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Numerical Modal Analysis of Reinforced Concrete Slab with Opening

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Abstract

Openings in reinforced concrete (RC) slabs are usually created as a result of variations in construction function, architectural or mechanical necessities. Heavy equipment loads resulting from mechanical system of any building are often carried by RC slabs. Even the static analysis and design of RC slabs with opening is not clearly stated in the available international Codes, dynamic analytical solution for such structure is complex.

In this paper, numerical analysis based on finite element approach is utilized to implement the modal analysis of RC slabs. Opening size and position was parametrically studied. Slab natural frequency or periods in addition to, the mode shape were registered. The results showed that the opening size and position involved in RC slab had a significant change in the value of natural frequency and period for the high level modes. The material nonlinearity affect on free vibration analysis of RC opening slab with different levels of stiffness modifiers was taken into account. The dynamic characteristics of RC opening slab as a function of stiffness modifier degree was numerically measured for six mode shapes. The paper found that a reduction in stiffness modifier value greatly reduce the natural frequency of RC opening slab.

Keywords: Modal analysis, Natural frequency, Mode shape, RC slab with opening, Stiffness Modifier, ANSYS (ANalysis SYStem) for dynamics.

Introduction

RC slab is the main structural part of any structural buildings. Opening with small size is commonly required in the slab to accommodate the mechanical and electrical services. While large size of opening in slab is frequently used by lift, stairways and elevator shafts. Usually RC slab is excreted to heavy loads due to HVAC and generator or any mechanical equipment. Due to these equipments, the RC slabs are subjected to harmonic sinusoidal loads. Definitely, the response of slabs will be different when that subjected to static load. RC slabs stiffness decrease when the slabs are subjected to dynamic loads due to continuous vibrations. When the slab stuffiness is changed, the dynamics characteristics of the slabs is also changes.

The used of analytical modal analysis of RC slab with opening is very tedious or impossible due to complex geometry in the presence of the hole which leads to a stress concentration problem. The available theoretical solution for modal analysis of slab is restricted for solid, homogenous, isotropic and linear plate with simple boundary conditions. Adopting the experimental concept to find the response of RC slab with opening under dynamic loading using full-scale experimental test is huge expensive due to difficulties in finding the suitable equipments and materials[1]. Due to the previous difficulties, the use of finite element methods becomes more popular.

Research Significant

A numerical attempt was made to predict the natural frequency of RC slab with opening. Two openings size 0.85x0.85m and 1.20x1.20m where located in center of the slab, near the mid edge and at the corner were taken into consideration. The overall analysis is carried out for four levels of stiffness modifier to represent the material nonlinearity effect on the dynamic characteristics of the slab. Due to missing experimental data on the natural frequency of the RC slab with opening, two verifications were made. The first one is based on checking the material behavior of RC opening slab with experimental done by other subjected to static load and the second one is based on verifying the dynamic behavior with the available analytical solution of solid linear, homogenous, isotropic plate.

Experimental Verification

To check the validity of the used numerical full model for modal analysis, a static solution for RC slab with opening was carried out. The reinforcement of slab used in this analysis is consisting of rebar of a diameter of 5 mm with spacing of 150 mm in two directions at the bottom of slab. This slab is tested monolithically by Piotr[2] and sufficient experimental data is available for their proper modeling by the finite element method. Figure 1 and Table 1illustrated the geometry and properties of the tested slab.

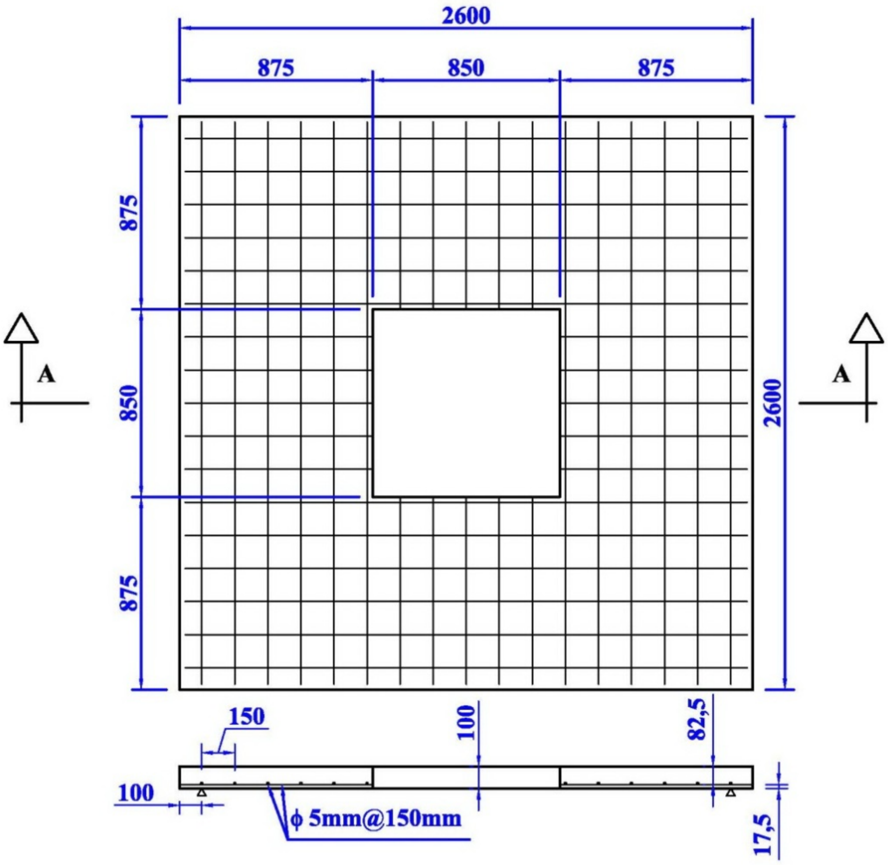
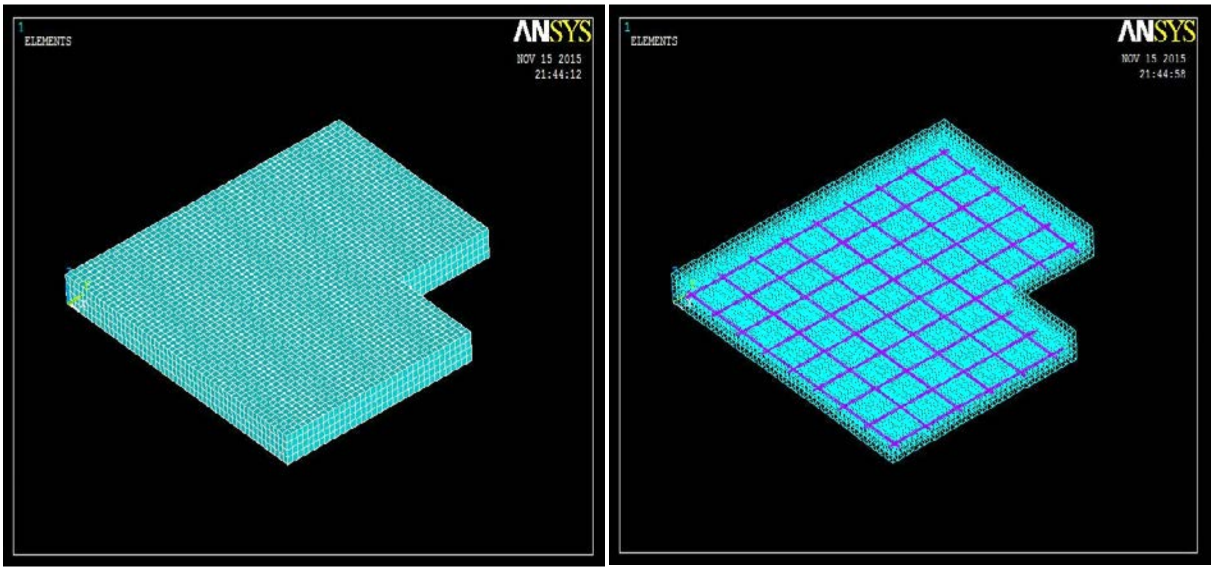


Figure 1: Slab Geometry.

Table 1: Material Properties.

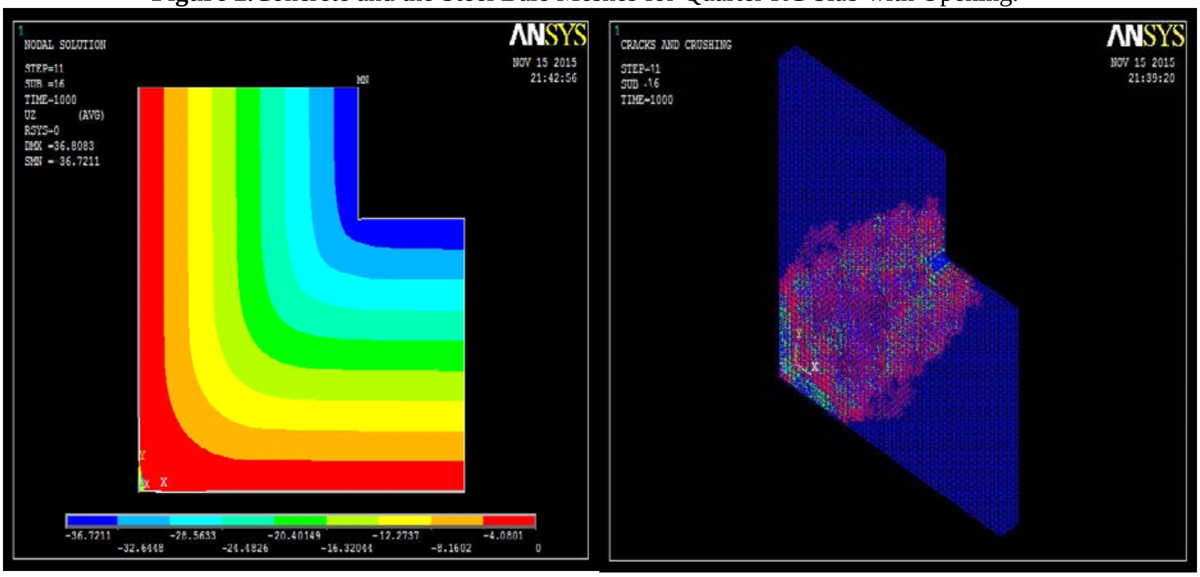
Symbol	Description	Value
h	Slab thickness	100 mm
f_{cu}	Cubic compressive strength of concrete	46.5 MPa
f_t	Splitting tensile strength of concrete	3.1 MPa
E_c	Modulus of elasticity of concrete	34 GPa
ν_c	Possion's ratio	0.2
ρ	Mass density of concrete	2500 kG/m^3
f_y	Yield strength of steel	510 MPa
E_s	Modulus of elasticity of steel	209 GPa
ν_s	Possion's ratio of steel	0.3

The concrete slabs are modeled using ANSYS as three-dimensional solid part to be capable to model the boundary conditions accurately and to simulate the reinforcement as a describe representation. The SOLID65 brick element was adopted to draw the geometrical shape of the concrete. This element has eight joints with three linear displacements in X, Y and Z axes for each joint. Plastic deformation, cracking and crushing capabilities are allowed in this element. In order to find the minimum number of element required to model the problem, numerical analysis was carried out for different meshes. Comparison with the analytical solution of different mesh arrangements indicates that the biggest influence on accuracy is due to the number of element through slab thickness. Therefore, the slab is meshed by four elements along its thickness. The total elements used in this analysis were (10458) elements. Spare element (LINK8) was used to represent the reinforcement of slab. This element three dimensional and it has two nodes with three degrees of freedom per each one[3]. Figures 2 show the concrete and rebar meshes considered in the analysis. Only quarter of a slab is considered due to symmetry to reduce the calculation time. Figure 3 displays the displacement field of the problem in terms of deflection contour and crack pattern. Finally the finite element solution in term of deflection at point A is verified with the experimental value recorded by Piotras shown in Figure 4[2].



(a) Concrete Mesh (b) Steel bar Mesh

Figure 2. Concrete and the Steel Bars Meshes for Quarter RC Slab with Opening.



(a) Deflection Contour (b) Crack Pattern

Figure 3. Deflection Contour and Crack Pattern of Quarter RC Slab with Opening.

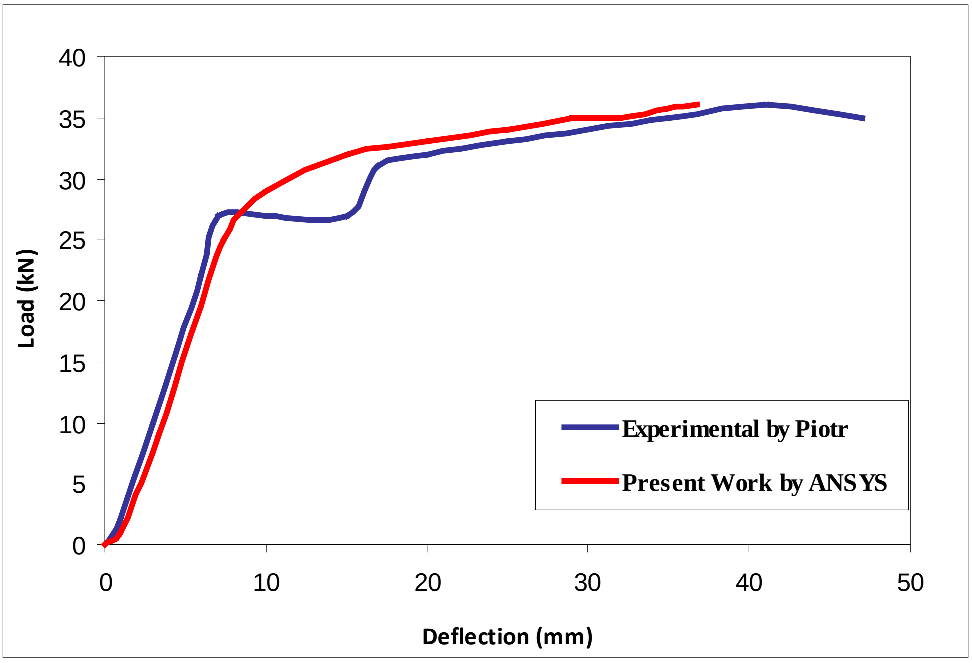


Figure 4. Load Deflection Curve at Mid Opening Side.

Analytical Verification

Modal analysis of a solid square slab whose dimensions are (2.6mx2.6m) of 100mm thickness) and simply supported on four edges was performed. The slab is assumed homogenous and isotropic while the analysis is carried out based on linear behavior. The aim of this simple analysis is to verify the numerical modal analysis adopted in this work with the available analytical results. The basic or governing equation of free vibration of plate in x and y axes plane is an eign value problem as[4]:

$$\nabla^4 w + \frac{\mu}{D}\frac{\partial^2 w}{\partial t^2} = 0 \quad \text{.... (1)}$$

Where,

$w = w(x,y,t)$ deflection in z-direction;

$\mu = \rho h$ the mass density per unit area of plate;

$\rho =$ mass density per unit volume of plate;

$h =$ thickness of plate

$D = \dfrac{Eh^3}{12(1-\nu^2)}$ flexural rigidity of plate;

$E =$ modulus of elasticity;

$\nu =$ possion's ratio.

The analytical solution of such problem is usually solved by using double Fourier sine series. The solution must satisfy the boundary conditions of plate at any time t. The two solution constants are obtained from the two initial conditions. The first initial condition is assumed the velocity is equal to zero when the time is equal to zero. While the second initial condition is assumed both the dynamic deflection and static deflection is coincide at time equal also zero. The final solution to the above equation is:

$$f = \frac{\pi}{2}\left(\frac{m^2}{a^2} + \frac{n^2}{b^2}\right)\sqrt{\frac{D}{\mu}} \quad \text{.... (2)}$$

Six theoretical and finite element natural frequencies are listed in Table 2 to determine the accuracy of the adopted numerical model.

Table 2 Theoretical and Numerical Natural Frequency of Solid Slab.

Mode No.	Analytical (Hz) Natural Frequency	Numerical (Hz) Natural Frequency
1	50.49	49.59
2	126.23	123.53
3	126.23	123.53
4	201.97	196.25
5	252.46	244.94
6	252.46	245.40

Case Study

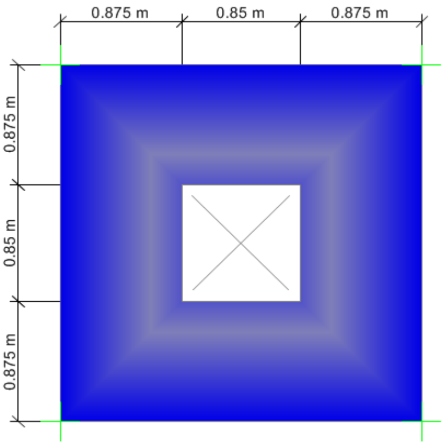
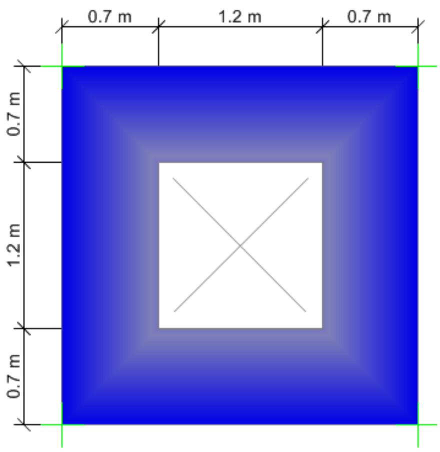
Two size of opening (0.85x0.85m and 1.20x1.20m) namely small and large openings respectively with three opening positions (Central, Mid Edge, Corner) relative to RC slab, as shown in Table 3, were investigated to check the feasibility of the proposed numerical model for measuring the natural frequency of RC opening slab. All the slabs are simply supported by its four corners to match the practical slab problem. Four levels of stiffness modifier, S.M, (S.M=1.00, S.M=0.75, S.M=0.50 and S.M=0.25) were used throughout this work to modeling the affect of the stiffness modifier degree on the dynamic characteristics of slab.

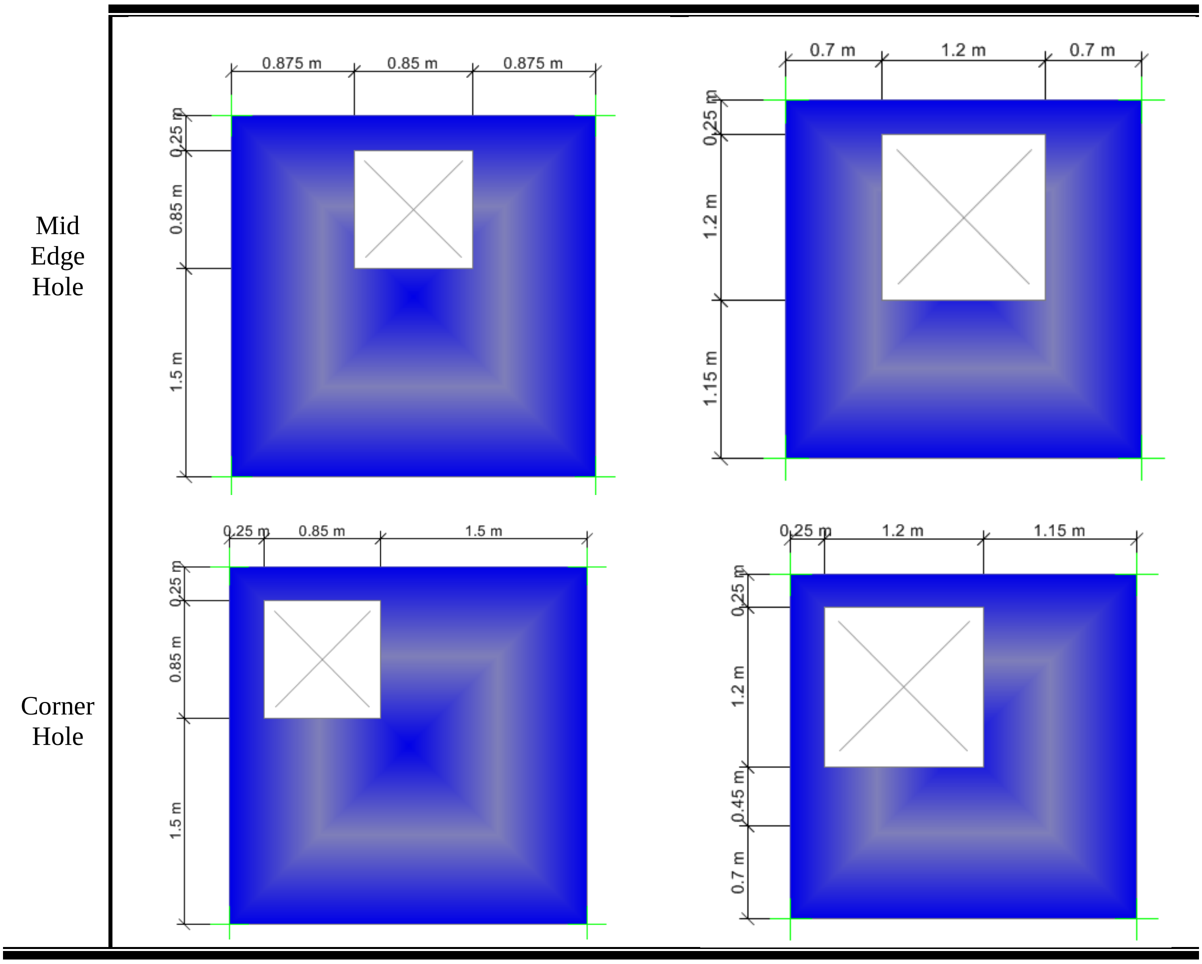
Figures 5 and 6 show the influence of the small and large openings location on the natural frequency of the first six mode shapes respectively. From these figures it is clear that the position of the same size opening in three different locations (Central, Mid Edge, and Corner) has a little affect on the natural frequency of the first three mode shapes, but this affect became a significant for the other higher mode shapes.

Figure 7 depicts the variation of the natural frequency with two different sizes of opening, small and large hole, which located in the same position either in the central of the RC slab or near the corner. Also it is obvious that the natural frequency has negligible effect in the first three levels of modes shape.

The influence of slab stiffens modifier (S.M) on slab frequency with small and large openings are shown in Figures 8 and 9 respectively for both central and corner location of opening. Four values of S.M (S.M=1.00, S.M=0.75, S.M=0.50 and S.M=0.25) based on slab flexural rigidity in both x and y axes of slab plane were considered. It is clearly from these figures how the reduction in stiffness of slab affects the dynamic characteristics in term of natural frequency of the RC slab. Also these figures depicted how the reduction in stiffness due to nonlinear analysis based on crack formulation is important in predication the natural frequency of the reinforced concrete slab. Finally Table 4 draws the six modes shapes in three columns. The first column represents the six modes shapes for solid slab, while the second one represents the modes shapes for the slab with central small opening and finally the last column illustrates the modes shapes of the slab with large opening.

Table 3 Opening Size and Location.

	Small Opening	Large Opening
Central Hole		



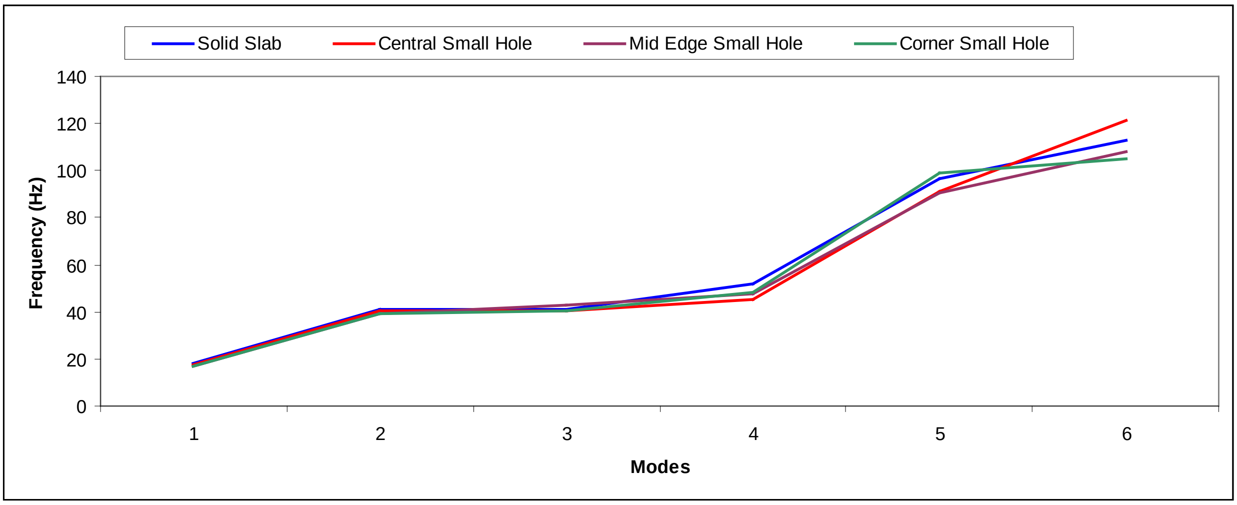


Figure 5. Influence of Small Opening Location on Frequency.

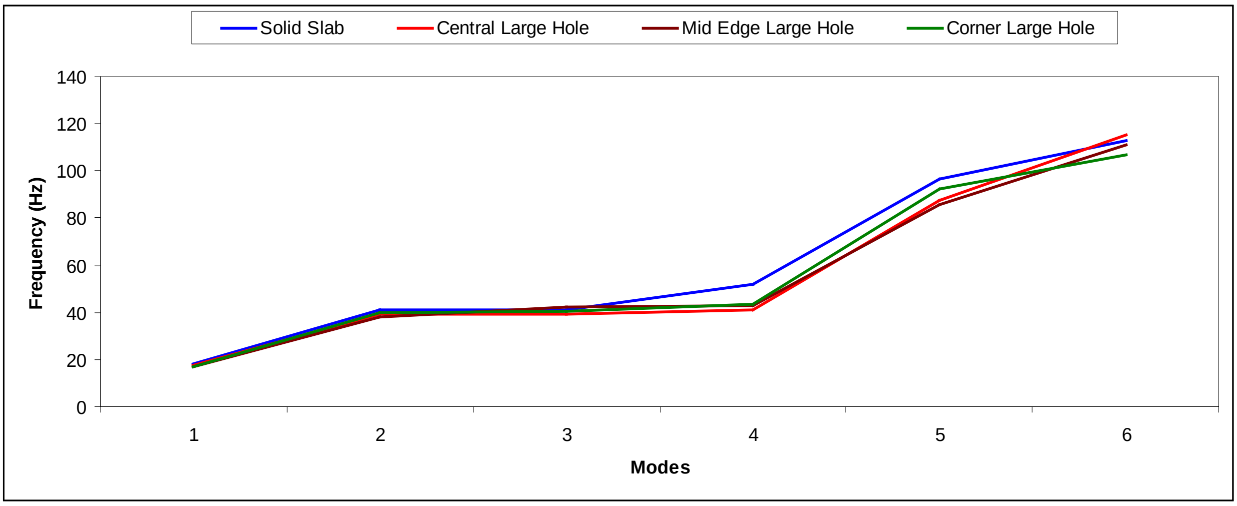
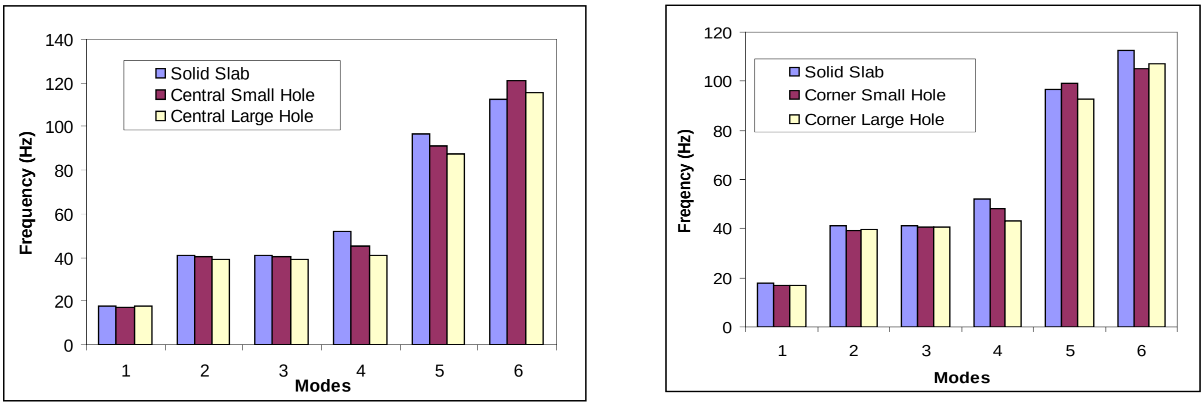
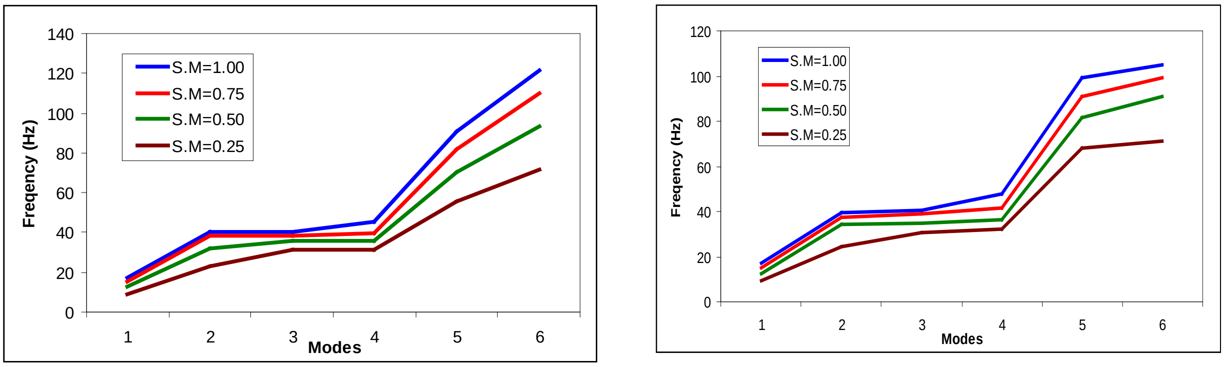


Figure 6. Influence of Large Opening Location on Frequency.



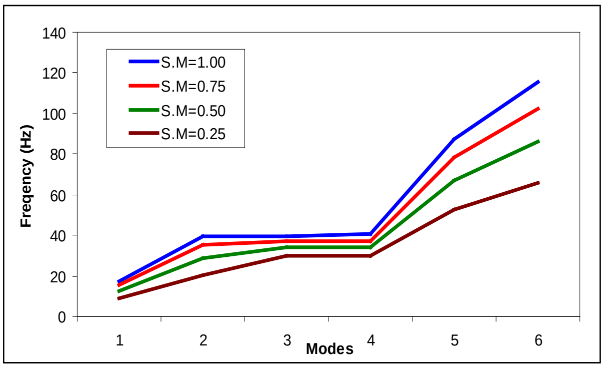
(a) Central Hole (b) Corner Hole

Figure 7. Effect of Opening Size on Frequency.

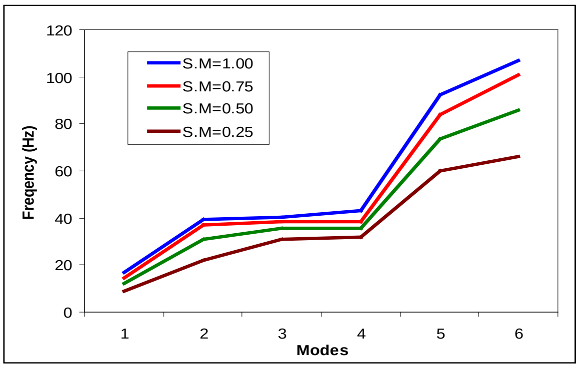


(a) Central Small Hole (b) Corner Small Hole

Figure 8. Effect of Stiffness Modifier on Frequency of Slab with Small Opening.



(a) Central Large Hole

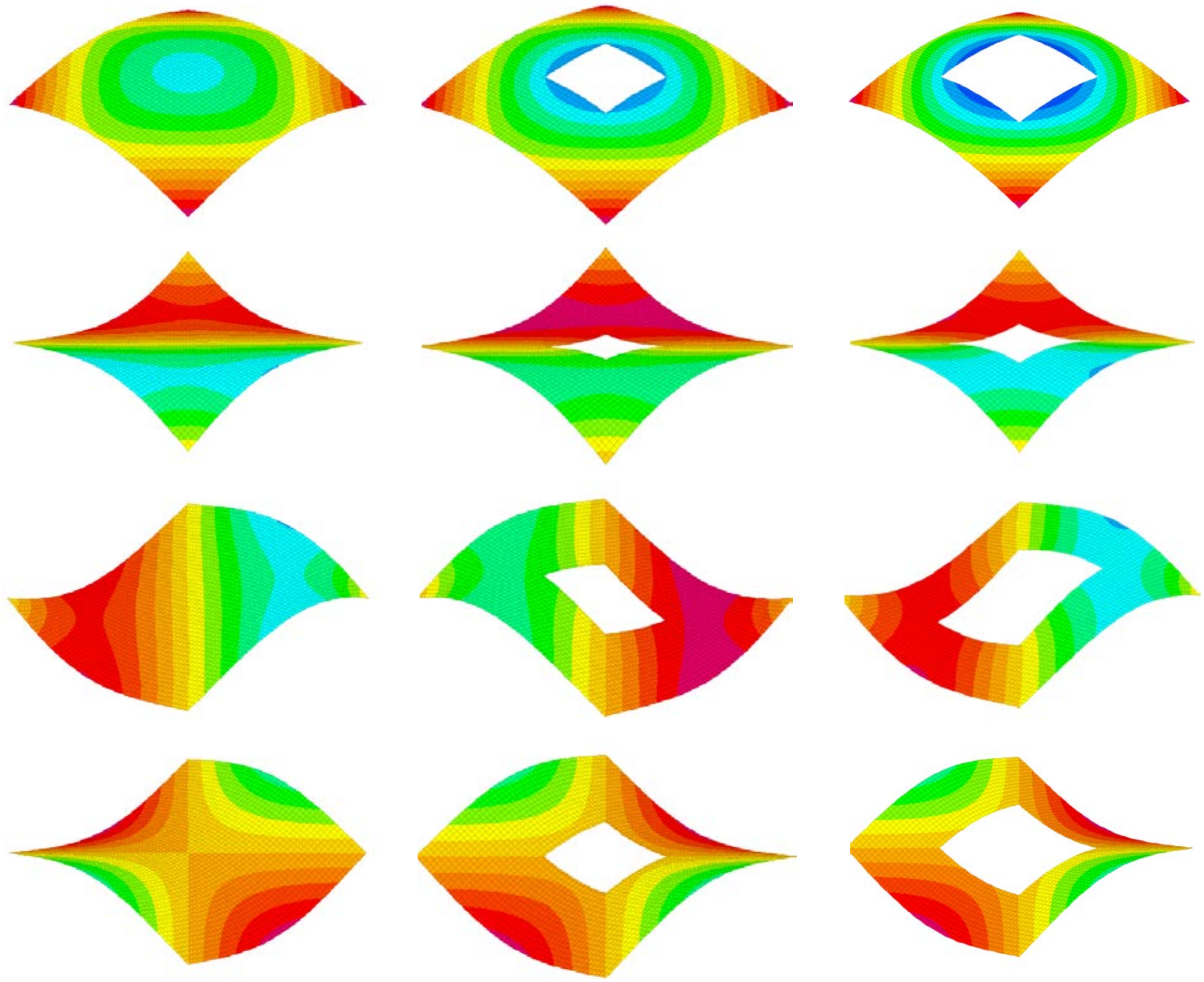


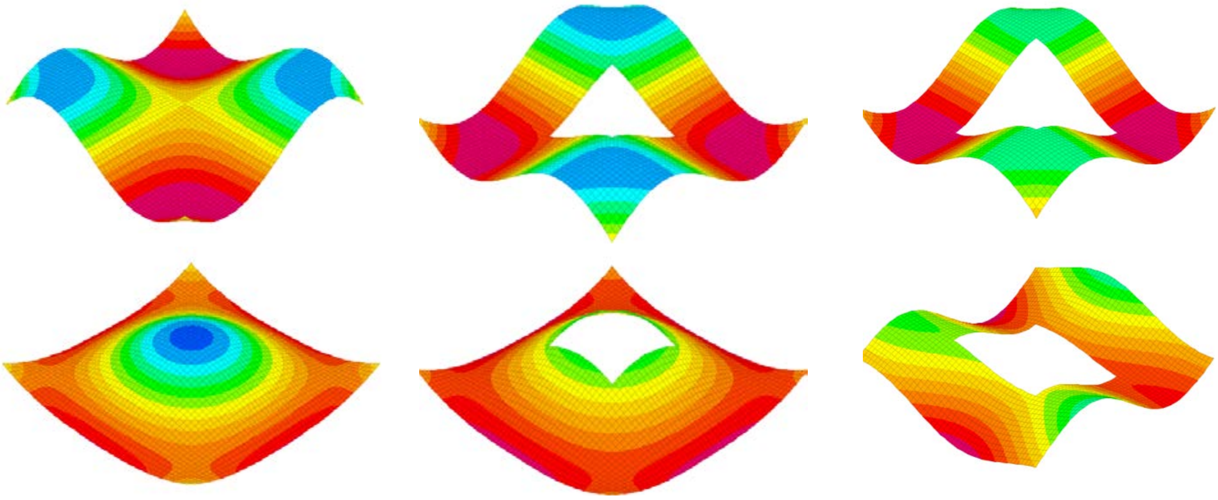
(b) Corner Large Hole

Figure 9 Effect of Stiffness Modifier on Frequency of Slab with Large Opening.

Table 4. Six Mode Shapes.

Solid Slab	Slab with Small Opening	Slab with Large Opening





Conclusion

1. The position of the same size opening in RC slab in three different locations (Central, Mid Edge, and Corner) has a little affect on the dynamics characteristics of the slabs representing in the natural frequency for the first three mode shapes, but this affect became significant for the other higher mode shapes.
2. The variation of the natural frequency with two different sizes of opening, small and large hole, which located in the same position either in the central of the RC slab or near the corner has a slightly affect the dynamic characteristics of RC slab.
3. The reduction in stiffness of slab in term of stiffness modifier of RC slab has a significant affects on dynamic characteristics of the slab in term of natural frequency. Also these figures depicted how the reduction in stiffness due to nonlinear analysis based on crack formulation is important in predication the natural frequency of the reinforced concrete slab.

References

1. R. Pinho, and A. S. Elnashai , Dynamic Collapse Testing of A Full-Scale Four Story RC Frame. ISET Journal of earthquake Technology, vol. 37, Issue 4, 2000, pp. 143 - 163.
2. Piotr Rusinowski, Two-way Concrete Slabs with Openings. Lulea University of Technology, Department of Civil and Environmental Engineering, Division of Structural Engineering, 2005.
3. Erdogan Madenci, Ibrahim Guven, The Finite Element Method and Applications in Engineering using ANSYS. 2006 by Springer Science-n Business Media, LLC.
4. Elia Efraim, Accurate formula for determination of natural frequencies of FGM plates basing on frequencies of isotropic plates, Sciencedirect Procedia Engineering 10 (2011) 242–247. Elsevier Ltd.

التحليل الاهتزازي العددي الحر للبلاطات الخرسانية المسلحة ذات الفتحات

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الخلاصة

أسباب وضع فتحات في البلاطات الخرسانية المسلحة عادة يكون نتيجة تغيير وظيفة البناء لتحقيق المتطلبات المعمارية او لضروريات تحقق الخدمات الميكانيكية في المباني. تتعرض عادة البلاطات الخرسانية المسلحة يشكل عام لاحمال المعدات الميكانيكية الثقيلة الضرورية لخدمة المباني. على الرغم من عدم وضوح طرق التحليل والتصميم الاستاتيكي للبلاطات الخرسانية ذات الفتحات في المدونات العالمية المختصة بالمنشات الخرسانية. لذا يعتبرالتحليل الديناميكي للبلاطات الخرسانية المسلحة ذات الفتحات معقد.

في هذا البحث تم استخدام طريقة العناصر المحددة لاجراء تحليل الاهتزاز الحر للبلاطات الخرسانية التي تحتوي على فتحات. حيث تم دراسة تاثير حجم الفتحة على التردد الطبيعي وفترته وشكله. أظهرت النتائج ان حجم الفتحة وموقعها له تاثير كبير في تغيير قيمة التردد الطبيعي وفترته في الفترات العالية. كما وجد ان التعرف اللاخطي للمادة له تاثير على الاهتزاز الحر الطبيعي للسقوف الخرسانية ذات الفتحات. حيث تم استخدام مستويات صلابة مختلفة في التحليل وتم تسجيل الخصائص الديناميكية لكل مستوى. وتم التوصل الى ان انخفاض مستويات صلابة السقوف الخرسانية المسلحة ذات الفتحات يقلل من التردد الطبيعي بشكل كبير.