let (X,T1,T2) be an H*- C- semi compact space, then (X, T1) and (X, T2) are both C- semi Compact

Example

Let(X,T1,T2) be a bitopological space where X= [0,1] and

T1= {X, Ǿ,{0}} $\cup$ {[0, $\frac{1}{n}$)|n$\in N$ } T2={X, Ǿ, (0,1]} $\cup$ { ($\frac{1}{n}$, 1]| n$\in N$ } Then (X,T1) and (X,T2) are C-semi compact but (X ,T1,T2) is not H*- C- semi compact

Remark

Let (X, T1,T2) be a bitopological space and let A $\subseteq$ X then

i) H-scl A $\subseteq$ H*-scl A

ii) H- Scl A $\subseteq$ T1-scl A

iii) H- scl A $\subseteq$ T2- scl A

The proof of the following proposition is clear

4-10 proposition

Every H-C-semi compact space is H*-C-semi compact

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الفضاءات شبه المرصوصة H-C والفضاءات شبه المرصوصة H*-C

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الخلاصة:

في هذا البحث قمنا بتعريف أنواع جديدة من الفضاءات شبه المرصوصة على الفضاءات الثنائية وقد أسميناها ((الفضاءات شبه المرصوصة -C H)والفضاءات شبه المرصوصة H*-C)) وقمنا بدراسة بعض خواص هذه الفضاءات ودراسة العلاقة بينهما