SOME PROPERTIES OF SINGULARITY OF N-ARY TREE AND LINE N-ARY TREE GRAPHS

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Abstract
The idea of the high zero sum weighting which uses a maximum number of non zero independent variables generalized which is equal the singularity of a graph. In a full n-ary tree **Th,a,n** , the edges adjacent to the root vertex are called its children (branches). The children of the root are said to be on level 1, if we have h levels, then for 0<i<h, each vertex on level i has children of its own which are on level i+1.The n external vertices are those on the last level called leafs (end vertices) .
In this paper the singularity of a full n-ary tree is studied with certain levels and external vertices(leafs) with deferent ary and proved that for ($T_{n,3,n}$) ,n ary Tree with level 3 to be δ ($T_{n,3,n}$)= $n^2(n-1)+(n-1)$. The results should that the singularity of line Graph of n-ary tree of any levels is always zero, and the construction of line n- ary tree is a copy of complete graph K_n which identifying each to others by a vertex or introducing by an edge depends on the duplicated vertices in the levels and on the leafs .
Keywords : Tree Graphs n-ary Tree singularity Line n-ary Tree .

Introduction
A **graph** G is a non empty set of vertices V together with a set of unordered pairs of vertices of V called edges. The **spectrum** of a graph G is the set of eigenvalues of the 0-1 adjacency matrix A(G) of G. An eigenvaluses $\lambda\in R$ of G satisfies the equation $Ax=\lambda x$ for some non-zero vector $x\in R^n$, called a λ-eigenvector of G; some eigenvaluse may be repeated. The number of eigenvalues of G that are equal to zero is called the singularity of the graph G and will denote by $\delta(G)$. If $\delta(G)>o$, then G is said to be singular. The characteristic polynomial of the graph G, defined n-ary as det(λI-A), will be denoted by $\phi(G)=\phi(G, \lambda)$. Thus the eigenvalues of G are just the solution of the equation $\phi(G, \lambda)$. Clearly, G is singular if and only if $\phi(G, 0)=0$.
Let r(A(G)) be the **rank** of A(G), clearly $\delta(G)=$ n-r(A(G)),(n is the number of vertices of G) . The rank of graph G is the rank of its adjacency matrix A(G),denoted by r(G), then $\delta(G)$=n-r(G). Each of $\delta(G)$ and r(G) determines the other.[1] and [5] . It is known that $0\leq\delta(G)\leq n-2$ if G is a simple graph on n vertices and G is not isomorphic to nK_1. L. Colatz and U.
Sinogowitz first posed the problem of characterizing all graphs G with $\delta(G)>o$, this question is of great interest in chemistry, because as has been shown in [2],[3] and [8], for a bipartite graph corresponding to an alternate hydrocarbon, if $\delta(G)>o$, then its indicates the molecule which such a graph represents is unstable . The problem has not yet been solved completely; only for bipartite graph some particular results are known [6] and [9]. In resent years, this problem has been investigated by many researchers [6],[8] and [10] .
Let G be a graph with n vertices and q edges, the **neighborhood** $N_G(v)$ of v is the set of all vertices of G which are adjacent to v. Two vertices with the same neighborhood are called **duplicate** vertices. The **degree** of a vertex v in a graph G, denoted by deg(v), is the number of edges incident with v. A graph G is **regular** if all vertices have the same degree An independent set of vertices in G is a set of vertices of G no two of which are adjacent, and the size of the largest such set is called **independence number** of G , and independent edge are these not adjacent by a vertex in G , and A matching of G is a collection of independent edges of G , A maximal matching is a matching with maximum number of independent edges in G and denoted by m(G).[3] and[4] .

Definition 1.1
[6](**Weighting vertex**) A vertex weighting of graph G is a function $f:V(G)\rightarrow R$ where R is the set of all real numbers , that assign a real number to each vertex of G , and a weighting of any graph G is said to be non-trivial if there is at least one vertex $v\in V(G)$ for which $f(v)\neq 0$.

Definition 1.2
A non-trivial vertex weighting of graph G is called a zero-sum weighting provided that for each $v\in V(G)$, $\sum_{w\in N_G(v)} f(w)=0$, where the summation is taken over all $w\in N_G(v)$.
It is not true that every graph has a non-trivial zero sum weighting, for example the complete graph (k_n) is such a graph has no non-trivial zero sum weighting sine its non singular.
M.Brown and et. in [6] and [11] proved an important relation between the singularity of a graph and existence of a zero sum weighting which is given in Th. 1.4 and its equivalent to the Theorem 1.3.

Theorem 1.3
[7] A graph is singular iff zero is an eigenvalue of the adjacency matrix A(G) of the graph G .

Theorem 1.4
[6] A graph G is singular iff there is a zero sum weighting for the graph G .

Definition 1.5

[13] The maximum number of non-zero distinct independent variables that can be used for a zero sum weighting of G is called a high zero-sum weighting and denoted by M(G).

Proposition 1.6

[13] $\delta(G) = M(G)$ i.e the singularity of a graph G equal to the number of non zero distinct independent variables that used in high zero-sum weighting in the graph hence M(G)=0 iff G is none-singular. In figure (1) , the weighting for the graph G is a high zero-sum weighting uses 3 distinct independent variables hence $\delta(G) = 3$.

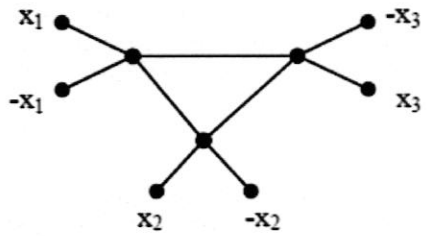

Fig .1, A non-trivial zero sum weighting for a graph G

Tree Graph and n-ary Tree

Acyclic graph is one that contains no circuit. A connected cyclic graph is called a tree. The trees on six vertices are shown in figure (2) Acyclic graph is usually called forests .

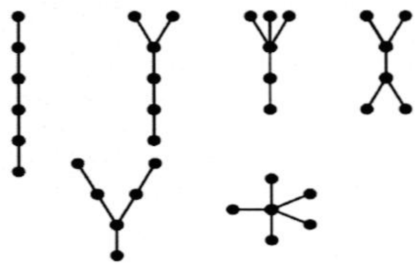

Fig .2, The tree on six vertices

Notice that in a tree, any two vertices are connected by exactly one path, morever, if the tree is non trivial, it must contain a vertex of degree exactly one, such a vertex is Called a **leaf** of the Tree [2]

Definition 2.1

[8] Spanning trees :

A sub tree of a graph is a sub graph which is a tree. If this tree is a spanning sub graph (contains all the vertices of the graph), is called a spanning tree.

Figure (3) shows a decomposition of the wheel W4 in to two spanning trees .

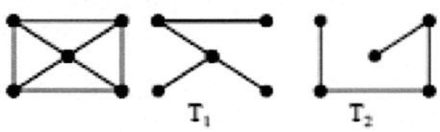

Fig .3,Two spanning trees of the wheel W_4

T_1 and T_2 are two Trees of 5 vertices and they are spanning trees of wheel W_4 (of 5 vertices) .

If a graph G has a spanning tree T, then G is connected because any two vertices of G are connected by a path in T.

Definition 2.2

n-ary tree $T_{h,a,n}$: A full n-ary tree has a special vertex in the tree called the root (v_0). The root has edges adjacent to other vertices called its children (branches). The children of the root are said to be on level 1, if we have h levels, then for 0<i<h, each vertex on level i has children of its own which are on level i+1. i.e $T_{h,a,n}$ denote The n ary Tree where h is the number of levels(the radius of the Tree) and a is the number of ary (the number of edges adjacent with each vertex in the level h) while n is the number leafs (end vertices) adjacent with the internal vertices in the last level .

The vertex on the last level, h, has no children (breaches) and is called leafs (end vertices). The vertices that are not leafs are said to be internal vertices. The root counts as an internal vertex. In figure (4) the root (v_0) is vertex 1 and n=2(binary ary), and the internal vertices are vertices 1 through 7 and the leafs are vertices 8 through 15. When as in our example, the tree is called a binary tree, so figure (4) is a full binary tree with 3 levels. This tree has 8 leafs .

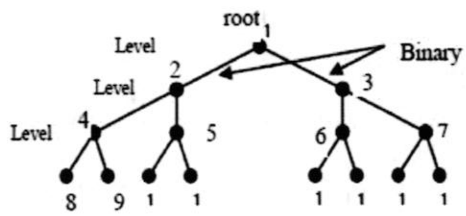

Fig .4, A full binary tree with 3 levels

The tree $T_{3,2,n}$ in figure(4) denotes the tree on level 3 an ary 2 with 8 leaves(end vertices) and 15 vertices.

Line tree graph and line n-ary tree

Generalized line graphs, represent the graph. Class of graphs with several remarkable spectral properties . Recall first that their least eigenvalue is greater than or equal to -2. It is worth mentioning that 0 or -1 can be in many instances, the eigenvalue of these graphs. One reason for these phenomena is the presence of the so called duplicate vertices (vertices with the same neighbors) . It is also note worthy that the number (0 and 1) as the eigenvalues of graphs have a special role in spectral graph theory. [7] and [8] .

In addition, graphs having 0 as an eigenvalues, i.e singular graphs are significant in mathematical chemistry. [8]

Definition 3.1

The **line graph** L(G) of a graph G is a graph that has a vertex for every edge of G, and two vertices of L(G) are adjacent if and only if they correspond to two edge of G with a common end vertex.

Example 3.2

For the cube graph Q_3 shown in figure (5), construct a line graph $L(Q_3)$ as in figure (6) .

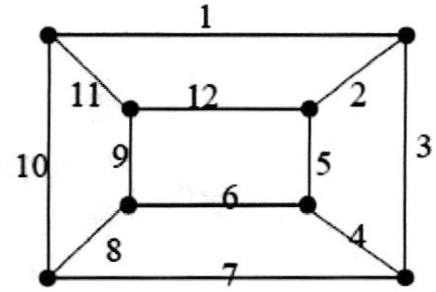

Fig -5- The cube graph Q_3

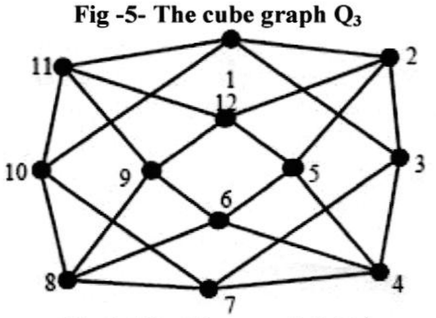

Fig-6- The Line graph $L(Q_3)$

Notice that the number of vertices in the line graph is equal to the number of edges in the cube. Vertex 1 in the line graph corresponds to the edge 1 in the cube graph and vertex 2 in the line graph correspond the edge 2 and so no.

Note 1: since need a non-empty set of vertices for a graph, the line graph of null graph is undefined .

Note 2: if graph G_1 and G_2 are isomorphic then $L(G_1)$ and $L(G_2)$ are isomorphic. However the converse is not true.

Singularity of n-ary tree

Singular graphs and tree graphs or topics closely related to them were investigated in a number of mathematical papers [9], [11]. The nullity of graph is important in chemistry, because $\delta(G)=0$ is necessary (but not sufficient) condition for a so called conjugated molecule to be chemically stable, where G is the graph representing the carbon-atom skeleton of this molecule, for details see[2] and[14] .

If T is a tree, then $\delta(T)$ is equal to the number of vertices of T, not belonging to perfect matching (the set of all independent edges contains all the vertices of the graph) .

Lemma 4.1

If T_n is an n-vertex tree and (m) is the set of its maximal matching, then its singularity is $\delta(T)=n-2m$ [14] .

From lemma 4.1 we see that the unique n-vertex tree with greatest nullity is the star, because this is the unique tree in which no two edges are independent thus $m(S_{1,n})=1$ and $\delta(S_{1,n})=n-2$.

It is also see that if n is even, then all n-vertex tree possessing a perfect matching (i.e a matching of size n/2) have singularity zero. If n is odd, then the minimal value of the singularity is one, achieved by all n-vertex trees for which m=(n-1)/2 .

In this paper determining the greatest singularity among n-vertex tree and n-ary tree by using a new technique used in papers, [6] ,[11], and [13].

Theorem 4.2

The singularity of n-ary tree with one level and n leafs $T_{1,n}$ equal to n-1.

Proof

since n-ary tree with one level is isomorphic to the star graph, tree with n vertices as shown in figure (7).Labeled the root vertex by v_0 and the other n vertices by $v_1,v_2,v_3,\ldots,v_n$. By using the new technique of zero sum weighting the weights of the vertices are of the form $x_1,x_2,\ldots,x_n$ for n leafs and zero for the root vertex .

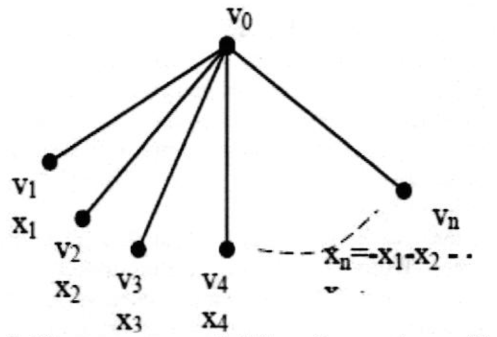

Fig .7, The zero sum weighting of n-ary tree with one level

To get the zero-sum weighting $\sum_{w_i\in N(v_i)} f(w_i)=0$ we have $x_1+x_2+\ldots+x_n=0 \Rightarrow x_n=-x_1-x_2-\ldots-x_{n-1}$, so the high zero sum weighting exist and uses exactly n-1 non-zero independent distinct variables, i.e. $M_v(T_{1,n})$=n-1,implies that this tree graph is singular,and $\delta(T_{1,n})=n-2$.

Theorem 4.3

Singularity of binary-ary tree with 2-level and n leafs denoted by $T_{2,2,n}$ equal to 2n-1 i.e $\delta(T_{2,2,n})=2n-1$.

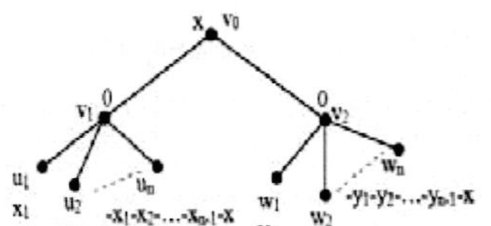

Fig .8,The zero sum weighting of binary-ary tree with level 2 and n leafs

Proof

The graph ($T_{2,2,n}$) which shown in figure(8), v_0 is the root and v_1,v_2 the second level vertices of $T_{2,2,n}$ where the leafs adjacent with each vertex $v_1$and v_2 are $u_1,u_2,\ldots,u_n$ and $w_1,w_2,\ldots,w_n$ respectively, then the weighting of the vertices has the form $f(v_0)=x,\ f(v_1)=f(v_2)=0$ and $f(u_i)=x_i\ \ \forall i=1,2,\ldots n$ $f(w_j)=y_j\ \ \forall j=1,2,\ldots n$.

Then to obtain the zero-sum weighting for the graph $T_{2,2,n}$, $\sum_{w_i \in N(v_i)} f(w_i) = 0$ must have $x+x_1+x_2+......+x_n = 0$ and $x+ y_1+y_2+......+y_n =0 \Rightarrow x_n = -x_1-x_2-.....-x_{n-1}-x$, $y_n = -y_1-y_2-....-y_{n-1}-x$.
That the zero sum weighting exist and uses exactly n-1+n-1+1 =2n-1 none zero independent variables, which implies that the graph ($T_{2,2,n}$) is singular, and $\delta(T_{2,2,n}) = 2n-1$.

Proposition 4.4
If the number of leafs adjacent with v_1 and v_2, in the binary-ary tree of level two of the Tree given in Th. 4.3 are deferent, n and m then $\delta(T_{2,2,n}) = n+m-1$. Where n and m are the number of vertices adjacent with v_1 and v_2 respectively.
Proof: The proof is similar than the proof of the theorem before but the variables used in a high zero-sum weighting are $x_n= -x_1-x_2-......-x_{n-1}-x$ and $y_m= -y_1-y_2-.....-y_{m-1}-x$, the total none zero distinct independent variables are exactly n-1+m-1+1=n+m+1.

If the root vertex v_0 has 3-branches, then the tree graph is called 3-ary $T_{3,2,n}$, means that the tree of three branches, 2 levels such that each vertex in level 2 adjacent with n leafs as shown in figure (9).

Proposition 4.5
The singularity of 3-ary(3 branch), 2 level with n leafs (end vertices) tree ($T_{3,2,n}$) equal to 3n-2.

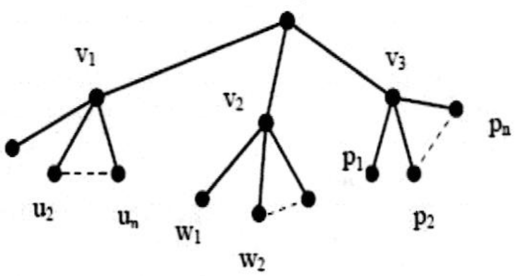

Fig .9, 3-ary , 2 level tree with n leafs $T_{3,2,n}$

Proof
To find the singularity of this tree, given the weights of the vertices of this type of ary tree graph as shown in figure (10) .The root has weight (x) and the weights of vertices of the level two by zero. And the weights of the other end vertices (leafs) be of the form $x_1,x_2,.....,x_n$, $y_1,y_2,.....,y_n$, and $z_1,z_2,.....,z_n$ respectively. Then to get the zero-sum weighting must have $x_1+x_2+.....+x_n+x=0$ $y_1+y_2+.....+y_n+x=0$ $z_1+z_2+.....+z_n+x=0$ Then $x_n=-x_1-x_2-....-x_{n-1}-x$ $y_n=-y_1-y_2-....-y_{n-1}-x$ $z_n=-z_1-z_2-....-z_{n-1}-x$

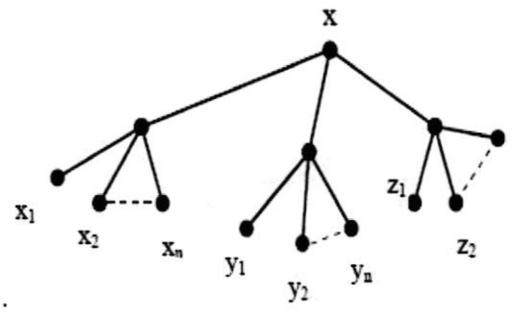

Fig .10, The zero sum weighting of $T_{3,2,n}$

The only non zero independent variables are used to get the high zero sum weighting are (n-1)+(n-1)+(n-1)+1=3n-2, so $\delta(T_{2,2,n}) = 3n-2$.

Corollary 4.6
If the number of end vertices (leafs) of each vertex in 3-ary, level two in ($T_{3,2,n}$) are n, m, p respectively, then $\delta(T_{3,2,n}) = n+m+p-2$.

Proof
Since the number of leafs(end vertices) are different in second level, then the weights are used for the vertices is of the form: $x_1,x_2,.....,x_n$, $y_1,y_2,.....,y_m$, and $z_1,z_2,....z_p$, respectively, and to get a non zero sum weighting, $x_n=x_1+x_2+.....+x_{n-1}+x$, $y_m= y_1+y_2+.....+y_{m-1}+x$, and $z_p =z_1+z_2+....+z_{p-1}$, implies that the singularity is n-1+m-1+p-1+1= n+m+p-2

level tree
Is a tree in which the root vertex v_0 has n-ray, may be incident with n vertices $v_1,v_2,....,v_n$, these vertices refers to level 2 of this type of tree, which are duplicate and each vertex v_i, i=1,2,....,n in level two are adjacent to an other set of vertices determine the third level, where they duplicated too, and ending with the end vertices called leafs. The 2-ary, 3-levels and 3 leafs tree graph denoted by $T_{2,3,3}$ shown as an example in figure (11).

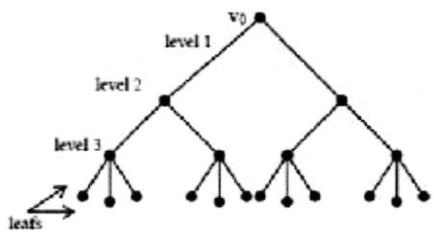

Fig.11,Binary ary 3-level tree of 19 vertices

Theorem 4.7
The singularity of binary-ary Tree with three levels ($T_{2,3,n}$) and n leafs is equal to (4n-3)

Proof
The n ary tree graph ($T_{2,3,n}$) is shown in figure (12).

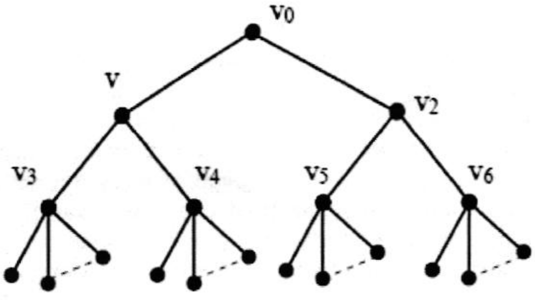

Fig.12, $T_{2,3,n}$

The root vertex v_0 has zero weights while the two adjacent vertices with v_0 in level 2, v_1 and v_2 has the weights x , -x , and the duplicate vertices in level 3, v_3, v_4, v_5 and v_6 also must have the zero weights , but the weights of end vertices of each vertex in level 3 as follow $x_1, x_2, \ldots, x_n$, $y_1, y_2, \ldots, y_n$, $z_1, z_2, \ldots, z_n$, $w_1, w_2, \ldots, w_n$ for v_3, v_4, v_5 and v_6 respectively to get zero sum weighting for this tree $\sum_{v_i \in N(w_i)} f(v_i) = 0$.

we must have

$x_n = -x - x_1 - x_2 - \ldots - x_{n-1}$

$y_n = -x - y_1 - y_2 - \ldots - y_{n-1}$

$z_n = -x - z_1 - z_2 - \ldots - z_{n-1}$

$w_n = -x - w_1 - w_2 - \ldots - w_{n-1}$

Then the number of non-zero distinct independent variable used is exactly 4(n-1)+1=4n-3 so $\delta(T_{2,3,n}) = 4n-3$ where n is the number of leafs adjacent with the vertices in last level of the graph tree .

Proposition 4.8

The singularity of ternary ary tree with 3 level and n leaf $T_{3,3,n}$ are equal to 9n-7.

i.e. $\delta(T_{3,3,n}) = 9n-7$.

Proof

The prove is obvious see figure (13), the vertices in level 2 v_1, v_2, v_3 takes the weights x ,y , -x-y respectively .

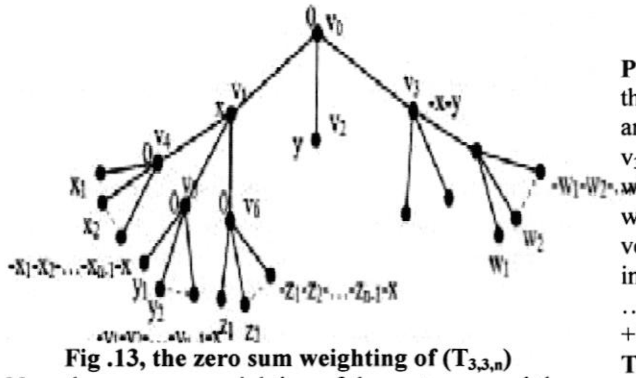

Fig .13, the zero sum weighting of ($T_{3,3,n}$)

Now the zero sum weighting of the vertex v_1 and the others are similar with small different, for v_1 uses 3(n-1)+1 non zero independent variables and in v_2 3(n-1)+1 (y instate of x) but in v_3 . we have 3(n-1), since in last weights we have $-w_1-w_2-\ldots-w_{n-1}+x+y$ to get zero sum weights but x and y are repeated then $\delta(T_{3,3,n}) = 3(n-1)+1+3(n-1)+1+3(n-1) = 9n-7$

Full n-ary tree

Full n-ary tree is in which each vertices in any level are duplicate i.e. if the root is adjacent with 3 or 4 vertices the other vertices must be adjacent to 3 or 4 vertices respectively till the leafs .

[Internet] and the degree of each vertices in the levels are equal except the root vertex v_0 of the degree n and the leafs of degree one i.e. each vertex from the root to the last level adjacent with exactly n vertices (has n branches) The figure (14) shows the full ternary tree with level 3.

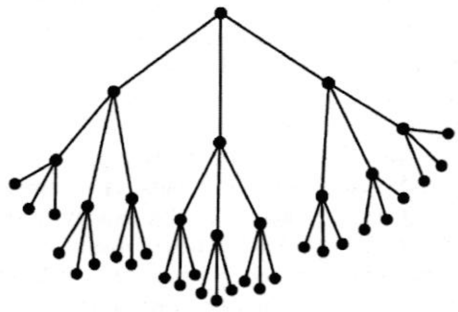

Fig .14, A full ternary tree with 3 levels

Theorem 5.1

The singularity of full n- ary of level two $T_{n,2,n}$ is equal n(n-1) +1. Where n is the number of ary and equal the number of leafs (end vertices) adjacent with each vertices in the second level .

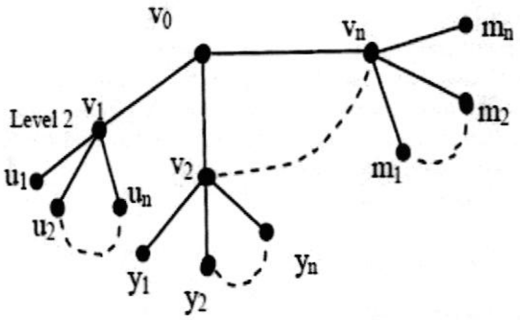

Fig (15) the full n-ary tree with level 2,$T_{n,2,n}$

Proof

the deem idea is that the weights of the root v_0 is (x) and the weights of the other vertices (v_1, v_2, $v_3, \ldots, v_n$) adjacent with v_0 in the level 2 are zero, where the way for giving weights to the leafs adjacent with vi, i=1, 2, 3…., n is shown in figure (15).the vertices in leafs, takes (n-1) non zero distinct independent variables for each vertex vi, i=1,2,3, ….,n in a high sum weighting So $\delta(T_{n,2,n}) = n(n-1) + 1 = n^2 - n + 1$.

Theorem 5.2

The singularity of full n-ary Tree ($T_{n,3,n}$) with level 3 is $n^2(n-1)+(n-1)$. Where n is the number of leafs adjacent to each vertex in level 3.

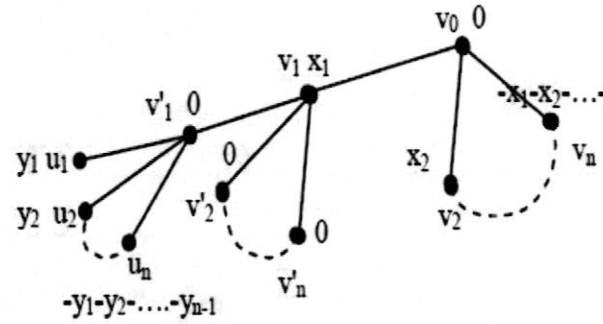

Fig .16, full n-ray with level 3

Proof

The base idea For giving weighting of this Tree (full ary with level three) is that the weight of the first vertex (root) v_0 is zero, and the adjacent vertices with v_0 are($v_1, v_2, ...,v_n$) in level two takes n-1 variables of weights. While each vertex vi, i= 1, 2, 3, ...,n are adjacent with exactly n other vertices $v'_1, v'_2, ...,v'_n$, with weights zero. In the level three each vertex v'j,j= 1, 2, ...,n are of adjacent with exactly n another vertices which is used n-1 non zero distinct variables. Then the total number of non zero independent variables used to get the zero- sum weighting of this Tree(Tn,3,n) is n(n-1) +n(n-1)+........+n(n-1) ,n times (n-1) +(n-1) ,which is equal to n[n(n-1)]+ (n-1).

Then $\delta(T_{n,3,n})=n^2(n-1)+(n-1)=n^3-(n^2-n+1)$.

Proposition 5.3

The singularity of n-ary with n level is so difficult and depend on the number of end vertices (leafs) of Tree $T_{n,n,n}$.

Singularity of Line n-ary Tree Graph

First observe that the singularity of line tree may assume any positive integer value. A trivial example for this is $L(K_2)$, whose singularity is one and $L(pK_2)$, whose singularity is p. See figure (17) .

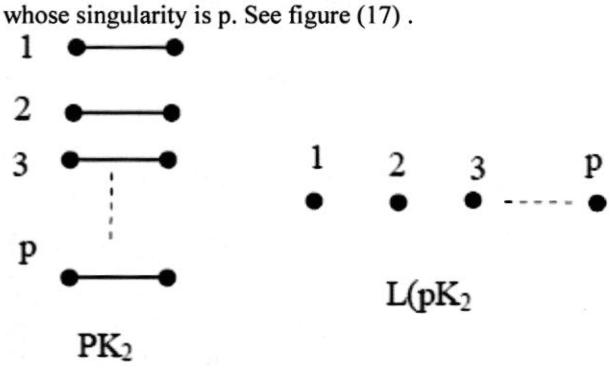

Fig. 17, The Line Tree with maximum singularity

With the line graph of other trees and especially with line n-ary tree the situation is different.

Theorem 6.1

If T is a tree of n vertices, then L(T) is either non singular or has singularity one. [10] .

Example 6.2

For path Pn (special case of tree), $\delta(L(T_n))=0$; if n is odd value $\delta(L(T_n))=1$; if n is even value .

In general the singularity of line tree graph is founded in the theorem below :

Theorem 6.3

If T is a tree, then the singularity of line tree L(T) is at most one.[14]

A generalized line graph can be viewed as the line graph of special type of n-ary tree with different levels. In general the line graph of n-ary tree its so special new graph consist of complete graph K_n or copies of K_n of different order depending on the number n in n- ary tree, on the levels and the number of leafs, end vertices of the n-ary tree. And these copies in the connected line n-ary tree graph are identifying in an edge or introduccing by a vertex. And at most those new line graphs are none singular which is proved in the following theorems.

We will give the results and the theorems in this section to discuss the singularity of line n-ary tree $T_{h,a,n}$.

Theorem 6.4

The line n-ary tree with one level and n leafs $T_{1,n}$ is none singular . To prove this theorem we have to show that the $L(T_{1,n})$ is complete graph of n vertices in the example below.

Example 6.5

Let $T_{1,5}$ be the 5-ary tree in level one with five leafs, then $T_{1,5}$ is complete graph K_5 as shown in figure (18).

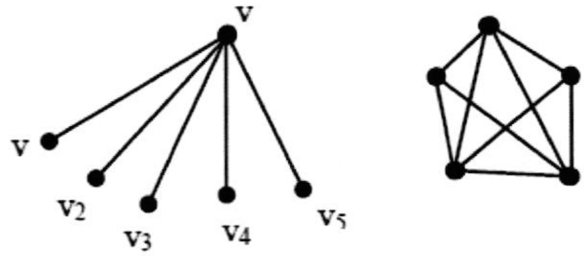

Fig .18, $T_{1,5}$ and $L(T_{1,5})$

Proof

Since the line n-ary tree $L(T_{1,n})$ is complete graph Kn, which is non singular because not possess the zero sum weighting, then $L(T_{1,n})$ in none singular. $\delta(L(T_{1,n}))=0$.

Theorem 6.6

The line binary - ary tree in level 2 is none singular. $\delta(L(T_{2,2,n}))=0$.

Proof

The line tree $L(T_{2,2,n})$ is a new graph constructed by introducing two complete graphs K_n by an edge depends on the duplicate vertices in the second level and end vertices (leafs) n. That is $L(T_{2,2,n})\cong K_{n+1}$: K_{n+1} for n=1,2,3,......, shows in figure (19), but K_{n+1} is not posses the zero sum weighting means it is non singular and has (-1) as an eigenvalue n+1 times in its spectra, then the introducing two copies of this graph K_{n+1} by an edge not causes the singularity.

Theorem 6.7

The singularity of Line ternary ary tree with 2-level and n leafs $L(T_{3,2,n})$ is zero.

بعض خواص شذوذية الأشجار من النمط n-ary والبيانات الخطية n-ary للاشجار

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الملخص

التوزين العالي للبيان، هو التوزين الذي يستخدم فيه أكبر عدد ممكن من المتغيرات المستقلة اللاصفرية المختلفة، و الذي يساوي درجة الشذوذ للبيان. أن الاشجار من النمط n-ary، الحافات المتصلة بالجذر تسمى بالفروع (أطفال) والتي تمثل المستوى الأولى وأذا لدينا h من المستويات فأن $0<i<h$ و كل رأس من المستوى i مرتبط مع فروع في مستوى i+1 و n من الرؤوس الخارجية عند مستوى الأخير تسمى بالأوراق (رؤوس النهائية) .

في هذا البحث درسنا درجة الشذوذ للأشجار من نمط ن-فروع لمستويات وفروع مختلفة مع أوراق n ، كذلك تم احتساب $\delta(T_{n,3,n})= n^2(n-1)+(n-1)$. كما برهنا بأن البيان الخطي لتلكم النمط من الأشجار عبارة عن ربط أو دمج بيانات تامة برتب مختلفة بحافة أو رأس مشترك، ورتبتها تتغير بتغير عدد الفروع ، المستويات والأوراق والتي تكون بدورها غير شاذة .

Proof: The idea of the proof is that the construction of line ternary-ary 2-level $T_{3,2,n}$ is representation of introducing copies of K_{n+1} each to others by an edge. And it's not posses the zero sum weighting. That is the uses of any number of variables as a weights to apply $\sum_{v_i \in N(w_i)} f(v_i) = 0$ for getting the high zero sum weighting, all variables must be zero, then its non singular . As seen in the Example below where the order of the complete graph depend on the n (the number of leafs adjacent to each internal vertex in the second level).
In general the line Tree graph of the form L(Tn,3,n) represents identifying copies of complete graphs each to others by an edge or an interlacing in a vertex to construct new form which have the role of none singularity.

Conclusion

1) The singularity of n-ary tree is changed (increased) directly with the number of branches in each level of the tree and depends on the number of leafs .

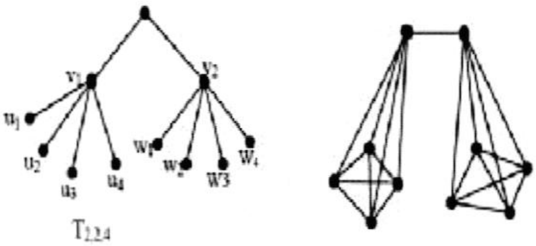

Fig .19,The line graph of binary ary level 2 and 4 leafs

2- the line graph of n-ary tree is always identifying of two complete graph of same order by a vertex or introducing of two complete graphs by a new edge and they are non-singular .

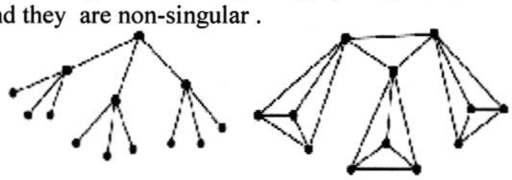

Fig .20,The line graph of ternary ary level 2 and 3 leafs

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