Some properties of weakly open functions in intuitionistic bitopological spaces

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Abstract
In 2006 Takashi Noiri1 and Valeriu Popa [10] introduced and studied characterizations and properties of weakly open functions in bitopological spaces. In this paper we generalized the concept of weakly open function to an intuitionistic bitopological spaces and we obtain some new characterizations of weakly open functions between intuitionistic bitopological spaces. Moreover, we investigate some properties of these functions comparing with the related functions.

Key words and phrases: intuitionistic bitopological spaces, weakly open function

1. Introduction
The notion of weakly continuous functions was introduced in [7] and [6]. It is proved in [7] that a function f: X $\rightarrow$Y is weakly continuous if and only if f^{-1}(V)$\subset$Int(f^{-1}(Cl(V))) for every open set V of Y.
In [8] and [9] Popa and Rose introduced the concept of weak openness which is a natural dual to that of weak continuity.
In [5] and [4] Jelic generalized the notion of weakly open functions in the setting of bitopological spaces. In this paper, we generalize the concept of weakly open function to an intuitionistic bitopological spaces and we obtain some new characterizations of weakly open functions between intuitionistic bitopological spaces. Moreover, we investigate some properties of these functions comparing with the related functions.

2. Preliminaries
First we present the fundamental definitions.

Definition 2.1. [1]. Let X be a non empty set. An intuitionistic set A is an object having the form A $=\langle x,A_1,A_2\rangle$, where A_1 and A_2 are subsets of X satisfying $A_1 \cap A_2=\varphi$.
The set A_1 is called the set of member of A, while A_2 is called the set of <u>nonmember</u> of A.

Remark. Every subset A of nonempty set X can be regarded as intuitionistic set having the form $\tilde{A}$= $\langle x,A,A^c\rangle$.

Definition 2.2: [2] Let X be a nonempty set, p $\in X$ a fixed element in X, and let A= $\langle x,A_1,A_2\rangle$ be an intuitionistic set (IS, for short). The IS $\dot{p}$ defined by $\dot{p}$=$\langle x,\{p\},\{p\}^c\rangle$ is called an intuitionistic point (IP for $short$) in X. The IS $\ddot{p}$=$\langle x,\emptyset,\{p\}^c\rangle$ is called a vanishing intuitionistic point (VIP, for short) in X. The IS $\dot{p}$ is said to be contained in A ($\dot{p}\in A$, for $short$) if and only if $p\in A_1$, and similarly IS $\ddot{p}$ contained in A. ($\ddot{p}\in A$, for $short$) if and only if p $\notin A_2$. For a given IS A in X, we may write A =($\cup\{\dot{p}:\dot{p}\in A\}$) $\cup$ ($\cup\{\ddot{p}:\ddot{p}\in A\}$), and whenever A is not a proper IS (i.e., if A is not of the form A = $\langle x,A_1,A_2\rangle$ where $A_1 \cup A_2 \neq X$), then A =$\cup\{\dot{p}:\dot{p}\in A\}$ hold. In general, any IS A in X can be written in the form A=$\dot{A}\cup\ddot{A}$ where $\dot{A}$=$\cup\{\dot{p}:\dot{p}\in A\}$ and $\ddot{A}$ =$\cup\{\ddot{p}:\ddot{p}\in A\}$.

Definition 2.3: [3] An intuitionistic topology on a nonempty set X is a family τ of an intuitionistic sets in X satisfying the following conditions.

(1) $\tilde{\Phi}$, X $\in\tau$.

(2) τ is closed under finite intersections.

(3) τ is closed under arbitrary unions.

The pair (X,τ) is called an intuitionistic topological space (ITS, for short). Any element in τ is usually called intuitionistic open set(IOS , for $short$). The complement of an IOS in an IT'S (X,τ) is called intuitionistic closed set (ICS, for short).

Definition 2.4. [2] Let (X,τ) be an IT'S and let A $=\langle x,A_1,A_2\rangle$ be an intuitionistic subset in a set X. The interior ($IntA$, for $short$) and closure (ClA , for $short$) of a set A of X are defined:

Int (A) = $\cup$\{G: G$\subseteq$ A, G$\in\tau$\},

Cl (A) = $\cap$\{F: A$\subseteq$ F, $\overline{F}\in\tau$\}.

In other words: The Int (A) is the largest intuitionistic open set contained in A, and Cl (A) is the smallest intuitionistic closed set contain A i.e., Int(A) $\subseteq$ A and A $\subseteq$ Cl(A).

Definition 2.5. An intuitionistic bitopological on a nonempty set X is a two family τ_1, τ_2 of an intuitionistic sets in X such that both (X, τ_1) and (X, τ_2) are intuitionistic topological space .The treble (X,τ_1 ,τ_2) is called an intuitionistic bitopological space.

Definition 2.6. Let (X,τ_1 ,τ_2) be an intuitionistic bitopological space and A=$\langle x,A_1,A_2\rangle$ be an intuitionistic subset in a set X. The interior

$(IntA\ , for\ short)$ and closure $(ClA\ , for\ short)$ of a set A of X with respect to t_i are defined by:

iInt(A) = $\cup\{ G : G \subseteq A, G \in t_i, for\ i = 1; 2\}$,

iCl(A) = $\cap\{ F : A \subseteq F, \overline{F} \in t_i, for\ i = 1; 2\}$.

Definition 2.7: A subset A of an intuitionistic bitopological space (X,τ_1 ,τ_2) is said to be:

(1) intuitionistic (i, j)-regular open if A = iInt(jCl(A)), where i $\neq$ j, i, j = 1; 2.

(2) intuitionistic (i, j)-semi-open if A $\subset$ jCl(iInt(A)), where i $\neq$ j, i, j = 1, 2.

(3) intuitionistic (i, j)-pre-open if A $\subset$ iInt(jCl(A)), where i $\neq$ j, i, j = 1, 2.

(4) intuitionistic (i, j)-α-open if A $\subset$ iInt(jCl(iInt(A))), where i $\neq$ j, i, j = 1, 2.

(5) intuitionistic (i, j)-β-open if A $\subset$ iCl(jInt(iCl(A))), where i $\neq$ j, i, j = 1, 2 .

Definition 2.8. A subset A of an intuitionistic bitopological space (X,τ_1 ,τ_2) is said to be :

(1) intuitionistic (i, j)-regular-closed if A = iCl(jInt(A)), where i $\neq$ j , i, j = 1; 2.

(2) intuitionistic (i, j)-semi-closed if jInt(iCl(A)) $\subset$ A , where i $\neq$ j, i, j = 1, 2.

(3) intuitionistic (i, j)-pre-closed if iCl(jInt(A)) $\subset$ A , where i $\neq$ j, i, j = 1, 2.

(4) intuitionistic (i, j)-α-closed if iCl(jInt(iCl(A))) $\subset$ A , where i $\neq$ j, i, j = 1, 2.

(5) intuitionistic (i, j)-β-closed if iInt(jCl(iInt(A))) $\subset$ A , where i $\neq$ j, i, j = 1, 2.

Definition 2.9. Let (X, τ_1 , τ_2) be an intuitionistic bitopological space and A a subset of X . A point x $\in$ X is said to be an intuitionistic (i, j)-θ-closure of A, denoted by (i;j)- Cl_θ (A) , if A $\cap$ jCl(U) $\neq \emptyset$, for every τ_i-intuitionistic open set U containing $\dot{x}$, where i, j = 1, 2 and i $\neq$ j .

A subset A of X is said to be intuitionistic (i, j)-θ-closure if A = (i, j)- Cl_θ (A). A subset A of X is said to be intuitionistic (i, j)-θ-open if X /A or A^c is an intuitionistic (i, j)-θ-closed. The intuitionistic (i, j)-θ-interior of A, denoted by (i, j)- Int_θ(A), is defined as the union of all intuitionistic (i, j)-θ-open sets contained in A.

Hence $\dot{x}$ $\in$ (i, j) -Int_θ(A) if and only if there exists a τ_i-intuitionistic open set U containing $\dot{x}$ such that $\dot{x}$ $\in$ U $\subset$ jCl (U) $\subset$ A.

Lemma 2.10. For a subset A of an intuitionistic bitopological space (X, τ_1 , τ_2) the following properties hold:

(1) X / (i,j)- Int_θ(A) = (i, j)-Cl_θ (X /A) ,

(2) X / (I, j)-Cl_θ (A) = (I, j)- Int_θ(X /A).

Proof: (1) suppose that (i , j) -Cl_θ (X /A) and to prove X / (i , j)- Int_θ(A). Since (i , j)-Cl_θ (X /A) is an intuitionistic (i, j)-θ-closed . Then X / (i , j)-Cl_θ is an intuitionistic (i , j)-θ-open , also X / (i , j)-Cl_θ (X/A) $\subset$ A . Hence X / (i , j) -Cl_θ (X/A) $\subset$ (i, j)- Int_θ(A). Conversely, let $\dot{x}$ $\in$ (i, j)- Int_θ(A). Then there exists an intuitionistic (i, j)-θ-open set G such that $\dot{x}$ $\in$ G $\subset$ A. Then X / G is an intuitionistic (i, j)- θ-closed and X /A $\subset$ X / G. Since $\dot{x}$ $\notin$ X / G, $\dot{x}$ $\notin$ (i, j) -Cl_θ (X /A) and hence X / (i, j)- Int_θ(A) $\subset$ (i, j) -Cl_θ (X /A) . Therefore X / (i, j)- Int_θ(A) = (i, j) -Cl_θ (X /A).

(2) This follows from (1) immediately.

Definition 2.11. A function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1,σ_2) is said to be:

(1) Intuitionistic (i, j) - semi-open if for each an intuitionistic τ_i-open set U of X, f (U) is intuitionistic (i, j)-semi-open in Y.

(2) Intuitionistic (i, j)-pre-open if for each an intuitionistic τ_i-open set U of X, f (U) is intuitionistic (i, j) - pre-open in Y.

(3) Intuitionistic (i, j)-α-open if for each an intuitionistic τ_i-open set U of X, f (U) is intuitionistic (i, j)-α-open in Y.

(4) intuitionistic (i, j)-β-open if for each an intuitionistic τ_i-open set U of X, f(U) is intuitionistic (i, j)- intuitionistic β-open in Y .

(5) weakly intuitionistic (i, j)-open if for each an intuitionistic τ_i-open set U of X, f(U) $\subset$ i-Int(f(jCl(U))).

A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1,σ_2) is said to be pair wise weakly intuitionistic open if f is weakly intuitionistic (1, 2)-open and weakly intuitionistic (2,1)-open .

3. Characterizations

Lemma 3.1. Let (X, τ_1 , τ_2) be an intuitionistic bitopological space . If U is an intuitionistic τ_i-open in X, then (i, j)- Cl_θ(U) = iCl (U).

Theorem 3.1. For a function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) , the following properties are equivalent:

(1) f is weakly intuitionistic (i, j)-open.

(2) f ((i, j)-Int_θ((A)) $\subset$ iInt(f(A)) for every intuitionistic subset A of X .
(3) (i, j)-Int_θ (f^{-1}(B)) $\subset$ f^{-1}(iInt(B)) for every intuitionistic subset set B of Y .
(4) f^{-1}(iCl (B)) $\subset$ (i, j)-Cl_θ (f^{-1}(B)) for every intuitionistic subset B of Y .
(5) For each $\dot{x} \in$ X and each an intuitionistic τ_i-open set U of X containing $\dot{x}$, there exists an intuitionistic σ_i-open set V of Y containing f($\dot{x}$) such that V $\subset$ f(jCl(U)).

Proof. (1)) $\Longrightarrow$(2): Let A be any intuitionistic subset of X and $\dot{x} \in$ (i, j)-Int_θ(A).

Then there exists an intuitionistic τ_i-open set U of X such that $\dot{x} \in$ U $\subset$ jCl (U) $\subset$ A.

Hence we have f($\dot{x}$) $\in$ f(U) $\subset$ f(jCl(U)) $\subset$ f(A). Since f is weakly intuitionistic (i, j)- open, f(U) $\subset$ iInt(f(jCl(U)))$\subset$ iInt(f(A)) and $\dot{x} \in f^{-1}$(iInt(f(A))). Thus (i, j)-Int_θ(A) $\subset f^{-1}$(iInt(A)) and hence f((i, j)-Int_θ(A)) $\subset$ iInt(f(A)).

(2)) $\Longrightarrow$ (3): Let B be any intuitionistic subset of Y. Then by (2), we have f((i, j)- Int_θ (f^{-1}(B))) $\subset$ iInt(f(f^{-1}(B))) $\subset$ iInt(B). Therefore, (i, j)-Int_θ(f¡1(B)) $\subset f^{-1}$(iInt(B)).

(3)) $\Longrightarrow$ (4): Let B be any intuitionistic subset of Y. Then we have X / (i, j) - Cl_θ (f^{-1}(B)) = (i, j) - Int_θ (X / f^{-1}(B)) = (i, j) - Int_θ (f^{-1}(Y /B)) $\subset f^{-1}$(iInt(Y / B)) = f^{-1}(Y / iCl (B)) = X / f^{-1}(iCl (B)). Therefore, f^{-1}(iCl (B)) $\subset$ (i, j) - Cl_θ (f^{-1} (B)).

(4)) $\Longrightarrow$ (5): Let $\dot{x} \in$ X and U be any an intuitionistic τ_i-open set containing $\dot{x}$. Set B=Y / f(jCl(U)). By (4), we have f^{-1}(iCl(Y / f(jCl(U)))) $\subset$ (i, j)-Cl_θ (f^{-1}(Y / f(jCl(U)))). Now, f^{-1}(iCl(Y / f(jCl(U)))) = X / f^{-1} (iInt(f(jCl(U)))). Moreover, we have (i, j)-Cl_θ(f^{-1}(Y / f(jCl(U)))) = (i, j)-Clµ(X / f^{-1}(f(jCl(U)))) $\subset$ (i, j)-Cl_θ(X / jCl(U)) = iCl(X / jCl(U)) = X / iInt(jCl(U)) $\subset$ X / iInt(U) = X / U. Therefore, we obtain U $\subset f^{-1}$(iInt(f(jCl(U)))) and f(U) $\subset$ iInt(f(jCl(U))). Since f(x) $\in$ f(U), there exists an intuitionistic σ_i-open set V such that f(x) $\in$ V $\subset$ f(jCl(U)).

(5)) $\Longrightarrow$ (1): Let U be any an intuitionistic τ_i-open set of X and $\dot{x} \in$ U. By (5), there exists an intuitionistic σ_i-open set V of Y containing f ($\dot{x}$) such that V $\subset$ f (jCl (U)).

Hence we have f(x) $\in$ V $\subset$ iInt (f (jCl (U))) for each $\dot{x} \in$ U. Therefore, we obtain f(U) $\subset$ iInt(f(jCl(U))). This shows that f is weakly intuitionistic (i, j) - open.

Theorem 3.2. For a function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) , the following properties are equivalent:
(1) f is weakly intuitionistic (i, j)- open.
(2) f (iInt(F)) $\subset$ iInt(f(F)) for each an intuitionistic τ_i-closed set F of X .
(3) f (U) $\subset$ iInt(f(jCl(U))) for every intuitionistic (i, j)- pre-open set U of X .
(4) f (U) $\subset$ iInt(f(jCl(U))) for every intuitionistic (i, j)-α-open set U of X .

Proof. (1)) $\Longrightarrow$ (2): Assume that f is weakly intuitionistic (i, j) - open. Let F be an intuitionistic τ_i-closed set of X. Then iInt (F) is τ_i-intuitionistic open and by (1) we have f(iInt(F)) $\subset$ iInt(f(jCl(iInt(F)))) $\subset$ iInt(f(F)).

(2)) $\Rightarrow$ (3): Let U be any intuitionistic (i, j)-pre-open set of X. Then by (2) we obtain f(U) $\subset$ f(iInt(jCl(U))) $\subset$ iInt(f(jCl(U))).

(3)) $\Rightarrow$ (4): This is obvious since every intuitionistic (i, j)-α-open set is intuitionistic (i, j)-pre-open.

(4)) $\Rightarrow$ (1): Let U be any τ_i-intuitionistic open set of X. Then U is intuitionistic (i, j) - α -open in X and hence f(U) $\subset$ iInt(f(jCl(U))). Therefore, f is weakly intuitionistic (i,j)-open .

Theorem 3.3. For a bijective function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) , the following properties are equivalent :
(1) f is weakly intuitionistic (i, j)-open.
(2) iCl (f(jInt(F))) $\subset$ f(F) for every an intuitionistic τ_i-closed set F of X.
(3) iCl(f(U)) $\subset$ f(iCl(U)) for every an intuitionistic τ_i-open set U of X .

Proof. (1)) $\Rightarrow$ (2): Let F be any intuitionistic τ_i-closed set of X. Then X / F is an intuitionistic τ_i-open and Y / f(F) = f(X / F) $\subset$ iInt(f(jCl(X / F))) = iInt(f(X / jInt(F)))= iInt(Y / f(jInt(F))) = Y / iCl(f(jInt(F))). This implies that iCl (f (jInt (F))) $\subset$ f(F).

(2)) $\Longrightarrow$ **(3):** Let U be any an intuitionistic τ_i-open set of X. By (2) we have iCl(f(U)) =iCl(f(jInt(U))) $\subset$ iCl(f(jInt(iCl(U)))) $\subset$ f(iCl(U)). Therefore, iCl (f(U)) $\subset$ f(iCl(U)).

(3)) $\Longrightarrow$ (1): Let U be any an intuitionistic τ_i-open set of X. Then, we have Y / iInt(f(jCl(U))) = iCl(Y / f(jCl(U))) = iCl(f(X / jCl(U))) $\subset$ f(iCl(X / jCl(U))) = f(X / iInt(jCl(U))) $\subset$ f(X / iInt(U)) = f(X / U) = Y / f(U). This implies f(U) $\subset$ iInt(f(jCl(U)).

Therefore, f is weakly intuitionistic (i, j)- open .

4. Relations with other forms of intuitionistic open functions:

Definition 4.1. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be pair wise intuitionistic open if the induced functions f_1: (X, τ_1) $\longrightarrow$ (Y, σ_1) and f_2 : (X, τ_2) $\longrightarrow$ (Y, σ_2) are an intuitionistic open .

Definition 4.2. A function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be almost intuitionistic (i, j)-open if f(U) is an intuitionistic σ_i-open set in Y for every intuitionistic (i, j)-regular open set U of X .

A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be almost pair wise intuitionistic open if it is almost intuitionistic (1, 2)- open and almost intuitionistic (2, 1)-open .

Remark 4.3. It is clear that pair wise intuitionistic openness $\Longrightarrow$ almost pair wise intuitionistic openness $\Longrightarrow$ weakly pair wise intuitionistic openness. But the converse is not true.

Example 4.4. Let X = $\{a,b\}$, τ_1 =$\{\tilde{\Phi},\tilde{X},A\}$, where A = $\langle x,\{a\},\{b\}\rangle$, τ_2=$\{\tilde{\Phi},\tilde{X},B\}$, where $\langle x,\{b\},\{a\}\rangle$. Also Y = $\{a,b\}$, σ_1 =$\{\tilde{\Phi},\tilde{X},A\}$ and σ_2 =$\{\tilde{\Phi},\tilde{X},B\}$.

Define a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) by f ($\dot{a}$, $\dot{b}$) = ($\dot{b}$, $\dot{a}$) . Then f is almost pair wise intuitionistic open , because f is almost intuitionistic (1,2)- open and almost intuitionistic (2, 1)-open . But f does not satisfy pair wise intuitionistic openness, because f_1: (X, τ_1) $\longrightarrow$ (Y, σ_1) and f_2 : (X, τ_2) $\longrightarrow$ (Y, σ_2) are not an intuitionistic open .

Example 4.5. Let X = $\{a,b\}$, τ_1 =$\{\tilde{\Phi},\tilde{X},A\}$, where A = $\langle x,\emptyset,\{a\}\rangle$, τ_2=$\{\tilde{\Phi},\tilde{X},B,C\}$, where $\langle x,\emptyset,\emptyset\rangle$, C = $\langle x,\emptyset,\{a\}\rangle$. Also Y = $\{a,b\}$, σ_1 =$\{\tilde{\Phi},\tilde{X},A\}$ and σ_2=$\{\tilde{\Phi},\tilde{X},B,C\}$.

Then a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is weakly pair wise intuitionistic open, because f is weakly intuitionistic (1, 2)-open and weakly intuitionistic (2, 1)-open . But f does not satisfy almost pair wise intuitionistic open , because f is not almost intuitionistic (1, 2)- open and almost intuitionistic (2, 1)-open .

Definition 4.6. An intuitionistic bitopological space (X, τ_1 , τ_2) is said to be intuitionistic (i, j)-semi-regular if for each $\dot{x} \in$ X and each an intuitionistic τ_i-open set U containing $\dot{x}$, there exists an intuitionistic (i ,j)-regular open set V of X such that $\dot{x}$ $\in$ V $\subset$ U. (X, τ_1 , τ_2) is said to be pair wise intuitionistic semi-regular if it is intuitionistic (1, 2)-semi-regular and intuitionistic (2, 1)- semi-regular.

Theorem 4.7. Let an intuitionistic bitopological space (X, τ_1 , τ_2) be pair wise intuitionistic semi-regular. Then a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is pair wise intuitionistic open if and only if it is almost pair wise intuitionistic open .

Proof. Suppose that f is almost intuitionistic (i, j) - open. Let U be any τ_i-intuitionistic open set of X. Since X is intuitionistic (i, j) - semi-regular, for each $\dot{x}$ $\in$ U there exists an intuitionistic (i, j)-regular open set $U_{\dot{x}}$ such that $\dot{x} \in U_{\dot{x}} \subset$ U. Since f is almost intuitionistic (i, j)-open, f ($U_{\dot{x}}$) is intuitionistic σ_i-open in Y. Since f (U) = $\{f(U_{\dot{x}}): \dot{x} \in U\}$ it follows that f(U) is intuitionistic σ_i-open. Therefore, f : (X, τ_i) $\longrightarrow$(Y, σ_i) is intuitionistic open for i = 1, 2 and hence f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is pair wise intuitionistic open .

The proof of conversely it is clear.

Definition4.8. An intuitionistic bitopological space (X, τ_1 , τ_2) is said to be intuitionistic (i, j)-almost regular if for each $\dot{x} \in$ X and each intuitionistic (i, j)-regular open set U containing $\dot{x}$, there exists an intuitionistic (i, j)-regular open set V of X such that $\dot{x}$ $\in$ V $\subset$ jCl(V) $\subset$ U . (X, τ_1 , τ_2) is said to be pair wise almost intuitionistic regular if it is intuitionistic (1, 2)-almost regular and intuitionistic (2,1)-almost regular.

Theorem 4.9. Let an intuitionistic bitopological space (X, τ_1 , τ_2) be pair wise almost intuitionistic regular. Then a function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is almost pair wise intuitionistic open if and only if it is weakly pair wise intuitionistic open .

Proof. Suppose that f is weakly pair wise intuitionistic open. Let U be any intuitionistic (i, j) - regular open set of X. Since X is intuitionistic (i, j)-almost regular, for each $\dot{x}$ $\in$ U there exists an intuitionistic (i, j)-regular open set $U_{\dot{x}}$ such that $\dot{x} \in$

$U_{\dot{x}} \subset$ jCl $(U_{\dot{x}}) \subset$ U. Since every intuitionistic (i, j)-regular open set is intuitionistic τ_i- open and f is weakly intuitionistic (i, j)- open, we obtain f(U) = $\cup$\{f($U_{\dot{x}}$) : $\dot{x} \in$ U\}$\subset\cup$\{iInt(f(jCl($U_{\dot{x}}$))) : $\dot{x} \in$ U \} $\subset$ \{iInt($\cup$f(jCl($U_{\dot{x}}$))) : $\dot{x} \in$ U \}= \{iInt(f(jCl($U_{\dot{x}}$))) : $\dot{x}\in$ U \} $\subset$ iInt(f(U)). Therefore, f(U) $\subset$ iInt(f(U)) and hence f(U) is an intuitionistic σ_i- open. Thus, f is almost intuitionistic (i, j) - open for i $\neq$j, i, j =1, 2.

The proof of conversely it is clear.

Definition 4.10. An intuitionistic bitopological space (X, τ_1 , τ_2) is said to be intuitionistic (i, j)-regular if for each $\dot{x} \in$ X and each an intuitionistic τ_i-open set U containing $\dot{x}$, there exists an intuitionistic τ_i-open set V such that $\dot{x} \in$ V $\subset$ jCl(V) $\subset$ U . (X, τ_1 , τ_2) is said to be pair wise intuitionistic regular if it intuitionistic (1, 2)- regular and intuitionistic (2, 1)-regular.

Corollary 4.11. Let (X, τ_1, τ_2) be a pair wise intuitionistic regular space . For a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) ,the following properties are equivalent:

1. f is pair wise intuitionistic open.
2. f is almost pair wise intuitionistic open.
3. f is weakly pair wise intuitionistic open .

Proof. This is an immediate consequence of Theorems 4.7 and 4.9 since every pair wise intuitionistic regular space is pair wise -intuitionistic semi-regular and pair wise almost -intuitionistic regular.

Definition 4.12. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be intuitionistic strongly continuous if f(Cl(A)) $\subset$ f(A) for every intuitionistic subset A of X .

Theorem 4.13. If a function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is weakly intuitionistic (i, j)- open and strongly intuitionistic j-continuous, then f is intuitionistic i-open .

Proof. Let U be any intuitionistic τ_i- open set of X. Since f is weakly intuitionistic (i, j)- open and strongly intuitionistic j-continuous, we have f(U)$\subset$ iInt(f(jCl(U))) $\subset$ iInt((f(U)) .

Therefore, f (U) = iInt (f (U)) and f (U) is σ_i-intuitionistic open. Hence f is intuitionistic i- open.

Definition 4.14. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to have the weak intuitionistic (i, j)-interiority condition if iInt(f(jCl(U))) $\subset$ f(U) for every intuitionistic τ_i-open set U of X .

Theorem 4.15. If a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is weakly intuitionistic (i, j)- open and satisfies the weak intuitionistic (i, j)- interiority condition, then f is intuitionistic i- open .

Proof. Let U be any intuitionistic τ_i-open set of X. Since f is weakly intuitionistic (i, j)-open and satisfies the weak intuitionistic (i, j) - interiority condition, we have f(U) $\subset$ iInt(f(jCl(U))) = iInt(iInt(f(jCl(U)))) $\subset$ iInt((f(U)). Therefore, f(U) = iInt(f(U)) and f(U) is intuitionistic σ_i- open . Hence f is intuitionistic i-open.

5. Some properties of weakly (i, j) - intuitionistic open functions

Definition 5.1. An intuitionistic bitopological space (X, τ_1 , τ_2) is said to be an intuitionistic (i, j)-hyper connected if jCl(U) = X for every intuitionistic τ_i-open set U of X .

Theorem 5.2. Let (X, τ_1 , τ_2) be intuitionistic (i, j)-hyper connected space . Then a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1,σ_2) is weakly intuitionistic (i, j)- open if and only if f(X) is intuitionistic σ_i-open in Y.

Proof. Let f be weakly intuitionistic (i, j)-open. Since X is intuitionistic τ_i-open, f(X) $\subset$iInt (f(jCl(X))) = iInt(f(X)). Therefore, f(X) is intuitionistic σ_i- open in Y, suppose that f(X) is intuitionistic σ_i- open in Y. Let U be intuitionistic τ_i-open in X. Then f(U) $\subset$ f(X) = iInt(f(X)) = iInt(f(jCl(U))). Therefore, f(U) $\subset$ iInt(f(jCl(U))). This shows that f is weakly intuitionistic (i, j) - open.

Definition 5.3. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be intuitionistic (i, j)- contra-closed if f(F) is intuitionistic σ_i-open in Y for every intuitionistic τ_j-closed set F of X .

Theorem 5.4. If a function f: (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is intuitionistic (i, j)-contra- closed, then f is weakly intuitionistic (i, j)- open .

Proof. Let U be any intuitionistic τ_i-open set of X. Then jCl (U) is intuitionistic τ_j- closed in X. Hence, we have f(U) $\subset$ f(jCl(U)) $\subset$ iInt(f(jCl(U))).Therefore, f is weakly intuitionistic (i,j)- open .

Definition 5.5. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be intuitionistic (i , j)- contra-open if f(U) is intuitionistic σ_j- closed in Y for every intuitionistic τ_i-open set U of X.

Theorem 5.6. If a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is intuitionistic (i, j)-pre-open and intuitionistic (i, j)-contra-open, then f is intuitionistic i-open **.**

Proof. Let U be any intuitionistic τ_i-open set of X. Since f is intuitionistic (i, j)-pre- open, f(U) $\subset$ iInt(jCl(f(U)))**.** Since f is intuitionistic (i, j)-contra-open, f(U) is intuitionistic σ_j-closed .Therefore, f(U) $\subset$ iInt(jCl(f(U))) = iInt(f(U)). Hence f(U) is intuitionistic σ_i-open in Y**.** This shows that f is intuitionistic i-open**.**

Corollary 5.7. If a function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is intuitionistic (i, j)- semi-open and intuitionistic (i, j)- contra-open, then f is intuitionistic i-open **.**
Proof. Obvious.

Definition 5.8. A function f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is said to be intuitionistic (i, j)- contra-continuous if $f^{-1}(F)$ is intuitionistic(i, j)-closed set in X for every intuitionistic σ_i- open set F of Y **.**

Lemma 5.9. If f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is a bijective and intuitionistic (i, j)- semi-open function, then jInt(iCl(f(F))) $\subset$ f(F) for every intuitionistic τ_i-closed set F of X**.**
Proof. Let F be any intuitionistic τ_i- closed set of X **.** Then X/ F is intuitionistic τ_i- open in X**.** Since f is intuitionistic (i, j) **-** semi-open, f(X / F) $\subset$ jCl (iInt (f(X / F)))**.** Therefore, Y / f(F) = f(X / F) $\subset$ jCl(iInt(f(X /F))) = jCl(iInt(Y / f(F))) = Y / jInt(iCl(f(F)))**.** Therefore jInt(iCl(f(F))) $\subset$ f(F)**.**

Theorem 5.10. If f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is an intuitionistic (i, j)- pre-open and intuitionistic (j, i)-semi-open bijection, then f is weakly intuitionistic (i, j)- open **.**
Proof. Let U be any intuitionistic τ_i-open set of X. Then jCl (U) is intuitionistic τ_j- closed in X. Since f is intuitionistic (j, i)-semi-open, by Lemma 5.7 iInt(jCl(f(jCl(U)))) $\subset$ f(jCl(U))**.** Since f is intuitionistic (i, j)-pre-open, f(U) $\subset$ iInt(jCl(f(U)))**.** Therefore, f(U) $\subset$ iInt(f(jCl(U)))**.** Hence f is weakly intuitionistic (i, j)-open**.**

Corollary 5.11. If f : (X, τ_1 , τ_2)$\longrightarrow$(Y,σ_1, σ_2) is an intuitionistic pre-open and intuitionistic semi-open bijection, then f is weakly intuitionistic open **.**
Proof. Obvious.

Definition 5.12. An intuitionistic bitopological space (X, τ_1 , τ_2) is said to be pair wise connected if it cannot be expressed as the union of two nonempty disjoint sets U and V such that U is intuitionistic τ_i-open and V is intuitionistic τ_j-open **.**

Theorem 5.13. If an intuitionistic bitopological space (Y, σ_1, σ_2) is pair wise connected and f : (X, τ_1 , τ_2) $\longrightarrow$(Y, σ_1, σ_2) is a pair wise weakly intuitionistic open bijection, then the an intuitionistic bitopological space (X, τ_1 , τ_2) is pair wise connected **.**

Proof. Suppose that the an intuitionistic bitopological space (X, τ_1 , τ_2) is not pair wise connected. Then there exist an intuitionistic τ_i-open set U_1 and intuitionistic τ_j-open set U_2 such that $U_1 \neq \emptyset$, $U_2 \neq \emptyset$ and $U_1 \cap U_2$ = X**.** Since f is bijective, we have f(U_1) $\neq \emptyset$, f(U_2) $\neq \emptyset$, f(U_1) $\cap$ f(U_2) $= \emptyset$ and f(U_1) $\cup$ f(U_2) = Y **.** Since f is pair wise weakly intuitionistic open f(U_1) $\subset$ iInt(f(jCl(U_1))) and f(U_2) $\subset$jInt(f(iCl(U_2)))**.** Since U_1 and U_2 are an intuitionistic τ_j-closed and intuitionistic τ_i-closed, respectively, we have f(U_1)$\subset$ iInt(f(U_1)) and f(U_2) $\subset$ jInt(f(U_2)) and hence f(U_1) = iInt(f(U_1)) and f(U_2) = jInt(f(U_2))**.** Therefore, f (U_1) is intuitionistic σ_i-open and f(U_2) is intuitionistic σ_i-open **.** This is contrary to the hypothesis that an intuitionistic bitopological space (Y, σ_1, σ_2) is pair wise connected **.** Therefore, the intuitionistic bitopological space (X, τ_1 , τ_2) is pair wise connected**.**

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بعض خواص الدوال المفتوحه الضعيفه في الفضاءات ألحدسيه ثنائية التبولوجي

رنا بهجت ياسين

قسم الرياضيات ، كلية التربية للبنات ، جامعة تكريت ، تكريت ، العراق

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الملخص

في عام (2006) تكاشي نويري وفاليروي بوبا [10] قدموا ودرسوا خصائص وخواص الدوال المفتوحه الضعيفه في الفضاءات ثنائية التبولوجي.
في هذا البحث عممنا هذا المفهوم الى الفضاءات ألحدسيه ثنائية التبولوجي وحصلنا على خصائص جديده للدوال المفتوحه الضعيفه بين الفضاءات ثنائية التبولوجي ألحدسيه وعلاوة على ذلك تحرينا بعض الخواص لهذه الدوال مقارنة بالدوال ذات العلاقة.