Using boundary integral equation method to solve Plane strain problems of asymmetric elasticity

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(Received: 19 / 1 / 2010 ---- Accepted: 1 / 10 / 2012)

Abstract

In this paper we try to solve plane strain problems of asymmetric elasticity by using boundary integral equation method and we compare the results which we obtained from this method with the results of other methods we see that accuracy of our result is more than other results.

Keywords : Symmetric , integral equation , boundary , strain .

1. Introduction

There are many methods to solve the boundary value problems for examples to these problems[5] Stockes's flow and creeping flow, Elasticity problems, plane stress problems, deflection of plates, Airy's function, Flexural rigidity, homogenous and non homogenous problems, harmonic and biharmonic equations, Contact problems with friction …. one of these methods is the boundary integral equation method .

2. plane strain problems of asymmetric elasticity:

As we known, in plane strain case, the displacement components u and v parallel to x and y axes respectively, and rotation component Ω_x and Ω_y are identically zero[3]. i.e.

$u = u(x , y)$, $v = v(x , y)$,

$w = 0$, $\Omega_x = 0$,

$\Omega_y = 0$, $\Omega_z = \Omega (x , y)$.

The non-vanishing components of strain E_{ij} (i , j = x , y) and curvature components M_{ij} (i,j = x,y,z) are related to the displacement gradients, rotation component

(1)

and the gradients of the rotation component by the relations:

$$E_{xx} = \frac{\partial u}{\partial x} , E_{xy} = \frac{\partial v}{\partial x} - \Omega , E_{yx} = \frac{\partial u}{\partial y} + \Omega$$

$$E_{yy} = \frac{\partial v}{\partial y} , M_{xz} = \frac{\partial \Omega}{\partial x} , M_{yz} = \frac{\partial \Omega}{\partial y} , \quad \ldots. (1)$$

In the absence of body forces and body couples, the components of stresses and couple-stresses satisfy the following equations of equilibrium:

$$\frac{\partial Pxx}{\partial x} + \frac{\partial Pxy}{\partial y} = 0 , \frac{\partial Pyx}{\partial x} + \frac{\partial Pyy}{\partial y} = 0 , \quad \ldots. (2)$$

They must satisfy the equations of compatibility, which in terms of stresses and couple-stresses are:

$$\nabla^2 (P_{xx} + P_{yy}) = 0 , \quad \ldots. (3)$$

Where the ∇^2 is the usual two-dimensional Laplacian operator. And from this equation [1], we obtain new equation:

$$\nabla^4 U = 0 , \quad \ldots. (4) .$$

Which is called the biharmonic equation.

The equations (1),(2),(3) are solved subject to prescribed boundary conditions [4].

3.The Boundary Conditions:

There are three types of boundary conditions [2] : -

The first type of boundary conditions involves the tractions P_{nx} , P_{ny} on the surface, where,

$P_{nx} = P_{xx} \cos (n,x) + p_{xy} \cos (n,y)$

$P_{nx} = P_{yx} \cos (n,x) + p_{yy} \cos (n,y)$.

The second type of boundary conditions displacement u,v and rotation component Ω_z are described on boundary of the region (area). In general, u, v, and Ω_z of the actual distance (s) for a fixed point on the boundary of the region, and , hence,

$U(x,y) = u(s)$, $v(x,y) = v(s)$, $\Omega_z (x,y) = \Omega_z (s)$, where (x,y) are points on the boundary of the region.

In the third type of boundary conditions, the displacements and rotation are described on one part of the boundary and the tractions and couple-stresses on the other part (on boundary).

These types of the boundary conditions explain what are known respectively as the first fundamental, the second fundamental and the mixed boundary value problems in the theory of elasticity.

The plane strain problems, in asymmetric elasticity theory, are concerned with the solution of equation (3)[2].

4.Boundary Integral Equation method (B.I.E):

In this method reduces a partial differential equations in (n) independent variables to one in (n-1) variables[1]; hence, a partial differential equations in two variables could be changed to an a ordinary differential equations. Or , we try to changes a partial differential equations to an integral equation /equations (an equation where the unknown is inside the integral), and the integral equation /equations is/are then solved by various techniques [5].

There are two formulas in this method [1]: direct formulation and indirect formulation, in the direct method, there are two kinds of governing equation: the homogenous and non-homogenous equation, which in formula:

$\nabla^4 U = F$,and $\nabla^4 U = 0$, respectively.

and after many of operations on the homogenous equation[Aweny,1993], we obtain the first boundary integral equation:

$$4\Pi W(x,y) = \int s \{ W \frac{\partial}{\partial n} (\nabla^2 V) + \nabla^2 W \frac{\partial v}{\partial n} - 4 \nabla^2 V \frac{\partial w}{\partial n} - V \frac{\partial \nabla^2 w}{\partial n} - \} ds , \quad \ldots. (5)$$

And the second boundary integral equation:

$$4\Pi \frac{\partial W(x,y)}{\partial n_o} = \int s \{ W(x,y) \frac{\partial^2}{\partial n_o \partial n} (\nabla^2 V) + \nabla^2 W \frac{\partial^2 V}{\partial n \partial n_o} - \frac{\partial(\nabla^2 V)}{\partial n_o} \frac{\partial w}{\partial n} - \frac{\partial V}{\partial n} \frac{\partial(\nabla^2 W)}{\partial n} \} ds \quad , \quad \ldots\ldots (6)$$

Where, W, V, are continuous and differentiable functions. This two integral equations contains four unknown (variables), and by boundary conditions, two from this variables are known [1].

And from a non- homogenous equation we obtain a new two formula of boundary integral equations after many processes :

The first boundary integral equation from non-homogenous equation

$$4\Pi W(x,y) = \int_G V(x,y, x_o , y_o) \frac{q(x,y)}{D} dA - \frac{1}{D} \int_S \{ K_n (V) W - M_n (V) \frac{\partial w}{\partial n} + \frac{\partial V}{\partial n} M_n (W) - V K_n (W)\} ds \quad , \quad \ldots. (7)$$

and the second boundary integral equation:

$$4\Pi \frac{\partial W(x,y)}{\partial n} = \int_G \frac{q(x,y)}{D} \frac{\partial V}{\partial n} dG - \frac{1}{D} \int_S \{ \frac{\partial K_n(V)}{\partial n} W - \frac{\partial M_n(V)}{\partial n} \frac{\partial W}{\partial n} + \frac{\partial^2 V}{\partial n^2} M_n (W) - \frac{\partial V}{\partial n} K_n (W) \} ds \, , \quad \ldots.. (8)$$

5. The Results

Now, we compare the results from our suggestion method (Boundary Integral Equation method (B.I.E)) and other result from different method and compare the two results which is obtain by running program which is built by FORTRAN language then we see the results of the boundary integral equation are more accuracy than the results of other methods and these clear in the table bellow:

	Classical case	Result (**Complementary solution**)	Result (**B.I.E**)
1.	-0.555555	-0.568663	-0.566423
2.	-0.459136	-0.469969	-0.460130
3.	-0.385802	-0.394904	-0.388623
4.	-0.328731	-0.336487	-0.330012'
5.	-0.283446	-0.290134	-0.287231
6.	-0.246913	-0.252739	-0.249976
7.	-0.217013	-0.222134	-0.220015
8.	-0.192223	-0.196769	-0.194435
9.	-0.171467	-0.175513	-0.172145
10.	-0.153893	-0.157524	-0.154578
11.	-0.138888	-0.142165	-0.140078
12.	-0.061728	-0.063184	-0.062987
13.	-0.034722	-0.035541	-0.035034
14.	-0.022222	-0.022744	-0.022456

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استخدام طريقة المعادلة التكاملية الحدودية لحل مسائل الإجهاد المستوي للمرونة غير المتماثلة

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(تاريخ الاستلام: 19 / 1 / 2010 ---- تاريخ القبول: 1 / 10 / 2012)

الملخص

حاولنا في هذا البحث حل (مسائل الإجهاد المستوي للمرونة غير المتماثلة) باستخدام طريقة المعادلة التكاملية الحدودية ، وتمت مقارنة النتائج التي حصلنا عليها عند الحل باستخدام هذه الطريقة مع نتائج الحل باستخدام الطرق الأخرى حيث لاحظنا إن النتائج الجديدة كانت أدق من النتائج الأخرى .