Contra-Continuous Functions in Bitopological Spaces

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Abstract

The aim of this paper is to define and study contra-continuous functions and some other forms of continuity such as perfect continuity, LC- continuity and RC- continuity in bitopological spaces and investigate the relations between contra-continuity and these kinds of continuity and their effects on some kinds of spaces such as locally indiscrete space, semi locally indiscrete and pre- locally indiscrete in bitopological spaces.

Key words: ij-contra-continuous function, ij-perfectly continuous function, ij-LC- continuous function, ij-RC-continuous function, ij- locally indiscrete space, ij-semi locally indiscrete space and ij-pre- locally indiscrete space.

1. Introduction

In 1963, J.C. Kelly [9] was first introduced the concept of bitopological spaces, where X is a nonempty set and τ_1, τ_2 are topologies on X. In 1992, F. H. khedr and S. M. Al.Areefi [8] introduced the concept pairwise precontinuous functions and pairwise semi precontinuous functions. In 1996, J. Dontchev [4] introduced the concept of contra-continuity in topological spaces.

In this paper we introduced and study new concept of a contra-continuity in bitopological spaces and some other forms of continuity such as perfect continuity, LC- continuity and RC- continuity in bitopological spaces. Finally we study and investigate the relations between contra-continuity and these kinds of continuity and their effects on some kinds of spaces such as locally indiscrete, semi locally indiscrete and pre- locally indiscrete in bitopological spaces.

2. Preliminaries

Let (X, τ_1, τ_2) or simply X be a bitopological space. For any subset $A \subseteq X$, $\tau_i - \text{int}(A)$ and $\tau_i - cl(A)$ denote the interior and closure of a set A with respect to the topology τ_i, respectively.

Definition 2.1: A set A of a bitopolgical space (X, τ_1, τ_2) is called

(a) $\tau_1\tau_2 -$semi open [1] if $A \subseteq \tau_2 - cl(\tau_1 - \text{int}(A))$ and $\tau_1\tau_2 -$semi closed if $\tau_2 - \text{int}(\tau_1 - cl(A)) \subseteq A$.

(b) $\tau_1\tau_2 -$preopen [8] if $A \subseteq \tau_1 - \text{int}(\tau_2 - cl(A))$ and $\tau_1\tau_2 -$preclosed if $\tau_1 - cl(\tau_2 - \text{int}(A)) \subseteq A$.

(c) $\tau_1\tau_2 - \beta -$open or $\tau_1\tau_2 -$semi preopen [8] if $A \subseteq \tau_2 - cl(\tau_1 - \text{int}(\tau_2 - cl(A)))$ and $\tau_1\tau_2 - \beta -$closed or $\tau_1\tau_2 -$semi preclosed if $\tau_2 - \text{int}(\tau_1 - cl(\tau_2 - \text{int}(A))) \subseteq A$.

(d) $\tau_1\tau_2 -$regular open [3] if $A = \tau_1 - \text{int}(\tau_2 - cl(A))$ and $\tau_1\tau_2 -$regular closed [3] if $A = \tau_1 - cl(\tau_2 - \text{int}(A))$.

(e) $\tau_1\tau_2 -$locally closed [2] if $A = G \cap F$ where G is $\tau_1 -$open and F is $\tau_2 -$closed.

Remark 2.2: [8] The following implications hold:

$$\begin{tikzcd}
\tau_i - \text{open (closed)} \arrow[r] \arrow[d] & \tau_i\tau_j - \text{semi open (semi closed)} \arrow[d] \\
\tau_i\tau_j - \text{preopen (preclosed)} \arrow[r] & \tau_i\tau_j - \text{semi preopen (semi preclosed)}
\end{tikzcd}$$

On each statement above, $i \neq j$ and $i, j = 1,2$.

Definition 2.3: [8] Let (X, τ_1, τ_2) and (Y, σ_1, σ_2) be a bitopological space. A function $f : (X, \tau_1, \tau_2) \to (Y, \sigma_1, \sigma_2)$ is called

(a) pairwise continuous if $f^{-1}(U)$ is $\tau_i -$open set in (X, τ_1, τ_2) for each $\sigma_i -$open set U of (Y, σ_1, σ_2), $i = 1,2$,

(b) pairwise semi continuous if $f^{-1}(U)$ is $\tau_i\tau_j -$semi open in (X, τ_1, τ_2) for each $\sigma_i -$open set U of (Y, σ_1, σ_2), $i \neq j$ and $i, j = 1,2$,

(c) pairwise precontinuous if $f^{-1}(U)$ is $\tau_i\tau_j -$preopen in (X, τ_1, τ_2) for each $\sigma_i -$open set U of (Y, σ_1, σ_2), $i \neq j$ and $i, j = 1,2$,

(d) pairwise semi precontinuous or pairwise $\beta -$continuous if $f^{-1}(U)$ is $\tau_i\tau_j -$semi preopen in (X, τ_1, τ_2) for each $\sigma_i -$open set U of (Y, σ_1, σ_2), $i \neq j$ and $i, j = 1,2$.

Remark 2.4: [8] The following implications hold:

$$\begin{tikzcd}
\text{Pairwise continuity} \arrow[r] \arrow[d] & \text{Pairwise semi continuity} \arrow[d] \\
\text{Pairwise precontinuity} \arrow[r] & \text{Pairwise semi precontinuity}
\end{tikzcd}$$

Definition 2.5: [7] A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called ij-perfectly continuous or ij-strongly continuous if $f^{-1}(U)$ is both τ_j-closed and τ_j-open in (X,τ_1,τ_2) for each σ_i-open set U of (Y,σ_1,σ_2), $i\neq j$ and $i,j=1,2$.

3. Pairwise Contra-Continuous Functions

First we recall the following known definitions.

Definition 3.1:
A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called
(a) pairwise $LC-$continuous if $f^{-1}(U)$ is $\tau_i\tau_j-$locally closed in (X,τ_1,τ_2) for each σ_i-open set U of (Y,σ_1,σ_2), $i\neq j$ and $i,j=1,2$.
(b) pairwise $RC-$continuous if $f^{-1}(U)$ is $\tau_i\tau_j-$regular closed in (X,τ_1,τ_2) for each σ_j-open set U of (Y,σ_1,σ_2), $i\neq j$ and $i,j=1,2$.

Definition 3.2: A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is called ij-contra-continuous if $f^{-1}(U)$ is τ_i-closed set in (X,τ_1,τ_2) for each σ_j-open set U of (Y,σ_1,σ_2), $i\neq j$ and $i,j=1,2$.

f is said to be pairwise contra-continuous if it is both ij-contra-continuous and ji-contra-continuous .

Example 3.3:
Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{b\},\{a,b\}\}$, $\tau_2=\{\phi,X,\{b,c\}\}$
$Y=\{1,2,3\}$, $\sigma_1=\{\phi,Y,\{1\}\}$, $\sigma_2=\{\phi,Y,\{2\}\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by $f(a)=1$, $f(b)=3$, $f(c)=2$. Then f is pairwise contra-continuous.

Remark 3.4: In fact ij-contra-continuous and j-continuity are independent notions. Example 3.3 above shows that ij-contra-continuous function need not be j-continuous while the reverse is show in the following example.

Example 3.5: A j-continuous function does not imply ij-contra-continuous function. Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{a\},\{a,c\}\}$, $\tau_2=\{\phi,X,\{c\}\}$, $Y=\{1,2,3\}$, $\sigma_1=\{\phi,Y,\{1\}\}$, $\sigma_2=\{\phi,Y,\{2\}\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be a function defined by $f(a)=1$, $f(b)=3$, $f(c)=2$. Then f is 1-continuous but not 21-contra-continuous, since $\{1\}$ is σ_1-open set in Y but $f^{-1}(\{1\})=\{a\}$ is not τ_2-closed in X and 2-continuous but not 12-contra-continuous, since $\{2\}$ is σ_2-open set in Y but $f^{-1}(\{2\})=\{c\}$ is not τ_1- closed in X.

Theorem 3.6:
A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$, the following are equivalent for:
(1) f is ij-contra-continuous.
(2) For each $x\in X$ and each σ_j-closed set V in Y with $f(x)\in V$, there exists an τ_i-open set U in X such that $x\in U$ and $f(U)\subset V$.
(3) The inverse image of each σ_j-closed set in Y is τ_i-open set in X.
On each statement a above, $i\neq j$ and $i,j=1,2$.

Proof: $(1)\to(2)$: Let $x\in X$ and V be σ_j-closed set in Y with $f(x)\in V$, then V^c is σ_j-open set in Y. Since f is ij-contra-continuous, we have $U^c=f^{-1}(V^c)=(f^{-1}(V))^c$ is τ_i-closed set in X. Hence, $U=f^{-1}(V)$ is τ_i-open set in X such that $f(U)\subset V$ and $x\in U$.
$(2)\to(3)$: This is obvious.
$(3)\to(1)$: Let V be σ_j-open set in Y, then V^c is σ_j-closed set in Y. Therefore, by our assumption, $f^{-1}(V^c)=(f^{-1}(V))^c$ is τ_i-open set in X. Hence, $f^{-1}(V)$ is τ_i-closed set in X, f is ij-contra-continuous.
$(2)\to(1)$: This is obvious.
$(1)\to(3)$: Let U be σ_j- closed set in Y, then U^c is σ_j- open set in Y. Since f is ij-contra-continuous, we have $f^{-1}(U^c)=(f^{-1}(U))^c$ is τ_i-closed set in X. Consequently, $f^{-1}(U)$ is τ_i-open set in X.

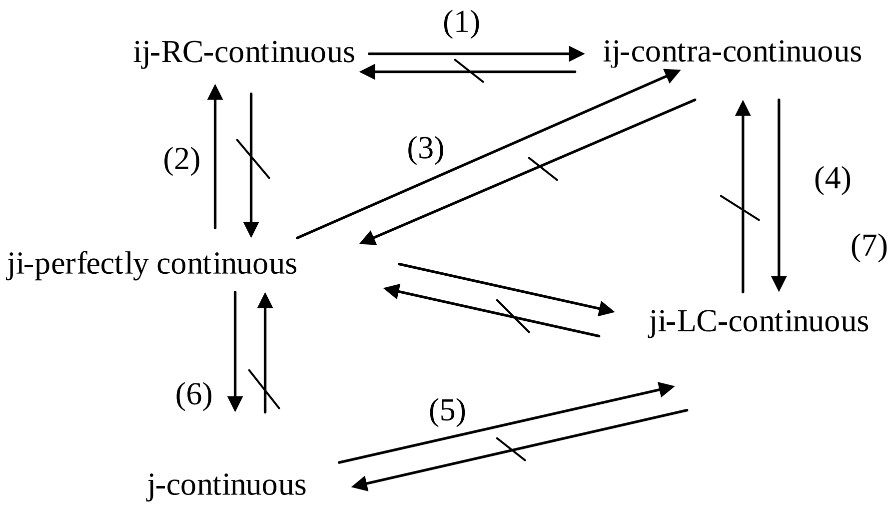
Lemma 3.7: [12] Every τ_i-open (τ_j-closed) subset of X is an $\tau_i\tau_j-$locally closed set.

Lemma 3.8: For a subset A of (X,τ_1,τ_2), the following are equivalent:-
(1) A is $\tau_1\tau_2-$regular closed,
(2) A is $\tau_1\tau_2-$preclosed and $\tau_2\tau_1-$semi open.

Proof: $(1)\to(2)$: Let A be $\tau_1\tau_2-$regular closed, then $A=\tau_1-cl\,(\tau_2-\text{int}\,(A))$. This implies that $\tau_1-cl\,(\tau_2-\text{int}\,(A))\subseteq A$ and $A\subseteq\tau_1-cl\,(\tau_2-\text{int}\,(A))$. Hence A is $\tau_1\tau_2-$preclosed and $\tau_2\tau_1-$semi open.

$(2)\to(1)$: This is obvious.

Theorem 3.9: The following implications are valid



Proof: **(1) ij-RC-continuous $\longrightarrow$ ij-contra-continuous**

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij-RC-continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ij-RC-continuous, $f^{-1}(U)$ is $\tau_i\tau_j-$regular closed in X, $i\neq j$ and $i,j=1,2$. Since every regular closed set is closed, then $f^{-1}(U)$ is τ_i-closed in X, $i=1,2$. Therefore, f is ij-contra-continuous.

(2) ji-perfectly continuous $\longrightarrow$ ij-RC-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ji- perfectly continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ji- perfectly continuous, $f^{-1}(U)$ is both τ_i-closed and τ_i-open in X, $i=1,2$, then $f^{-1}(U)$ is $\tau_i\tau_j-$regular closed in X, $i\neq j$ and $i,j=1,2$. Therefore, f is ij-RC-continuous.

(3) ji-perfectly continuous $\longrightarrow$ ij-contra-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ji- perfectly continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ji- perfectly continuous, $f^{-1}(U)$ is both τ_i-closed and τ_i-open in X, $i=1,2$. Hence $f^{-1}(U)$ is τ_i-closed in X, $i=1,2$. Therefore, f is ij-contra-continuous.

(4) ij-contra-continuous $\longrightarrow$ ji-LC-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ij-contra-continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ij-contra-continuous, $f^{-1}(U)$ is τ_i-closed in X, $i=1,2$. By lemma 3.7, $f^{-1}(U)$ is $\tau_j\tau_i-$locally closed in X, $i\neq j$ and $i,j=1,2$. Therefore, f is ji-LC-continuous.

(5) j-continuous $\longrightarrow$ ji-LC-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be j-continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is j-continuous, $f^{-1}(U)$ is τ_j-open in X, $j=1,2$. By lemma 3.7, $f^{-1}(U)$ is $\tau_j\tau_i-$locally closed in X, $i\neq j$ and $i,j=1,2$. Therefore, f is ji-LC-continuous.

(6) ji-perfectly continuous $\longrightarrow$ j-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ji-perfectly continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ji-perfectly continuous, $f^{-1}(U)$ is both τ_i-closed set and τ_i-open set in X, $i=1,2$. Therefore, f is j-continuous.

(7) ji-perfectly continuous $\longrightarrow$ ji-LC-continuous

Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be ji-perfectly continuous. Let U be σ_j-open set in Y, $j=1,2$. Since f is ji-perfectly continuous, $f^{-1}(U)$ is both τ_i-closed set and τ_i-open set in X, $i=1,2$. By lemma 3.7, $f^{-1}(U)$ is $\tau_j\tau_i-$locally closed in X, $i\neq j$ and $i,j=1,2$. Therefore, f is ji-LC-continuous.

The converse of theorem 3.9 is not true in general. The following counter examples show the cases.

Example 3.10:

Let $X = Y = \{a,b,c\}$,
$\tau_1 = \{\phi, X, \{a\}, \{a,c\}\}$,
$\tau_2 = \{\phi, X, \{c\}, \{b,c\}\}$,
$\sigma_1 = \{\phi, Y, \{a\}, \{a,b\}\}$, $\sigma_2 = \{\phi, Y, \{b\}\}$.

(a) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be the identity function. Then f is 12-contra-continuous but not 21- perfectly continuous, since $\{b\}$ is σ_2-open set in Y but $f^{-1}(\{b\})=\{b\}$ is not τ_1-open set in X.

(b) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (a). Then f is 12-contra-continuous but not 2-continuous, since $\{b\}$ is σ_2-open set in Y but $f^{-1}(\{b\})=\{b\}$ is not τ_2-open set in X.

(c) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (a). Then f is 12-contra-continuous but not 12-RC-continuous, since $\{b\}$ is σ_2-open set in Y but $f^{-1}(\{b\})=\{b\}$ is not $\tau_1\tau_2-$regular closed in X.

(d) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be defined by $f(a)=f(c)=a$, $f(b)=c$. Then f is 1-continuous but not 12- perfectly continuous, since $\{a\}$ is σ_1-open set in Y but $f^{-1}(\{a\})=\{a,c\}$ is not both τ_2-closed set and τ_2-open set in X.

(e) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (d). Then f is 1-continuous but not 21-RC-continuous, since $\{a\}$ is σ_1-open set in Y but $f^{-1}(\{a\})=\{a,c\}$ is not $\tau_2\tau_1-$regular closed in X.

(f) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (a). Then f is 12- LC-continuous but not 1-continuous, since $\{a,b\}$ is σ_1-open set in Y but $f^{-1}(\{a,b\})=\{a,b\}$ is not τ_1-open set in X.

(g) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (a). Then f is 12- LC-continuous but not 12- perfectly continuous, since $\{a,b\}$ is σ_1-open set in Y but $f^{-1}(\{a,b\})=\{a,b\}$ is not τ_2-open set in X.

(h) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (a). Then f is 12- LC-continuous but not 21-RC-continuous, since $\{a,b\}$ is σ_1-open set in Y but $f^{-1}(\{a,b\})=\{a,b\}$ is not $\tau_2\tau_1-$regular closed in X.

(i) Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ as in (d). Then f is 21- LC-continuous but not 12-contra-continuous, since $\{b\}$ is σ_2-open set in Y but $f^{-1}(\{b\})=\{c\}$ is not τ_1-closed set in X.

Example 3.11:

Let $X=\{a,b,c\}$, $\tau_1=\{\phi,X,\{b\},\{c\},\{b,c\}\}$, $\tau_2=\{\phi,X,\{a\},\{a,c\}\}$, $Y=\{1,2\}$, $\sigma_1=\{\phi,Y,\{2\}\}$, $\sigma_2=\{\phi,Y,\{1\}\}$. Let $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ be defined by $f(a)=f(c)=1$, $f(b)=2$. Then f is 21-RC-continuous but not 12- perfectly continuous, since $\{2\}$ is σ_1-open set in Y but $f^{-1}(\{2\})=\{b\}$ is not both τ_2-closed set and τ_2-open set in X.

Theorem 3.12: If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is ji-contra-continuous and ij-$\beta-$continuous, then f is ij-semi continuous.

Proof: Let U be σ_i-open set of Y, $i=1,2$. Then $f^{-1}(U)$ is τ_j-closed set and $\tau_i\tau_j-$semi preopen in X, $i\neq j$ and $i,j=1,2$, since f is ji-contra-continuous and ij-$\beta-$continuous. We have $f^{-1}(U)\subseteq\tau_j-cl(\tau_i-\text{int}(\tau_j-cl(f^{-1}(U))))$, since $f^{-1}(U)$ is τ_j-closed set then $f^{-1}(U)\subseteq\tau_j-cl(\tau_i-\text{int}(f^{-1}(U)))$. This show that $f^{-1}(U)$ is $\tau_i\tau_j-$semi open in X, $i\neq j$ and $i,j=1,2$. Therefore, f is ij-semi continuous.

Theorem 3.13: If a function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is ji-contra-continuous and ij-precontinuous, then f is i- continuous.

Proof: Let U be σ_i-open set of Y, $i=1,2$. Then $f^{-1}(U)$ is τ_j-closed set and $\tau_i\tau_j-$preopen in X, $i\neq j$ and $i,j=1,2$, since f is ji-contra-continuous and ij-precontinuous. We have $f^{-1}(U)\subseteq\tau_i-\text{int}(\tau_j-cl(f^{-1}(U)))=\tau_i-\text{int}(f^{-1}(U))$. This show that $f^{-1}(U)$ is τ_i-open in X, $i=1,2$. Therefore, f is i-continuous.

Theorem 3.14:
A function $f:(X,\tau_1,\tau_2)\to(Y,\sigma_1,\sigma_2)$ is ij-RC-continuous if and only if it is ij-contra-continuous and ji-semi continuous.

Proof: "Necessity". Since every ij-RC-continuous function is ij-contra-continuous and every $\tau_i\tau_j-$regular closed set is $\tau_j\tau_i-$semi open, ij-RC-continuous functions are ji-semi continuous, $i\neq j$ and $i,j=1,2$.

"Sufficiency". Let U be σ_j-open set of Y, $j=1,2$. Then $f^{-1}(U)$ is τ_i-closed set and $\tau_j\tau_i-$semi open in X, $i\neq j$ and $i,j=1,2$, since f is ij-contra-continuous and ji-semi continuous. Since every τ_i-closed set is $\tau_i\tau_j-$preclosed set then $f^{-1}(U)$ is $\tau_i\tau_j-$preclosed and $\tau_j\tau_i-$semi open in X, $i \neq j$ and $i, j = 1,2$. We have $\tau_i - cl(\tau_j - \mathrm{int}(f^{-1}(U))) \subseteq f^{-1}(U) \subseteq \tau_i - cl(\tau_j - \mathrm{int}(f^{-1}(U)))$ .Therefore, we obtain $\tau_i - cl(\tau_j - \mathrm{int}(f^{-1}(U))) = f^{-1}(U)$, this implies that $f^{-1}(U)$ is $\tau_i\tau_j -$ regular closed and hence f is ij-RC-continuous.

Corollary 3.15: A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is ij-RC-continuous if and only if it is ij-contra-continuous and ji-$\beta -$ continuous.

Proof: This is obvious by theorem above and every $\tau_j\tau_i -$ semi open is $\tau_j\tau_i -$ semi preopen.

Theorem 3.16:

A function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is i-continuous if and only if it is ij-precontinuous and ij-LC-continuous.

Proof: "Necessity". This is obvious.

"Sufficiency". Let U be $\sigma_i -$ open set of Y, $i = 1,2$. Then $f^{-1}(U)$ is $\tau_i\tau_j -$ locally closed ($f^{-1}(U)$ is $\tau_i -$ open and $f^{-1}(U)$ is $\tau_j -$ closed) and $\tau_i\tau_j -$ preopen in X, $i \neq j$ and $i, j = 1,2$, since f is ij-LC-continuous and ij-precontinuous. We have $f^{-1}(U) \subseteq \tau_i - \mathrm{int}(\tau_j - cl(f^{-1}(U))) = \tau_i - \mathrm{int}(f^{-1}(U))$.

Then $f^{-1}(U)$ is $\tau_i -$ open in X. Therefore, f is i-continuous.

Definition 3.17: A bitopolgical space (X,τ_1,τ_2) is called

(a) ij-locally indiscrete if every $\tau_i -$ open set of X is $\tau_j -$ closed set in X, $i \neq j$ and $i, j = 1,2$,

(b) ij-semi locally indiscrete if every $\tau_i\tau_j -$ semi open set of X is $\tau_j -$ closed set in X, $i \neq j$ and $i, j = 1,2$,

(c) ij-pre-locally indiscrete if every $\tau_i\tau_j -$ preopen set of X is $\tau_j -$ closed set in X, $i \neq j$ and $i, j = 1,2$.

Example 3.18: Let $X = \{1,2,3\}$, $\tau_1 = \{\phi, X, \{1\}\}$ and $\tau_2 = \{\phi, X, \{2,3\}\}$. Then (X,τ_1,τ_2) is ij-locally indiscrete space, $i \neq j$ and $i, j = 1,2$.

The space (X,τ_1,τ_2) in Example 3.10 is not ij-locally indiscrete space, $i \neq j$ and $i, j = 1,2$, since $\{a,c\}$ is $\tau_1 -$ open set in X but it not $\tau_2 -$ closed set in X and $\{c\}$ is $\tau_2 -$ open set in X but it not $\tau_1 -$ closed set in X.

Theorem 3.19: Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ and $g:(Y,\sigma_1,\sigma_2) \to (Z,\mu_1,\mu_2)$ be two functions such that $g \circ f:(X,\tau_1,\tau_2) \to (Z,\mu_1,\mu_2)$. Then

(a) $g \circ f$ is ji-contra-continuous, if g is ji-contra-continuous and f is j-continuous.

(b) $g \circ f$ is ji-contra-continuous, if g is i-continuous and f is ji-contra-continuous.

(c) $g \circ f$ is ji-contra-continuous, if f is j-continuous and g is i-continuous and Y is ij-locally indiscrete.

On each statement a above, $i \neq j$ and $i, j = 1,2$.

Proof: (a) Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be j-continuous and $g:(Y,\sigma_1,\sigma_2) \to (Z,\mu_1,\mu_2)$ be ji-contra-continuous. Let U be $\mu_i -$ open set in Z, $i = 1,2$. Since g is ji-contra-continuous, $g^{-1}(U)$ is $\sigma_j -$ closed set in Y. Since f is j-continuous, $(g \circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_j -$ closed set in X. Therefore, $g \circ f$ is ji-contra-continuous.

(b) Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be ji-contra-continuous and $g:(Y,\sigma_1,\sigma_2) \to (Z,\mu_1,\mu_2)$ be i-continuous. Let U be $\mu_i -$ open set in Z, $i = 1,2$. Since g is i-continuous, $g^{-1}(U)$ is $\sigma_i -$ open set in Y. Since f is ji-contra-continuous, $(g \circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_j -$ closed set in X. Therefore, $g \circ f$ is ji-contra-continuous.

(c) Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be j-continuous and $g:(Y,\sigma_1,\sigma_2) \to (Z,\mu_1,\mu_2)$ be i-continuous. Let U be $\mu_i -$ open set in Z, $i = 1,2$. Since g is i-continuous, $g^{-1}(U)$ is $\sigma_i -$ open set in Y. Since Y is ij-locally indiscrete, $g^{-1}(U)$ is $\sigma_j -$ closed set. Since f is j-continuous, $(g \circ f)^{-1} = f^{-1}[g^{-1}(U)]$ is $\tau_j -$ closed set in X. Therefore, $g \circ f$ is ji-contra-continuous.

Example 3.20: The composition of two ij-contra-continuous functions need not be ij-contra-continuous, $i \neq j$ and $i, j = 1,2$.

Let $X = Y = Z = \{a,b,c\}$, $\tau_1 = \{\phi, X, \{a\}, \{a,b\}\}$, $\tau_2 = \{\phi, X, \{b\}, \{b,c\}\}$, $\sigma_1 = \{\phi, Y, \{a\}\}$, $\sigma_2 = \{\phi, Y, \{c\}\}$, $\mu_1 = \{\phi, Z, \{a,b\}\}$, $\mu_2 = \{\phi, Z, \{b,c\}\}$.

Let $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ be the identity function. Then f is 21-contra-continuous.

Let $g:(Y,\sigma_1,\sigma_2) \to (Z,\mu_1,\mu_2)$ be the identity function. Then g is 21-contra-continuous.

But $(g \circ f)^{-1}(\{a,b\}) = f^{-1}[g^{-1}(\{a,b\})] = \{a,b\}$ is not $\tau_2 -$closed set in X. Hence $g \circ f$ is not 21-contra-continuous.

Theorem 3.21:
If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is i-continuous and X is ij-locally indiscrete, then f is ji-contra-continuous.

Proof: Let U be $\sigma_i -$open set of Y, $i = 1,2$. Then $f^{-1}(U)$ is $\tau_i -$open set in X since f is i-continuous. Since X is ij-locally indiscrete, $f^{-1}(U)$ is $\tau_j -$closed set, $i \neq j$ and $i, j = 1,2$. Therefore, f is ji-contra-continuous.

Theorem 3.22:
If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is ij-semi continuous and X is ij-semi locally indiscrete, then f is ji-contra-continuous.

Proof: Let U be $\sigma_i -$open set of Y, $i = 1,2$. Then $f^{-1}(U)$ is $\tau_i\tau_j -$semi open in X, $i \neq j$ and $i, j = 1,2$, since f is ij-semi continuous. Since X is ij-semi locally indiscrete, $f^{-1}(U)$ is $\tau_j -$closed set, $i \neq j$ and $i, j = 1,2$. Therefore, f is ji-contra-continuous.

Theorem 3.23:
If a function $f:(X,\tau_1,\tau_2) \to (Y,\sigma_1,\sigma_2)$ is ij-precontinuous and X is ij-pre-locally indiscrete, then f is ji-contra-continuous.

Proof: Let U be $\sigma_i -$open set of Y, $i = 1,2$. Then $f^{-1}(U)$ is $\tau_i\tau_j -$preopen in X, $i \neq j$ and $i, j = 1,2$, since f is ij-precontinuous. Since X is ij-pre-locally indiscrete, $f^{-1}(U)$ is $\tau_j -$closed set, $i \neq j$ and $i, j = 1,2$. Therefore, f is ji-contra-continuous.

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الدوال ضد المستمرة في الفضاءات الثنائية التبولوجية

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الملخص

الفكرة الأساسية لبحثنا هو تعريف ودراسة الدوال ضد المستمرة وبعض الصيغ الأخرى للاستمرارية كالاستمرارية التامة ، استمرارية – LC و استمرارية – RC في الفضاءات الثنائية التبولوجية ونتحرى العلاقات بين ضد الاستمرارية وهذه الأنواع من الاستمرارية وتأثيراتِهم على بعض أنواع الفضاءات كفضاء غير مبعثر محلياً ، شبه غير مبعثر محلياً وقبلي غير مبعثر محلياً في الفضاءات الثنائية التبولوجية .