A.K. Hamoudi
Department of Control and Systems Engineering, University of Technology
ahmed_khk22@yahoo.com

N.O. AbdulRahman
Department of Control and Systems Engineering, University of Technology
rb_6_1981@yahoo.com

Received on: 01/09/2016
Accepted on: 29/12/2016

Improved Sliding Mode Controller for a Nonlinear System Based on a Particle Swarm Optimization Technique

Abstract*- In this paper, the performance of the classical Sliding Mode Controller (CSMC) is improved by using the Particle Swarm Optimization (PSO) technique. The PSO technique is used to obtain the optimal values of the CSMC parameters such as the gain K and the slope of sliding surface λ, and as a result, the response of the system is improved by decreasing the settling time of the system and makes the steady state error equal to zero. A single inverted pendulum, which is considered an example of a nonlinear system, has been used for testing the proposed controller. The simulation results are implementing by using the Programming and Simulink of Matlab, and the result shows a good validity of the proposed controller.*

Keywords*- Classical sliding mode controller, the particle swarm optimization (PSO), chattering phenomenon, boundary layer.*

How to cite this article: A.K. Hamoudi and N. Abdulrahman, "Improved Sliding Mode Controller for a Nonlinear System Based on a Particle Swarm Optimization Technique," *Engineering and Technology Journal*, Vol. 35, Part A, No. 2, pp. 155-162, 2017.

1. Introduction

In this work, the design of CSMC is presented to control a nonlinear system. The Classical Sliding Mode Controller (CSMC), which is a particular type of Variable Structure Control system (VSCS), is provided an effective and robust control method for controlling nonlinear systems. In 1950, the Classical Sliding Mode Controller was first introduced and many researchers have developed this method in recent years. The classical sliding mode control is considered as a newly popular and strong method to control various nonlinear control system problems; because this controller gives robustness to nonlinear systems with good response properties [1]. The advantages of the CSMC when it's used, is stand to its insensitive to the parameters uncertainty and external disturbance, reduction in order by one from the original system, and finally the system that used the sliding mode controller will be satisfied the case of the asymptotically stable (i.e., the error and the derivative error equal zero). The disadvantages of the CSMC when it has used the system was suffering from the "chattering phenomenon". Chattering phenomenon is considering as undesirable property that appear in the control action as shown in Figure 1 and this problem was affected the stability of the controlled system [2]. For this reason, reducing the chattering is a very important matter. Many methods were proposed to reduce this problem such as replacing the Sign function that was used in CSMC by a saturation function [3]. Other researchers introduced to use the intelligent controller such as fuzzy logic with SMC to get a modified controller named a sliding mode fuzzy controller (SMFC) [4]. In addition, some researchers propose to reduce the chattering phenomenon by using a genetic algorithm such as in [5]. The coefficient (λ) which is the slope of switching surface is considered as a very important parameter in designing the CSMC due to its effective on the performance of the system, so its value must be carefully choice. For this reason, many methods were proposed in [6] to find a suitable value of the CSMC parameters. The PSO technique is one method that was proposed to obtain the optimal values of the controller parameters. This method was introduced by Reference [7]. The PSO method is adopted in this work to find the best values of the CSMC parameters. The PSO method is used to the find suitable value of the switching surface slope (λ) and the switching gain (k) [8]. These optimal values of λ and k improved the performance of the system by reducing the chattering phenomenon and reducing the settling time.

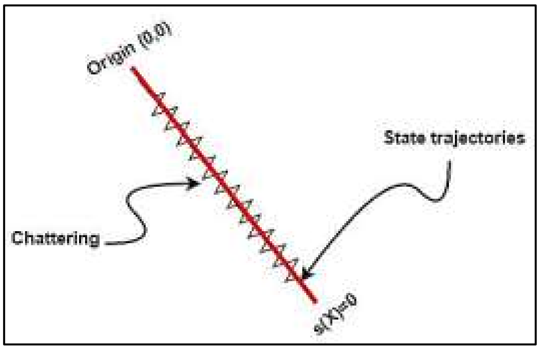

Figure 1: The chattering phenomenon in SMC [9]

https://doi.org/10.30684/etj.35.2A.10
2412-0758/University of Technology-Iraq, Baghdad, Iraq

2. Classical Sliding Mode Control (CSMC)

The Sliding Mode Controller is a simple and effective method. It is considered as a one of the most powerful technique and a very strong mathematical tool that gives robust control with a perfect performance for control system. This method was proposed to solve the most challenging problem in the designing a control procedure for a nonlinear system, additionally it has solved the problems for a large variation in dynamic system parameters and nonlinearities [10].

The Sliding Mode Controller is consisting from two main phases:

Reaching phase: the trajectory in this phase is oriented toward the sliding surface. When the state trajectory hits the sliding surface, at this instant, the sliding phase will be started.

Sliding phase: The system state trajectory is enforced in this phase to stay on the sliding surface and to slide towards the origin in finite time as shown in Figure 2.

The switching surface can be described as:

$$s(x,t)=\lambda_e+e;\lambda>\dot{\theta} \quad (1)$$

Where, λ is constant parameter with a positive value. Let the error and its derivatives defined as $x_1 = e = \theta - \theta_f$ and $x_2 = \dot{e} = \dot{\theta}$

Where θ_f is the final angle position (set point) that can be considered as the step input. So the equation (1) rewritten as:

$$s(x,t)=\lambda x_1+x_2 \quad (2)$$

Let the value of λ equal one**,** therefore equation 2 will be rewritten as below

$$s(x,t)=x_1+x_2 \quad (3)$$

The control law of the SMC is defined as:

$$u=u_{eq}+u_{dis} \quad (4)$$

Where, u_{eq} is the equivalent (nominal) control part which is responsible to orient the system state trajectory toward sliding surface ($s = 0$), and u_{dis} is the discontinuous control part that required to maintain the system state trajectory close to the switching surface if it's want to go out this surface.

The discontinuous control u_{dis} is described as below [3]:

$$u_{dis}=-k(x)sign(s) \quad (5)$$

Where, $k(x)$ is a discontinuous gain and $sign(s)$ is a signum function that is described in Figure 3.

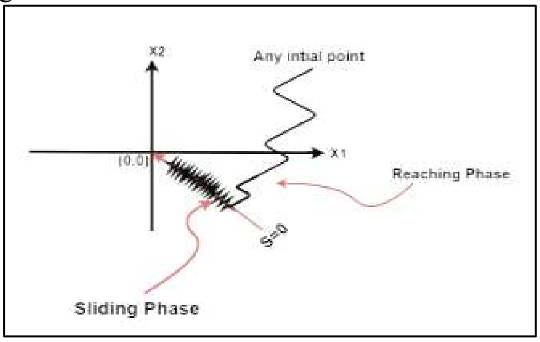

Figure 2: The phase plane of the State trajectory in SMC [3]

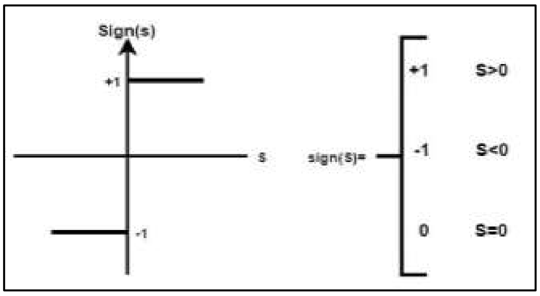

Figure 3: The signum function

Therefore, equation (4) for the control action can be rewritten as below:

$$u=u_{eq}-k(x)\times sign(s) \quad (6)$$

And the derivative of the sliding variable

$$\dot{s} = \dot{x}_1 + \dot{x}_2 \quad (7)$$

The major idea is to keep the $s(x,t)$ close to the sliding surface (i.e., s=0).

The general equation for any nonlinear system can be described as below:

$$\dot{x} = f(x)+B(x)u+d(x,t) \quad (8)$$

In order to make the right side in equation (7) equal to zero, the switching gain $k(x)$ must be assumed as below:

$$k(x) = k_o + \left(\frac{\frac{\partial x}{\partial s}*f(x) + \frac{\partial x}{\partial s}*d(x,t)}{\frac{\partial x}{\partial s}*B(x)}\right) \quad (9)$$

The reason that the chattering phenomenon is present along the switching surface is comes from using the "$sign$ function" in the discontinuous control part in equation (6). This phenomenon was considering as a drawback property in classical SMC, which is effected on the stability of the system.

3. SMC with Boundary Layer (using sat function)

In order to reduce the chattering phenomenon, the boundary layer will be used. In boundary layer, the $sat\ (s)$ function is used instead of $sign\ (s)$ function in control law of equation (6), and the control law will be rewritten as below [3]:

$$u = u_{eq} - k(s) \times sat(s)$$
(10)

The $sat\ (s)$ function is defined as:

$$sat\ (s) = \begin{cases} +1 & if\ \ (s \geq +\varphi) \\ \frac{s}{\varphi} & if\ \ (-1 > s\ < +1 \\ -1 & if(\ \ (s \leq -\varphi) \end{cases} \quad (11)$$

Where φ is represent the thickness of the boundary layer or the thickness of the $\boldsymbol{sat}\ (\boldsymbol{s})$ function as it is shown in Figure 4.

4. Plant Description

A single inverted pendulum is usually modeled as nonlinear system and it widely used in many researches for testing the control performance. The single inverted pendulum is adopted in this paper in order to control its position without any losses in accuracy and stability of the system performance. The nonlinear equation, which described the behavior of the single inverted pendulum, is [4]:

$$\ddot{\theta} = \frac{gsin\theta - aml(\dot{\theta})^2\ \sin\theta\ cos\theta + a\ cos\theta\ u}{l(4/3 - amcos^2\theta)} + d(t) \quad (12)$$

Where:

θ the angular position of the pendulum (in radian) and it is considered as the controlled variable (the output).

$\dot{\theta}$ the angular velocity (in radian/ second).

$k(x)$M the mass of the pendulum cart (in kilogram).

m the mass of the pendulum (in kilogram),

l the length of the pendulum (meter),

a a constant coefficient and it is equal to the $(\frac{m}{M+m})$.

g the gravity acceleration (and it value is equal to 9.8 meter/second ^2). Finally $d(t)$ is the external disturbance applied to the system.

u the control action (in volt).

The above system is suffering from presence of the disturbances and variation of parameters.

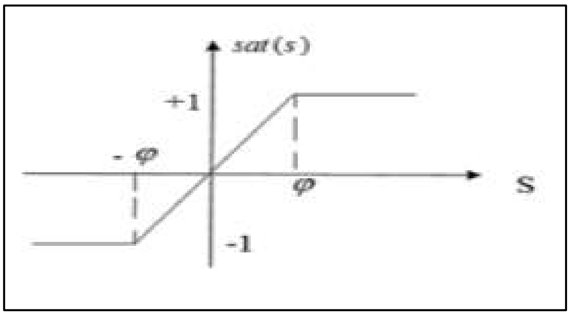

Figure 4: The $sat\ (s)$ function

The uncertainties values of above parameters are $\delta M = 0,\ \delta l = \mp 0.2,\ \delta m = \mp 0.08$.

Table 1, shows the nominal value, minimal value and maximal value when the parameters uncertainty affected the above parameters. The error in position between the desired and the actual angle of the single inverted pendulum system can be described in the state equation as below:

Let the error of position is x_1 = e $=\theta - \theta_f$, Where, θ_f is the reference position and is considered as a step input.

And let the error of angular velocity is written as; $x_2 = \dot{e} = \dot{\theta} - \dot{\theta}_f = \dot{\theta}$, since $\dot{\theta}_f = 0$ (because θ_f has constant value).

The above equation can be rewritten as:

$$\dot{x}_1 = \dot{x}_2$$

$$\dot{x}_2 = \frac{gsin(x_1 + \vartheta_f) - am_n l_n (x_2 + \dot{\vartheta}_f)^2 \sin(x_1 + \vartheta_f)\cos(x_1 + \vartheta_f) + a\cos(x_1 + \vartheta_f)\,u}{l_n\left((4/3) - am_n cos^2(x_1 + \vartheta_f)\right)} + d(x, u) \quad (13)$$

To obtain the switching gain$k(x)$. It is must assumed that the derivative of the sliding mode variable equal to zero

$$\dot{s} = \dot{x}_1 + \dot{x}_2 = 0$$
(14)

And by substituting equation (12), (13) and (5) in equation (14), we find;

$$k_o + \left(\frac{\frac{-|x_2| maxl(4/3 - a\,maxm + g - a\,maxl\,(x_2+\theta_f)^2 + d(t)maxl}{a}}{\frac{d(t)maxl\,(4/3 - a\,maxm)}{a}} \right) \quad (15)$$

Table 1: The parameters values under the effect of the parameters uncertainty

Parameter value	Nominal value	Minimal value	Maximal Value
M	1	1	1
l	0.5	0.3	0.7
m	0.2	0.12	0.28

5. Design the SMC for a Single Inverted Pendulum System

The equation (6) for design of CSMC controller can be written as:

$$u = [\frac{-x_2 - g sin(x_1+\theta_f) - am\,(x_2+\dot{\theta_f})^2 \sin(x_1+\theta_f)\cos(x_1+\theta_f)}{l(4/3 - amcos^2(x_1+\theta_f))}] - k(x) * sign(x) \quad (16)$$

Where $u_{eq} =$

$$[\frac{-x_2 - g sin(x_1+\theta_f) - am\,(x_2+\dot{\theta_f})^2 \sin(x_1+\theta_f)\cos(x_1+\theta_f)}{l(4/3 - amcos^2(x_1+\theta_f))}]$$

Also when using the saturation function, the control action will be rewritten as described in equation (10) by:

$$u = [\frac{-x_2 - g sin(x_1+\theta_f) - am\,(x_2+\dot{\theta_f})^2 \sin(x_1+\theta_f)\cos(x_1+\theta_f)}{l(4/3 - amcos^2(x_1+\theta_f))}] - k(x) * \text{sat} \quad (17)$$

6. Overview of PSO

In 1995, Dr. Eberhart and Dr. Kennedy developed the optimization technique called PSO, which is inspired by social behavior of bird flocking, or fish schooling [11]. The particle swarm optimization (PSO) is pointing to many similarities with evolutionary computation techniques such as Genetic algorithms [5]. The inertia weight was added in the modified PSO in order to improve the original PSO [12]. In several years ago, the PSO has been used for a wide range of applications and has been successfully applied in many researches and applications. The PSO technique gives better results in a short time compared with other methods.

7. Tuning the CSMC Parameters Based on PSO

When using the PSO algorithms, each particle may be considered as a solution to the problem. Each particle keeps track with the best solution (best particle fitness) in the problem space (population size). The position of each particle (i^{th}) is represented by XI= (xi1, xi2, … , xiD). Where D represent the dimensional space of each particle (population size).

The set that contains the best position of the particles is called pbest. The pbest represent the particles of minimum fitness value after a comparison is made between its current fitness value-and its previous fitness value. The pbest is defined as PI = (pi1, pi2, …, piD). The gbest is the particle of minimum fitness value among all particles in pbest set. The velocity of each i^{th} particle is represented as VI = (vi1, vi2, viD). The position of each i^{th} particle is calculated by the following equation:

$$v_{id}^{n+1} = w \times v_{id}^{n} + c_1 \times rand(\ ) \times (p_{id}^{n} - x_{id}^{n}) + c_2 \times rand(\ )(p_{gd}^{n} - x_{id}^{n}) \quad (18)$$

$$x_{id}^{n+1} = x_{id}^{n} + v_{id}^{n+1} \quad (19)$$

Where, c_1 and c_2 are constants with positive value and its value in this paper are taken as c_1 = c_2= 1.494 [7].

Rand () represent the random function and its value is between 0 and 1.

n is representing the number of iteration.

w is the inertia weight which is used in equation (18) to make a balance between the global and local search. It can be represented as a positive constant value. In order to guarantee the convergence of PSO, the value of w is assumed 0.724 as in [18].

To use the PSO algorithms, the performance index is firstly used to measure the initial fitness of each particle. There are four type of performance index, which can be defined as below:

$$ISE = \int_0^\infty e^2(t)dt \quad (20)$$

Where, c_1 and c_2 are constants with positive value and its value in this paper are taken as c_1 = c_2= 1.494 [7].

Rand () represent the random function and its value is between 0 and 1.

n is representing the number of iteration.

w is the inertia weight which is used in equation (18) to make a balance between the global and local search. It can be represented as a positive constant value. In order to guarantee the convergence of PSO, the value of w is assumed to be 0.724 as in [18].
To use the PSO algorithms, the performance index is firstly used to measure the initial fitness of each particle. There are four type of performance index which can be defined as below:

$$IAE = \int_0^\infty |e|\ dt \qquad 21)$$

$$ITAE = \int_0^\infty t|e|\ dt \qquad (22)$$

$$ITSE = \int_0^\infty t\ e^2\ dt \qquad (23)$$

In this work, the performance index in equation (20) was adopted to find the initial fitness of each particle in the population size. After that, the pbest and gbest must be calculated as it is explained above.
At each time instant, equation (18) is used to obtain the velocity of each particle according to its previous velocity and its previous position. Equation (19) is then used to calculate the new position of each particle. This new position represents the new fitness of each particle. The pbest and gbest must be calculated again.
After that, equations (18) and (19) are used again to obtain the new velocity and new position of each particle. The pbest and gbest are calculated again. This procedure is repeated repeatedly until the number of iteration is completed.
In this paper, the PSO algorithms is used to obtain an optimal value of the CSMC parameters such as the switching gain K and the switching surface slope λ. In order to tune the CSMC parameters (K and λ), the PSO algorithm introduces a matrix of initial particles in search problem space. Every initial particle can be assumed as a candidate solution for the CSMC parameters.
In this paper, the values of these parameters (K and λ) are assumed to be from (0.1 to 10). The swarm size (number of particles) is assumed 50. A matrix of size (2 x Swarm size) is used to find the position and the velocity of each particle in PSO algorithm by using equations (18) and (19) which is repeated times (=100) is considered as a number of iteration. The PSO method for tuning the parameters of CSMC, which is explained above, is used to find optimal values of (K and λ). The elements of the gbest represent the optimal values of (K and λ). These optimal values will give a good response with suitable performance as it is shown in the simulation results. In this work the maximal uncertainty that is mentioned in Table 1, is adopted as well as the external disturbance equal to 0.25.

8. The Simulation Results

I. Case (A)

The classical sliding mode controller with *sign* function were shown in Figure 5-10.

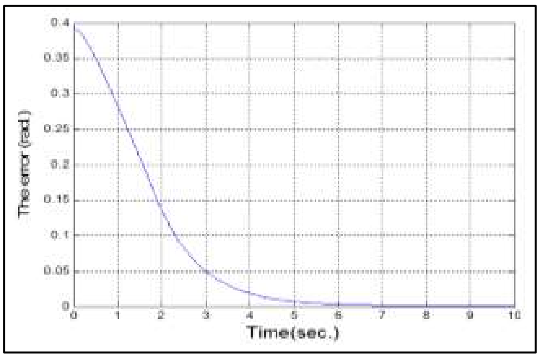

Figure 5: The derivative of error x_2 (rad.) vs. time

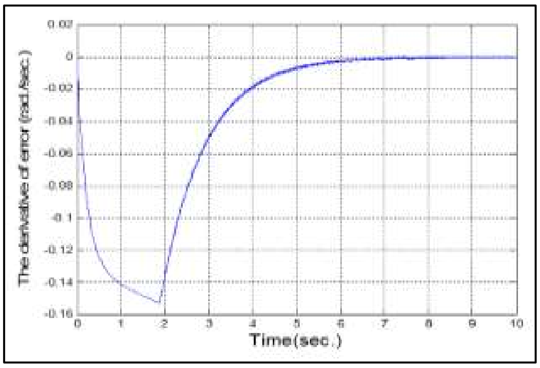

Figure 6: The error x_1(rad.) vs. time

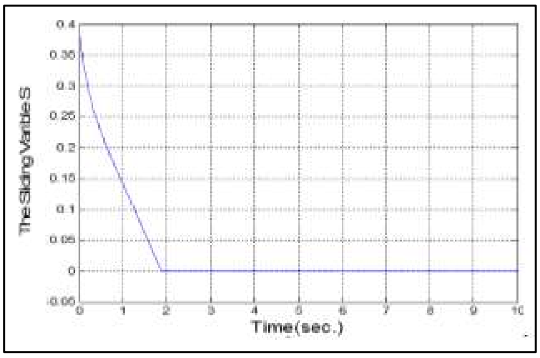

Figure 7: The sliding variable s vs. time

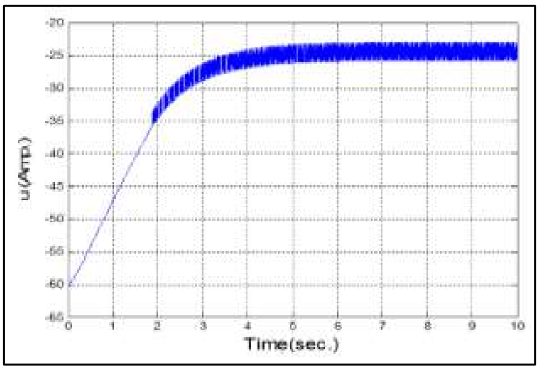

Figure 8: The control action u (Amp.) vs. time

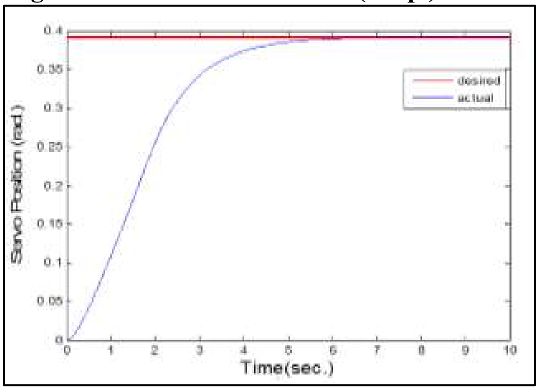

Figure 9: Actual and desired position vs. time

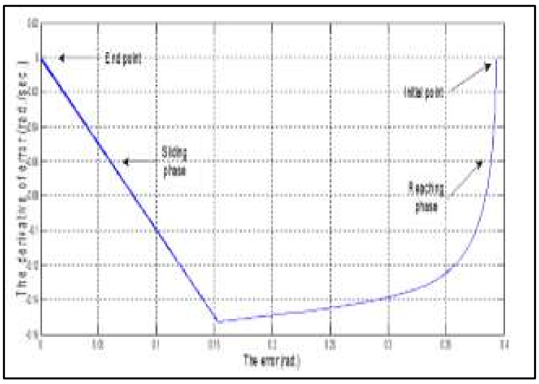

Figure 10: The graph between the error x_1 and the derivative of error x_2

II. Case (B)

The CSMC with the boundary layer were shown in Figure 11-16.

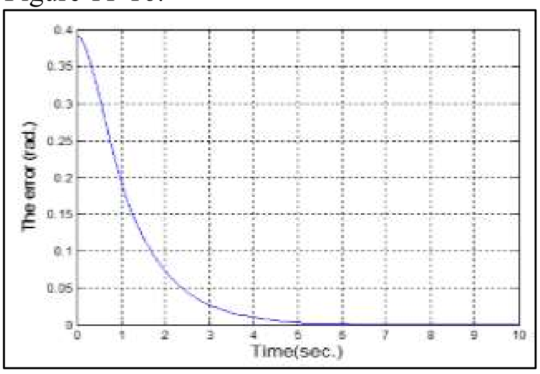

Figure 11: The derivative of error x_2 (rad.) vs

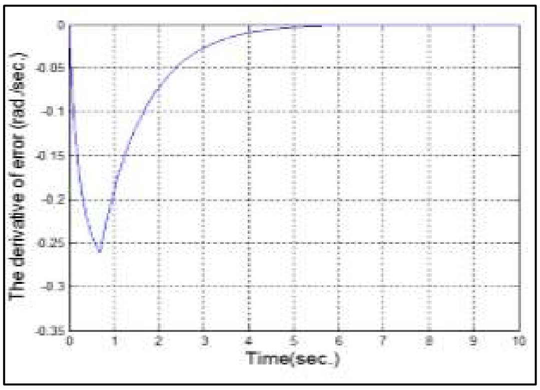

Figure 12: The error x_1(rad.) vs. time

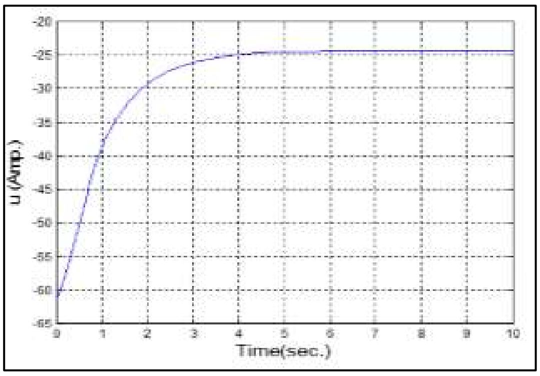

Figure 13: The sliding variable s vs. time

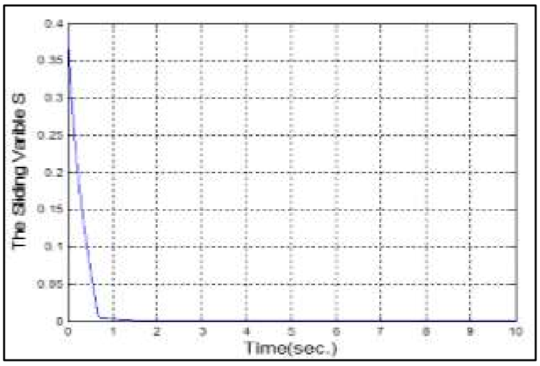

Figure 14: The control action u (Amp.) vs. time

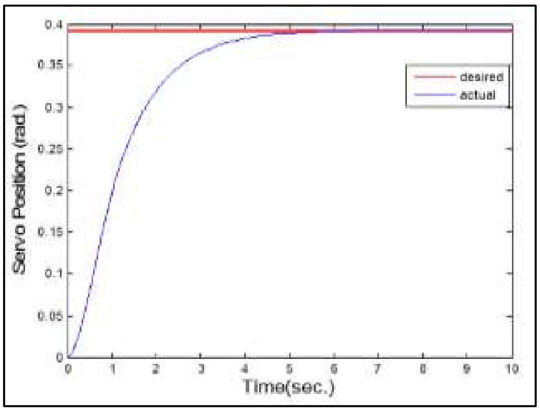

Figure 15: Actual and desired position vs. time

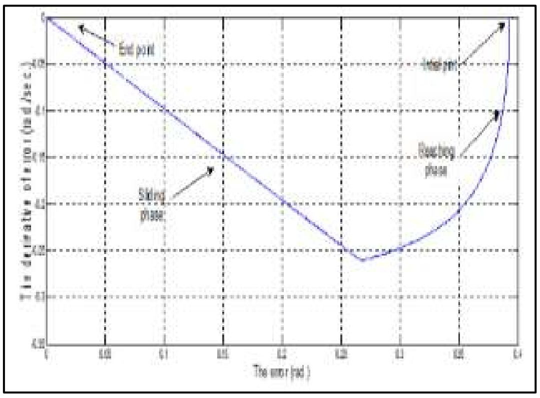

Figure 16: The graph between the error x_1 and the derivative of error x_2

III. Case (C)

The results of CSMC with the boundary layer when using a PSO for tuning K and λ were shown in Figure 17-22.

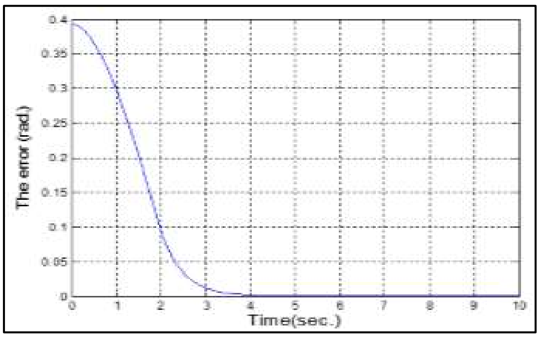

Figure 17: The derivative of error x_2 (rad.) vs. time

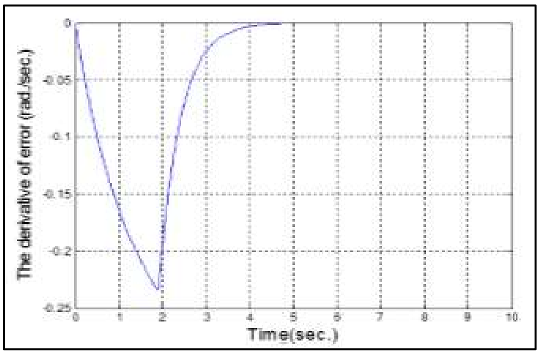

Figure 18: The error x_1(rad.) vs. time

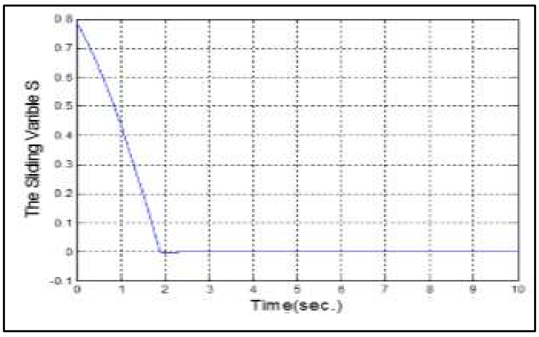

Figure 19: The sliding variable s vs. time

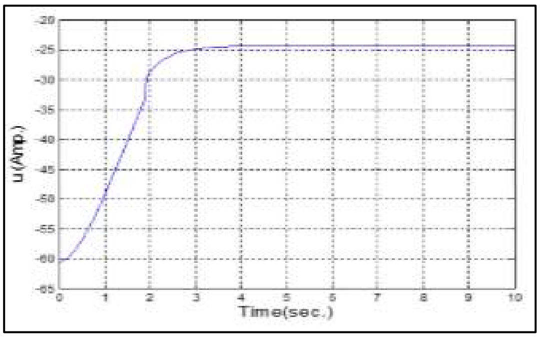

Figure 20: The control action u (Amp.) vs. time

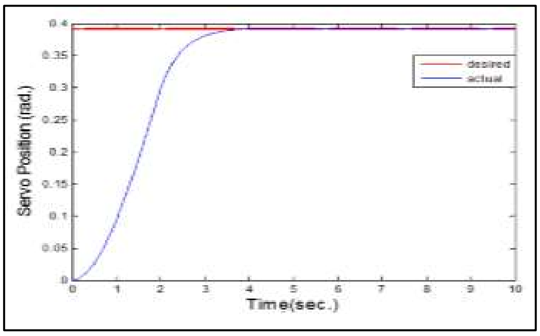

Figure 21: Actual and Desired position vs. time

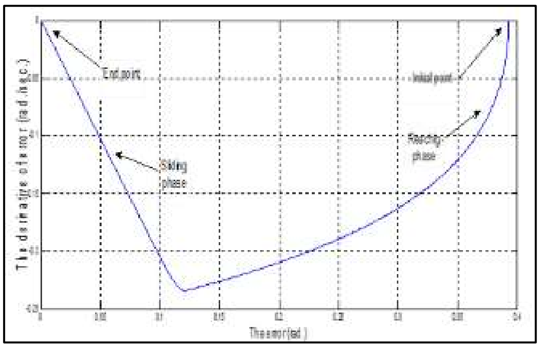

Figure 22: The graph between the error x_1 and the derivative of error x_2

9. Conclusions and Discussion

In this work, the SMC is used because its ability to make the system asymptotically stable under the effect of external disturbance and parameters variation as shown in Figures 5 and 6. In CSMC, the chattering phenomenon is considered as a severe problem. The chattering phenomenon is considered as undesired property in CSMC because it is affected the stability of the system. For this reason, the boundary layer is used to solve this problem by replacing the $sign$ function by a sat function. When using the boundary layer, the chattering problem is reduced and the control action will be smooth and as a result, a stable response and good performance is obtained. Finally, the case of using the PSO algorithm to tune the CSMC parameters such as

K and **λ,** gives a good response with shorter settling time as it is shown in Figure 17. Different values of **K** and **λ** are obtained when using different methods as shown in Tables 2.

Finally, the improvement of using PSO algorithms with SMC is reducing the settling time comparing with the other cusses of using a CSMC without PSO as shown in Figure 9, 15 and 21. Table 3. Shows the values of settling time with different controllers.

Table 2: The optimized parameters of CSMC parameters

Controllers Type	K	λ
CSMC with sign function	2.656	1
CSMC with sat function	2.656	1
CSMC with sat function based on PSO	2	2

Table 3: The values of settling time when using different controllers

Controllers Type	Settling Time *(ts)*
CSMC with sign function	9sec
CSMC with sat function	6.5sec
CSMC with sat function based on PSO	3sec

References

[1] C.A. Yfoulis, A. Muir and P.E. Wellstead, “A New Approach for Estimating Controllable and Recoverable Regions for Systems with State and Control Constraints,” *International Journal of Robust and Nonlinear Control*, Vol. 12, No. 7, 561–589, 2002.

[2] S.A. AL-Samarraie, “Design of a Continuous Sliding Mode Controller for the Electronic Throttle Valve System,” University of Baghdad. *Journal of Engineering*, No. 4, Vol. 17, 859-871, 2011.

[3] S.A. AL-Samarraie, “Invariant Sets in Sliding Mode Control Theory with Application to Servo Actuator System with Friction,” WSEAS Transactions on Systems and Control, Issue 2, Vol. 8, 33-45, 2013.

[4] F. Plestan, Y. Shtessel, V. Br´egeault, and A. Poznyak, “Adaptive Sliding Mode Control for a Class of MIMO Nonlinear Systems Application to an Electro Pneumatic Actuator,” *Control Engineering Practice*, Vol. 21, No. 5, 679–688, 2013.

[3] V.I. Utkin, J. Guldner, and J. Shi, “Sliding Mode Control in Electromechanical Systems,” CRC Press. Taylor & Francis Group, 2009.

[4] A.K. Hamoudi, “Design and Simulation of Sliding Mode Fuzzy Controller for Nonlinear,” *Journal of Engineering*, College of Engineering, University of Baghdad, Vol. 22, No. 103, 66–76, 2016.

[5] A.K. Hamoudi, “Sliding Mode Control for Nonlinear System Based on Genetic Algorithm,” *Engineering and technology*, Vol. 32, No. 11A, 2745-2759, 2014.

[6] H. Lee and V.I. Utkin, “Chattering Suppression Methods in Sliding Mode Control Systems,” *Annual Reviews in Control*, Vol. 31, No. 2, 179–188, 2007.

[7] M. Clerc, “The Swarm and the Queen: Towards a Deterministic and Adaptive Particle Swarm Optimization. Proceedings of the Congress on Evolutionary Computation, 1951-1957, 1999.

[8] F. Piltan, N. Sulaiman, S. Soltani, M. H. Marhaban, R. Ramli, “An Adaptive Sliding Surface Slope Adjustment in PD Sliding Mode Fuzzy Control for Robot Manipulator*,” International Journal of Control and Automation,* Vol. 4 No. 3, 65- 76, September. Piscataway, NJ: IEEE Service center, 2011.

[9] J. Zhao, T. Li and J. Qian, “Application of Particle Swarm Optimization Algorithm on Robust PID controller Tuning,” Springerlink-Verlag Berlin Heidelberg, 948-957, 2000.

[10] R.C. Eberhart and Y.H. Shi “Comparing Inertia Weights and Constriction Factors in Particle Swarm Optimization,” Proc. The Congress on Evolutionary Computation. Vol. 1, 84-88, 2000.

[11] Z. Chen, W. Meng, Z. Wang and J. Zhang, “Sliding Mode Variable Structure Control Based on Particle Swarm Optimization,” Taiyuan University of Science and Technology, pp.692-696, Taiyuan, Shanxi, 030024, China, 2009.

[12] M. I. Solihin, L.F. Tack and M.L. Kean, Tuning of PID Controller Using Particle Swarm Optimization (PSO). Proceeding of the International Conference on Advanced Science, *Engineering and Information Technology*, Vol.1, 458-461, 2011.