

Comparison of Theoretical Ultimate Bearing Capacity of Cohesionless Soils with Experimental And Field Data

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Abstract

Estimation of the ultimate bearing capacity of the soil supporting foundation is one of the important requirements to develop safe design of foundations. There are many methods used to estimate bearing capacity of soil such as theoretical, empirical and field tests. There are different methods proposed to estimate the theoretical ultimate bearing capacity. In this work, a statistical analysis is carried out to compare the obtained results using theoretical equations with the results of experimental and field tests. Data of ninety seven experimental and field tests on cohesionless soils were used to conduct this analysis. The acceptance of the results has been studied for theoretical equations. This revealed that all theoretical equations can be used to estimate ultimate bearing capacity. Whereas, the results of statistical tests showed that the results of Meyerhof's bearing capacity equation gives more accurate results than that obtained by other theoretical equations (Terzaghi, Hansen and Vesic' equations).

Keywords: Ultimate bearing capacity, Meyerhof's equation, Terzaghi's equation, Hansen's

الخلاصة

إن تقدير قيمة قابلية التحمل القصوى للتربة تحت الأساس السطحية أحد أهم المسائل التي تواجه المهندسين للحصول على تصميم آمن لأسس المنشآت الهندسية. هناك العديد من الطرق الشائعة الاستخدام لحساب قابلية التحمل القصوى للتربة ومن أهمها الطرق النظرية التي تعتمد على خصائص التربة والطرق التجريبية التي تعتمد على نتائج الفحوصات الحقلية. في هذا البحث تم إجراء تحليل إحصائي لدراسة نتائج المعادلات النظرية ومقارنتها بنتائج الفحوصات المختبرية والحقلية لـ (٩٧) نموذج. لقد تمت دراسة مقبولة نتائج أربع معادلات نظرية في حساب قابلية التحمل القصوى للتربة من خلال استخدام مبدأ اختبار الفروض، وكذلك تم حساب مجموعة من المعاملات الإحصائية لمقارنة نتائج المعادلات النظرية مع نتائج الفحوصات المختبرية والحقلية لتحديد المعادلة الأكثر دقة في حساب قابلية التحمل القصوى للتربة. ومن خلال نتائج البحث فإن المعادلات النظرية الأربع تعتبر مقبولة لاستخدامها في حساب قابلية التحمل القصوى، كما أن نتائج البحث أظهرت أن النتائج المستحصلة باستخدام المعادلة المقترحة من قبل ماير هوف تعتبر أكثر دقة من نتائج بقية المعادلات النظرية.

الكلمات المفتاحية: قابلية التحمل القصوى، معادلة ماير هوف، معادلة ترزاكي، معادلة هانسن

1. Introduction

Foundation is a part of the structure which is used to transmit the structural loads to the soil layer(s). Foundations are classified mainly based on the depth to width ratio into two categories: shallow foundations and deep foundations. Shallow foundations are the most common type of foundation used in the traditional structures. Any safe foundation must be designed to ensure that there is no risk of shear failure in the supporting soil and there is no successive settlement more than the tolerable amount (Jumikis, 1971).

The first function is satisfied by applying a total pressure not more than the allowable soil capacity. The term allowable capacity is meant the ultimate soil capacity divided by an amount of safety factor. Hence, the determination of the ultimate soil capacity is a very important mission of the geotechnical engineers (Bowles, 1996).

Prandtl in 1921 and Reissner in 1924 were the pioneers who considered a rigid loaded strip. Terzaghi, Meyerhof and DeBeer studies were directed towards understanding the bearing capacity of shallow foundations in saturated or dry conditions using the

conventional soil mechanics. However, shallow foundations are found to be built close to the ground surface where the soils are unsaturated (Mohamed, 2014).

There are several methods proposed to determine the ultimate soil capacity. These methods can be categorized as: theoretical methods based on the soil properties and empirical methods based on the data of field tests such as SPT, CPT and PLT (Bowles, 1996).

The most accurate tool to predict the footing behavior and also to check the validity of the methods mentioned above is the field test of the footing. This technique is considered impractical due its cost and it's a time consume method. So, the experimental models are used to check the validity of the theoretical and empirical methods.

The aim of this paper is to answer the following two questions about theoretical equations: firstly, which equation can give acceptable values of ultimate bearing capacity? Secondly, which one can give more accurate results? So, statistical tests for the results of the theoretical equations with experimental and field tests were conducted in this paper. Ninety seven results of experimental and field tests published in different literatures were adopted.

2. Collecting Data

The data used for comparison were collected from literatures. These data include load test data of experimental and field models. The collected data consist of footing geometry {width of footing (B), footing shape (L/B) and footing depth (D)}, soil properties {unit weight (γ) and angle of internal friction (ϕ)} and finally the corresponding ultimate soil capacity (q_u).

The ultimate soil capacity reported in the literatures was computed in different ways for large scale footing and for small scale models. For large scale footing the ultimate soil capacity was defined as the load corresponding to the point of the minimum slope on the load – settlement curve. On the other hand, the ultimate soil capacity of the small scale model was defined as the load corresponding to the breaking point of the load – settlement curve in log – log scale.

Table (1) shows the references of the data and all parameters related to them (Padmini *et al.*, 2007).

Table (1): Useful data about used results (After Padmini *et al.*, 2007)

No. of tests	Test Type	B (m)	D (m)	L/B	γ (kN/m ³)	ϕ (deg)	q_u (kPa)
5 ^a	LSF*	0.6	0 – 0.3	2	9.85 – 10.85	34.9 – 44.8	270 – 1760
11 ^b	LSF*	0.5 – 0.52	0 – 0.3	1 – 3.85	10.2	37.7	154 – 681
24 ^c	LSF*	0.5	0 – 0.5	1 – 4	11.7 – 12.41	37 – 44	109 – 2847
2 ^d	LSF*	1	0 – 0.2	3	11.93 – 11.97	39 – 40	630 – 710
5 ^e	LSF*	0.991 – 3.016	0.711 – 0.889	1	15.8	32	1019.4 – 1773.7
50 ^f	SSM#	0.0585 – 0.152	0.029 – 0.15	1 – 6	15.7 – 17.1	34 – 42.5	58.5 – 423.5

^a Muhs *et al.* (1969)

^b Weiß (1970)

^c Muhs & weiß (1971)

^d Muhs & weiß (1973)

^e Briaud & Gibbens (1999)

^f Gandhi (2003)

* LSF: Large scale footing.

Small scale model.

3. Theoretical Methods of Ultimate Capacity

There are many different equations proposed to predict the ultimate bearing capacity of soil supporting shallow foundation. Among these, four equations are chosen to conduct the comparison in this study. These equations were proposed by Terzaghi, Meyerhof, Hansen and Vesic'. These equations are selected because they are commonly

used in practice. Figure (1) illustrates the scattering of the results of the estimated ultimate soil capacity by the four equations with that of the selected experimental and field tests.

It can be noted from the figure above that there is a high scattering between the observed ultimate bearing capacity from experimental or field models with the computed values using the theoretical equations. The ultimate bearing capacity computed using Terzaghi or Hansen or Vesic equations is sometimes overestimated compared with the observed ultimate capacity of the soil under shallow foundations. Due to the high scattering of the results indicated in Figure (1), statistical tests will be used for checking the validity and the accuracy in the following section.

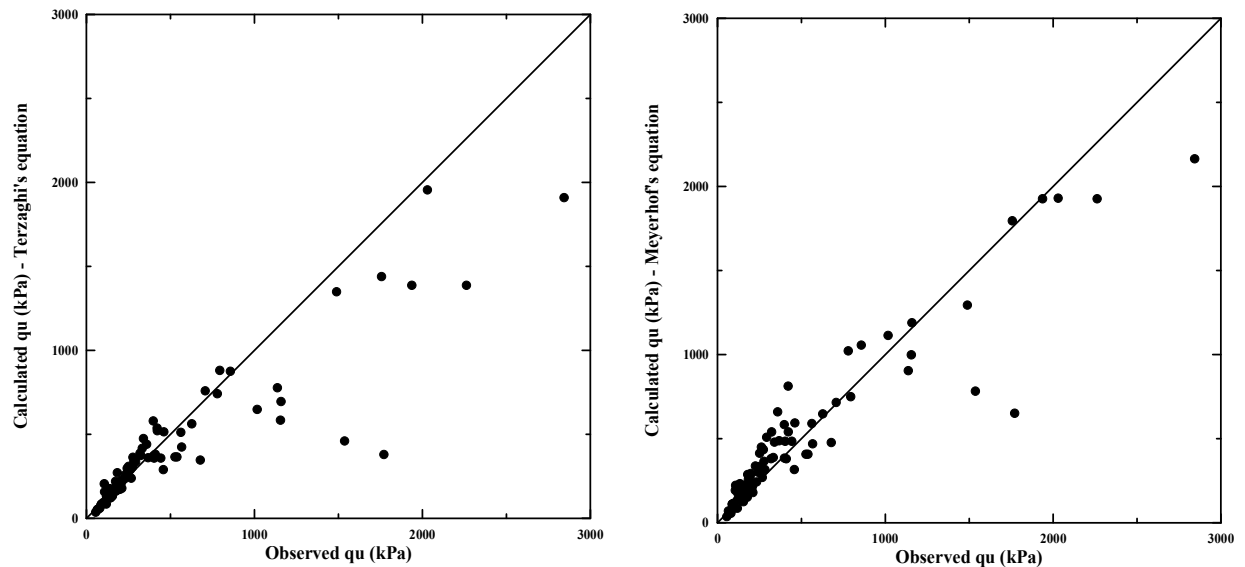


Figure (1): Scattering of ultimate bearing capacity predicted by theoretical equations with the results of experimental and field tests

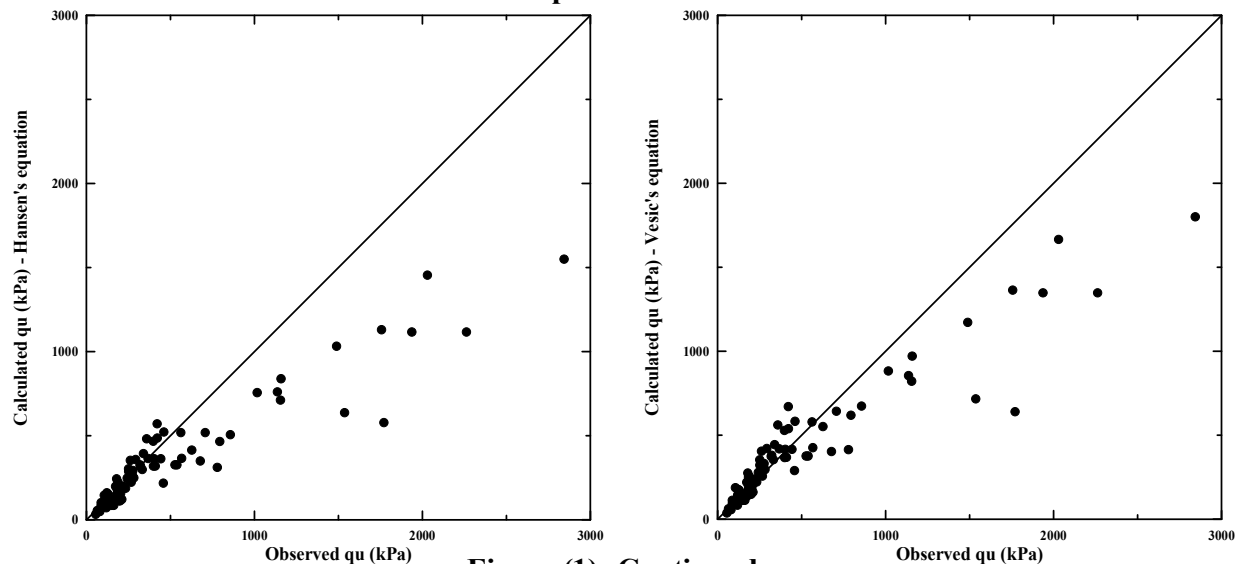


Figure (1): Continued.

4. Statistical Analysis

In order to make a decision about the validity and accuracy of the theoretical equations used to estimate the ultimate bearing capacity of the soil supporting shallow foundation, statistical analysis was carried out as follows:

4.1 Acceptance of Theoretical Equations

In order to compare the results of the theoretical equations with that of experimental or field tests, inferential study with the concept of hypothesis test was used in this study. The collected data and the computed ultimate bearing capacities were assumed to be random samples. The hypothesis was implemented to determine whether these samples are from the same or equal populations. In other words, it is required to see if there is a difference in the mean of collected values and the mean of the computed values of the ultimate bearing capacity. Thus, a two-tailed test was applied with significant level of 0.05. The null and alternative hypotheses are as following:

$$H_0 : \mu_1 = \mu_2$$

$$H_1 : \mu_1 \neq \mu_2$$

where: μ_1 and μ_2 : the mean values of actual bearing capacity and computed bearing capacity respectively.

The hypothesis test needs the mean and standard deviation of the samples used in the study. There are several statistical tests used in the hypothesis test such as Z-test, t-test and Chi-test. The selection of the suitable statistical test depends on the size of the sample and the nature of the sample (e.g., normal or not) (Sullivan, 2005).

In the present study all samples (observed and computed ultimate bearing capacity) were tested to check their normality using the statistical package MINITAB 17.0 by plotting the probability plots. Figure (2) demonstrates the probability plots of all data. It can be seen that all data samples are not normal. Nevertheless, the t-test was used because the sample size is more than 30 observations in all samples. Table (2) demonstrates the statistical parameters and the results of the hypothesis test of the collected data with the four selected theoretical equations of ultimate bearing capacity.

From the results shown in the Table (2) it can be noted that for all theoretical equations the null hypothesis can be accepted. Hence, there is significant evidence at the $\alpha = 0.05$ level of significant to support the claim that there is no significant difference in the ultimate bearing capacity between the field or experimental tests and the theoretical equations. So, all theoretical equations mentioned in this work are accepted to estimate the ultimate bearing capacity of shallow foundations.

Table (2): Statistical parameters and test hypothesis of the collected and computed ultimate bearing capacity

Parameters	Data of experimental and filed tests	Data of q_u by:			
		Terzaghi's equation	Meyerhof's equation	Hansen's equation	Vesic's equation
min.	58.5	32.59	32.06	28.86	33.26
max.	2847	1952.17	2161.33	1546.16	1797.18
$\bar{x}$	439.62	363.06	439.41	310.15	365.24
s	530.88	370.07	445.94	300.58	351.06
Df	-	171	186	152	166
t – value	-	1.165	0.003	2.090	1.151
t – limit	right-side	2.26	2.26	2.262	2.423
	left-side	-2.26	-2.26	-2.26	-2.42
Conclusion of about null hypothesis (H_0)		Accept	Accept	Accept	Accept

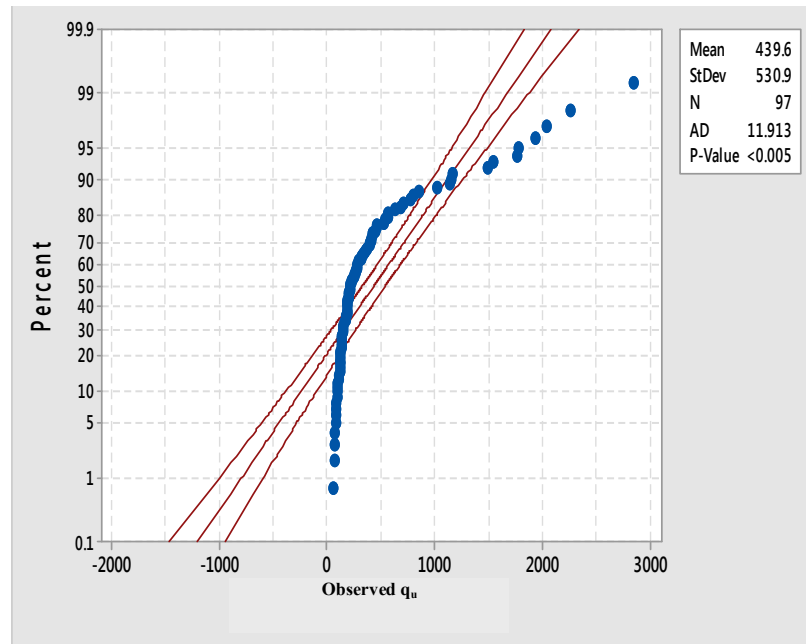
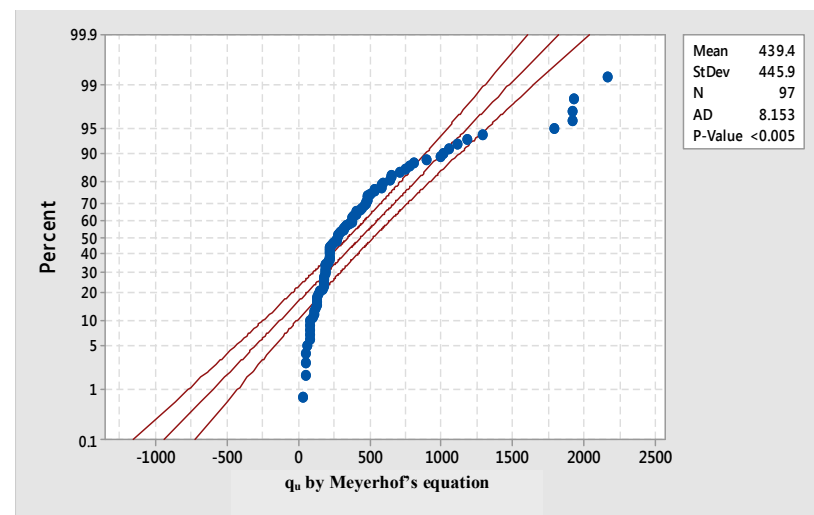
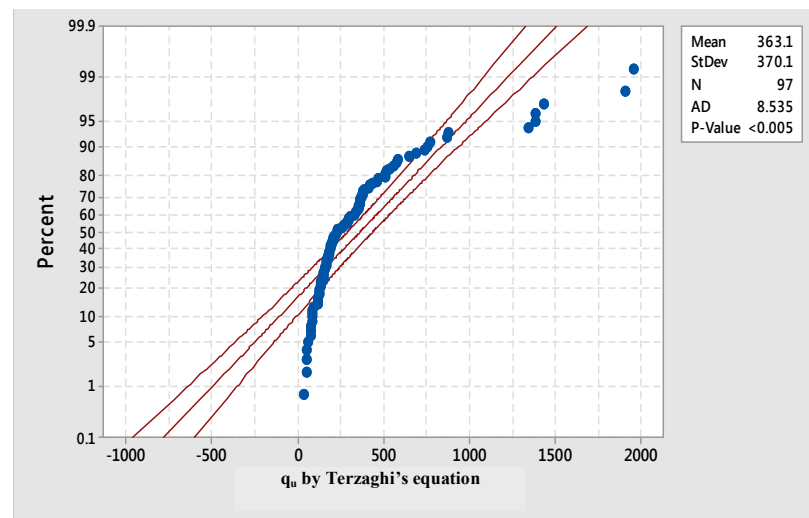


Figure (2): Normality test of the ultimate bearing capacity predicted by theoretical equations and observed by experimental and field tests



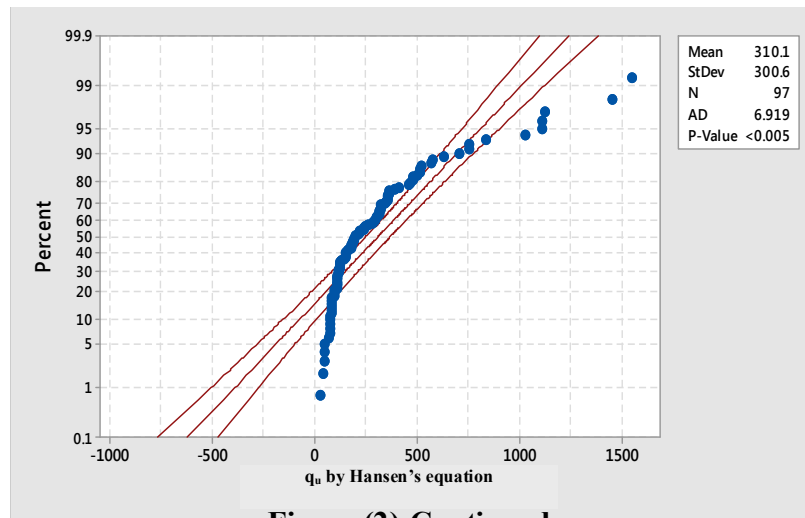


Figure (2):Continued.

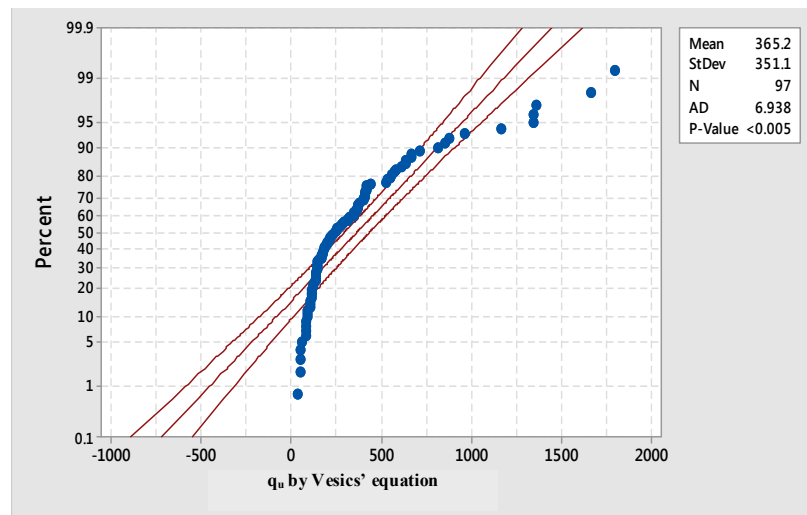


Figure (2):Continued.

4.2 Accuracy of Theoretical Equations

It is still necessary to decide which one of the four theoretical equations is more suitable in computing the ultimate bearing capacity of the soil supporting shallow foundations. So, there are several tests can be used to assess the accuracy of the results obtained by theoretical equations comparing with the experimental and field tests. Table (3) shows the used tests throughout this work and the expressions used for computing them. Table (3) indicates different types of statistical tests. Theil's Inequality Coefficient has been considered to be more sensitive and accurate than the others (Hourdakis *et al.*, 2003).

Table (3): Statistical coefficients and their expressions

Coefficient			Expression
No.	Name	Symbol	
1	Root mean square error	RMSE	$RMSE = \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{predicted} - x_{observed})_i^2}$
2	Root mean square percent	RMSP	$RMSP = \sqrt{\frac{1}{N} \sum_{i=1}^N \left(\frac{x_{predicted} - x_{observed}}{x_{observed}} \right)_i^2}$
3	Theil's inequality coefficient	U	$U = \frac{\sqrt{\frac{1}{N} \sum_{i=1}^N (x_{predicted} - x_{observed})_i^2}}{\sqrt{\frac{1}{N} \sum_{i=1}^N (x_{predicted})_i^2} + \sqrt{\frac{1}{N} \sum_{i=1}^N (x_{observed})_i^2}}$
4	Bias proportion	U _m	$U_m = \frac{N \cdot (\bar{x}_{predicted} - \bar{x}_{observed})^2}{\sum_{i=1}^N (x_{predicted} - x_{observed})_i^2}$
5	Variance proportion	U _s	$U_s = \frac{N \cdot (s_{predicted} - s_{observed})^2}{\sum_{i=1}^N (x_{predicted} - x_{observed})_i^2}$

In light of what's mentioned above, the subscript (predicted) referred to the computed ultimate capacity computed by equations and the subscript (observed) referred to the ultimate capacity from experimental or field tests.

Table (4) illustrates the results of all tests mentioned above. It can be stated that the results of ultimate capacity using Meyerhof's equation show good matching with the results of experimental and field tests. The results obtained using Meyerhof's equation give the lowest value of RMSE (188.24 kPa) when it's compared with other equations. Also, it gives the lowest values of U, U_m and U_s coefficients 0.14, 0.00000126 and 0.204 respectively which approach to the recommended value (zero). Whereas, the use of Hansen's equation in the estimating the ultimate bearing capacity of the shallow foundation gives scattered results compared with the experimental and field tests. It reveals a high value of RMSE (296.05) and highest values of U, U_m and U_s coefficients (0.26, 0.191 and 0.605), respectively.

Table (4): Values of statistical coefficients for theoretical equations compared with results of experimental and field tests

		RMSE	RMSP	U	U _m	U _s
Predicted data by:	Terzaghi's eq.	258.16	2.50	0.21	0.088	0.388
	Meyerhof's eq.	188.24	3.46	0.14	1.265E-06	0.204
	Hansen's eq.	296.05	2.98	0.26	0.191	0.605
	Vesic's eq.	240.63	2.60	0.20	0.096	0.558

5. Conclusions

The following points can be concluded from the results of the present study:

1. The estimated ultimate bearing capacity of the cohesionless soils using theoretical equations proposed by Terzaghi, Meyerhof, Hansen and Vesic' is acceptable compared with the results of experimental and field tests.
2. The use of equation proposed by Meyerhof to estimate the ultimate bearing capacity gives more accurate results than that obtained using the other equations (Terzaghi, Hansen and Vesic' equations).

6. References

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LIST OF SYMBOLS

B	<i>width of footing</i>	$\bar{x}$	<i>Mean of the sample</i>
D	<i>Depth of footing</i>	df	<i>Degree of freedom</i>
L/B	<i>Width to length ratio</i>	s	<i>Sample standard deviation</i>
γ	<i>Soil unit weight</i>	q_u	<i>Ultimate bearing capacity</i>
ϕ	<i>Angle of internal friction of the soil</i>	μ	<i>Mean of the population</i>