P- ISSN 1991-8941 E-ISSN 2706-6703
2011,(5), (1) :52-55

Journal of University of Anbar for Pure Science (JUAPS)

Open Access

Some Types of Lindelof in Bitopological Spaces

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ARTICLE INFO

Received: 2 / 9 /2010
Accepted: 14 / 3 /2011
Available online: 14/6/2012
DOI: 10.37652/juaps.2011.15408

Keywords:
Lindelof ,
Bitopological
Spaces.

ABSTRACT

In this paper, we define another types of Lindelof on bitopological space, namely N-Lindelof , S-Lindelof and pair-wise Lindelof spaces, and we introduce some properties about these types.

Introduction:

In 1963, the term of bitopological space was used for the first time by Kelly [1].A set equipped with two topologies is called a bitopological space and denoted by (X,τ,τ'), where (X,τ) and (X,τ') are two topological spaces. In 2010 N.A.Jabbar and A.I.Nasir introduced N-open set [2]. A subset A of a bitopological space (X,τ,τ') is called an N-open set if and only if it is open in the space $(X,\tau\vee\tau')$, where $\tau\vee\tau'$ is the supremum topology on X contains τ and τ'.In this paper, we introduce the concept of N-Lindelof space. A bitopological space (X,τ,τ') is said to be an N-Lindelof space if and only if every N-open cover of X has a countable subcover, we study some properties of this kind of Lindelof space. Also we introduce the concept of S-Lindelof and pair-wise Lindelof. We also study the relationships among the three kinds of Lindelof spaces.

1. N-Lindelof Space

In this section, we give the definitions of N-open set, N-Lindelof space in bitopological spaces.

open cover of X which is an N-Lindelof space .Therefore, there exists a countable number of $\{U_\alpha : \alpha\in\Delta\}$ $\ni$ $\{X-A\}\cup\{U_{\alpha_i} : i\in\Delta\subset\mathrm{N}\}$ is a countable subcover of X .Since $A\subseteq X$ and $X-A$ covers no part of A , then $\{U_{\alpha_i} : i\in\Delta\subset\mathrm{N}\}$ is a countable subcover of A .So A is N-Lindelof set .

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1.1 Definition [1]

Let X be a non-empty set, let τ,τ' be any two topologies on X , then (X,τ,τ') is called bitopological space.

1.2 Definition [2]

A subset A of a bitopological space (X,τ,τ') is called an N-open set if and only if it is open in the space $(X,\tau\vee\tau')$, where $\tau\vee\tau'$ is the supremum topology on X contains τ and τ' .

1.3 Definition [2]

The complement of an N-open set in a bitopological space (X,τ,τ') is called N-closed set.

1.4 Remark

Let (X,τ,τ') be a bitopological space, then:

Every open set in (X,τ) or in (X,τ') is an N-open set in (X,τ,τ').

Every closed set in (X,τ) or in (X,τ') is an N-closed set in (X,τ,τ').

1.5 Note:

The opposite direction of remark 1.4 may be untrue as the following example shows :

1.11 Definition [2]

A function $f:(X,\tau,\tau')\to(Y,T,T')$ is said to be an N-continuous function if and only if the inverse image of each N-open subset of Y is an N-open subset of X .

1.12 Theorem

The N-continuous image of an N-Lindelof space is an N-Lindelof space .

Proof:

Let (X,τ,τ') be an N-Lindelof space, and let

$f:(X,\tau,\tau')\to(Y,T,T')$ be an N- continuous, onto function. To show that (Y,T,T') is an N-Lindelof space ,let $\{U_\alpha:\alpha\in\Delta\}$ be an N- open cover of Y, then $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ is an N-open cover of X, which is N-Lindelof space .So there exists a countable number of $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ such that the family $\{f^{-1}(U_{\alpha_i}):i\in\Delta\subset N\}$ covers X and since f is onto, then $\{U_{\alpha_i}:i\in\Delta\subset N\}$ is a countable subcover of Y .Hence Y is N-Lindelof space.

2. S-Lindelof Space

In this section, we give the definition of S-Lindelof space, then we study pair-wise Lindelof space in bitopological spaces and relationships between them and the N-Lindelof.

2.1 Definition [3]

A subset A of a bitopological space (X,τ,τ') is said to be S-open set if it is τ-open or τ'-open. The complement of the S-open set is called S-closed set.

Example

Let $X=\{1,2,3\}$, $\tau=\{\phi,\{1\},X\}$ and $\tau'=\{\phi,\{2\},X\}$ then $\tau\vee\tau'=\{\phi,\{1\},\{2\},\{1,2\},X\}$ is the family of all N-open subsets of (X,τ,τ'). $\{1,2\}$ is an N- open set in (X,τ,τ') but it is not open in both (X,τ) and (X,τ'). So $\{3\}$ is an N-closed set in (X,τ,τ') which is not closed in both (X,τ) and (X,τ').

1.6 Definition [2]

Let (X,τ,τ') be a bitopological space, let A be a subset of X .A subcollection of the family $\tau\vee\tau'$ is called an N-open cover of A if the union of members of this collection contains A.

1.7 Definition

A bitopological space (X,τ,τ') is said to be an N-Lindelof space if and only if every N-open cover of X has a countable subcover.

1.8 Corollary

If (X,τ,τ') is an N-Lindelof space. Then both (X,τ) and (X,τ') are Lindelof spaces.

Proof:

Follows from remark 1.4.

1.9 Corollary

If τ is a subfamily of τ'. Then (X,τ,τ') is an N-Lindelof space if and only if (X,τ) and (X,τ') are Lindelof spaces.

Proof :

Necessity , follows from corollary (1.8). Sufficiency, in view of τ is a subfamily of τ', then $\tau\vee\tau'=\tau'$. So (X,τ,τ') is N-Lindelof .

1.10 Corollary

The N-closed subset of an N-Lindelof space is N-Lindelof .

Proof:

Let (X,τ,τ') be an N-Lindelof space and let A be an N-closed subset of X. To show that A is an N-Lindelof set .Let $\{U_\alpha:\alpha\in\Delta\}$ be an N-open cover of A. Since A is N-closed subset of X, then $X-A$ is an N-open subset of X, so $\{X-A\}\cup\{U_\alpha:\alpha\in\Delta\}$ is an N-

there exists a countable number of $\{U_\alpha:\alpha\in\Delta\}$ ∍ $\{X-A\}\cup\{U_{\alpha_i}:i\in\Delta\subset\mathrm{N}\}$ is a countable subcover of X .Since $A\subseteq X$ and $X-A$ covers no part of A, then $\{U_{\alpha_i}:i\in\Delta\subset\mathrm{N}\}$ is a countable subcover of A .So A is S-Lindelof set .

2.8 Definition [4]

Let $f:(X,\tau,\tau')\to(Y,T,T')$ be a function .Then f is said to be a bicontiunous function if and only if $f^{-1}(U)\in\tau$ for each $U\in T$, and $f^{-1}(V)\in\tau'$ for each $V\in T'$

2.9 Example

Let $X=\{1,2,3\}$, $\tau=\{\phi,X,\{1\},\{2\},\{1,2\}\}$ and $\tau'=\tau_D$

Let $Y=\{a,b,c\}$, $T=\{\phi,Y,\{a\}\}$ and $T'=\tau_I$ .Define $f:(X,\tau,\tau')\to(Y,T,T')$ ∍ $f(1)=a,\ f(2)=b$ and $f(3)=c$ .Then f is bicontiunous function .Where τ_D and τ_I are the discrete and indiscrete topologies on X and Y respectively.

2.10 Lemma

A bicontiunous image of an S-Lindelof space is an S-lindelof space.

Proof:

Let $f:(X,\tau,\tau')\to(Y,T,T')$ be a bicontiunous, onto function and let (X,τ,τ') be an S-Lindelof space. To show that (Y,T,T') is an S-Lindelof, let $\{U_\alpha:\alpha\in\Delta\}$ be an S-open cover of Y, then $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ is an S-open cover of X ,which is an S-Lindelof space. Therefore there exists a countable number of $\{f^{-1}(U_\alpha):\alpha\in\Delta\}$ such that the family $\{f^{-1}(U_{\alpha_i}):i\in\Delta\subset N\}$ covers X and since f is onto ,then $\{U_{\alpha_i}:i\in\Delta\subset N\}$ is a countable subcover of Y .Hence Y is S-Lindelof space.

2.11 Corollary

Every N-Lindelof space is an S-Lindelof space.

Proof:

Follows from remark 2.2.

2.2 Remark

Every S-open (S-closed) set in bitopological space (X,τ,τ') is an N-open (N-closed) set.

The converse of the remark 2.2 need not be true, see the example of note (1.5), where the set $\{1,2\}$ is N-open set which is not S-open. So $\{3\}$ is N-closed set which is not S-closed set.

2.3 Definition [3]

Let (X,τ,τ') be a bitopological space, let A be a subset of X .A subcollection of the family $\tau\cup\tau'$ is called an S-open cover of A if the union of members of this collection contains A.

2.4 Definition

A bitopological space (X,τ,τ') is called an S-Lindelof if and only if every S-open cover of X has a countable subcover.

2.5 Corollary

If (X,τ,τ') is an S-Lindelof space. Then both (X,τ) and (X,τ') are Lindelof spaces.

Proof:

Clear.

2.6 Corollary

If τ is a subfamily of τ' .Then (X,τ,τ') is an S-Lindelof space if and only if (X,τ) and (X,τ') are Lindelof spaces.

Proof:

Clear.

2.7 Lemma

The S-closed subset of an S-Lindelof space is S-Lindelof .

Proof:

Let (X,τ,τ') be an S-Lindelof space and let A be an S-closed subset of X . To show that A is an S-Lindelof set .Let $\{U_\alpha:\alpha\in\Delta\}$ be an S-open cover of A . Since A is S-closed subset of X , then $X-A$ is an S-open subset of X, so $\{X-A\}\cup\{U_\alpha:\alpha\in\Delta\}$ is an S-open cover of X which is an S-Lindelof space .Therefore, IF τ is a subfamily of τ' and (X,τ') is a Lindelof space, then (X,τ,τ') is a pair-wise Lindelof space.

Proof:

Suppose that τ is a subfamily of τ' and (X,τ') be a Lindelof space. Then by corollary (1.9) , (X,τ,τ') is an N-Lindelof and by corollary (2.11), (X,τ,τ') is S-Lindelof.Therefore (X,τ,τ') is a pair-wise Lindelof .

2.19 Lemma

If (X,τ) and (X,τ') are Lindelof spaces .Then (X,τ,τ') is S-Lindelof if and only if it is a pair-wise Lindelof.

Proof:

Necessities,follows from corollary (2.17).

Sufficiency, suppose (X,τ,τ') is a pair-wise Lindelof space, to prove it is an S-Lindelof space, let $\{U_\alpha:\alpha\in\Delta\}$ be an S-open cover of X ,then there are three probabilities .

IF $\{U_\alpha:\alpha\in\Delta\}$ is a τ - open cover, since (X,τ) is Lindelof space, then $\{U_\alpha:\alpha\in\Delta\}$ has a countable subcover of X , so X is an S-Lindelof.

IF $\{U_\alpha:\alpha\in\Delta\}$ is a τ' - open cover, since (X,τ') is Lindelof space, then $\{U_\alpha:\alpha\in\Delta\}$ has a countable subcover of X , so X is an S-Lindelof.

If $\{U_\alpha:\alpha\in\Delta\}$ is a pair-wise open cover, since (X,τ,τ') is a pair-wise Lindelof space ,then $\{U_\alpha:\alpha\in\Delta\}$ has a countable subcover, so X is an S-Lindelof.

From the above three probabilities we have X is an S-Lindelof.

2.12 Corollary

Let (X,τ,τ') be a bitopological space. If τ is a subfamily of τ'.Then the concepts of S-lindelof and N-Lindelof are coincident.

Proof:

Follows from corollary (1.9) and corollary (2.6).

2.13 Definition [3]

Let (X,τ,τ') be a bitopological space and let $A \subseteq X$ .An S-open cover of A is called a pair-wise open cover if it contains at least one non-empty element from τ and at least one non-empty element from τ' .

2.14 Example

Let $X = \{1,2,3\}$, $\tau = \{\phi, X, \{1\}\}$.

Let $\tau' = \{\phi, X, \{2\}, \{3\}, \{2,3\}\}$.

Then the cover $C = \{\{1\}, \{2\}, \{3\}\}$ is a pair-wise open cover of X .

2.15 Remark

It follows from the definition (2.13) that every pair-wise open cover of the bitopological space (X,τ,τ') is an S-open cover .

The converse of the remark 2.15 need not be true, for example [2]:

Let $X = \{1,2,3\}$, $\tau = \{\phi, X, \{1\}\}$.

Let $\tau' = \{\phi, \{2\}, \{3\}, \{2,3\}, \{1,2\}, X\}$.

Then the cover $C = \{\{1,2\}, \{3\}\}$ is an S-open cover of X , but it is not pair-wise open cover

2.16 Definition

A bitopological space (X,τ,τ') is called a pair-wise Lindelof space if every pair-wise open cover of X has a countable subcover.

2.17 Corollary

Every S-Lindelof space is a pair-wise Lindelof space .

Proof:

Follows from remark (2.15).

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بعض أنواع فضاءات ليندلوف على الفضاءات الثنائية

فوزي نوري نصار

الخلاصة:

في هذا البحث ، قمنا بتعريف أنواع أخرى من فضاءات ليندلوف على الفضاءات الثنائيه أسميناها فضاءات ليندلوف – N وفضاءات ليندلوف . S وفضاء ليندلوف الثنائي الأولي .ودراسة بعض خواص هذه الفضاءات والعلاقة بينها .