A Non PGL (3,q) k-arcs in the projective plane of order 37

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Abstract

A k-arc in the plane PG(2,q) is a set of k points such that every line in the plane intersects it in at most two points and there is a line intersects it in exactly two points. The k-arc is complete if there is no k+1-arc containing it. The main purpose of this paper is to study and find the projectively distinct k- arcs, k=4,5,6,7 in PG(2,37) through the classification and construction of the projectively distinct k-arcs and finding the group of projectivities of each projectively distinct k-arc and describing it. Also it was found that PG(2,37) has no maximum arc .

Introduction

A k-arc in projective plane PG (2,q) is a set of k points such that every line in the plane intersects it in at most two points and there is a line intersects it in exactly two points. The k-arc is complete if there is no k+1-arc containing it.

The purpose of this thesis is to classify and construct the projectively distinct k-arcs in PG(2,37) over the Galois field GF(37). Also it was shown that PG(2,37) has no maximum k-arc. Hirschfeld [7] and Sadeh [10] showed the classification and construction of k-arcs over the Galois field GF(q), with q≤11. Younis [12]first studied classification of k-arcs in the projectivety plane PG (2,16). The classification of the complete k-arcs in PG (2,q) ,for q=23,25,27 has been given by Coolsaet and Sticker [5],[6] .

Chao ,Kaneta [4] classification of the complete k-arcs in PG(2,q) ,for 23≤q≤29.

AL-Taee [1] classified the projective distinct k-arcs for k=5,6 in the projective plane PG(2,32).

Also this paper represents an extra step in this side ، which include first study for construction and classification k-arcs in the projective plane PG (2,37). The subject matter in this paper needs long time for running computer programs which are used to find projectively distinct k-arcs in the projective plane of order 37 and finding groups for these k-arcs for values k=5,6.

(1-2) Definition(Companion Matrix) [7]

Let $f(x)= x^n \_ a_{n-1} x^{n-1} \_ \dots \_ a_o$ be any monic polynomial ,then its Companion matrix ,C(f),is given by the n x n matrix :

$$C(f)=\begin{bmatrix} 0 & 1 & 0 & 0 & \dots & 0 \\ 0 & 0 & 1 & 0 & \dots & 0 \\ . & . & . & . & \dots & . \\ 0 & 0 & 0 & 0 & \dots & 1 \\ a_0 & a_1 & a_2 & a_3 & \dots & a_{n-1} \end{bmatrix}$$

In particular case ,when n=3 then $f(x)= x^3 \_ a_2 x^2 \_ a_1 x^1 \_ a_o$,

$$\text{and } C(f)=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ a_0 & a_1 & a_2 \end{bmatrix}$$

(1-2-1) Definition

A Projectively S : α → α is a bijection given by a non-singular matrix T such that if P (x′)= P(x) S , then tx= xT where x′and x are coordinate vectors for P(x′)and P (x),and t ∈ Ko.

(1-2-2) Fundamental theorem of projective geometary [7]

(1) If $\{p_1,p_2,\dots,p_{n+2}\}$,$\{p_1^*,p_2^*,\dots,p_{n+2}^*\}$are two sets of points of PG(n ,q) such that no n+1 points chosen from the same set lie in a prime(a prime in PG(2,q) is a line) , then there exists a unique projectivity T such that $p_i^*= p_iT$,for all i in $N_{n+2}=\{1,2,...,n+2\}$.

(2) Let S = PG(n ,q), and £: S ⟶S be a collineation , then £=σT, where σ is an automorphism and T is a projectivity. This means that if K= GF (p^h),and P(Y)=P(X)£; then there exists m in N_h, t_{ij} in GF$(p^h)$for (i,j)in $N_h^* \times N_h^*$ and t in GF $(p^h)\backslash\{0\}$ such that $tY=X^r$ T, where $r= p^m$ and $X^r=(x_0^r,x_1^r,\dots,x_n^r)$ and $T=(t_{ij})$;i,jin N_h^*,where$N_h^*=\{0,1,2,\dots,h\}$and $N_h=\{1,2,\dots,h\}$.

(1-2-3) Cyclic projectivities [7]

A projectivity T which permutes the points of PG (n,q) which is denoted byθ (n) in a single cycle is called a cyclic projectivity , where $\theta(n)=(q^{n+1}-1)/(q-1)$. For example ,the projectivity represented by the matrix:

$$T=\begin{bmatrix} 0 & 1 & 0 \\ 0 & 0 & 1 \\ 1 & 0 & 1 \end{bmatrix}$$

on PG(2 ,5) is a cyclic projectivity which is given by right multiplication on the points of PG (2 ,5).

(1-2-4) Definitions [7,8]

A line l of PG(2,q) is an i-secant of a (k ,n)-arc K if ,|l∩k|=i ,A 0-secant is called an external line of k-arc ,a 1-secant is called unisecant and a 2-secant is called a bisecant.

(1-2-5) Theorm [11]

Let t(p) be the number of unisecants through P, where P is a point of the k-arc K. Let T_i be the total number of i-secants of K in the plane ,then:

1-t(p)=q+2-k=t

2-T_2=k(k-1)/2,T_1=kt, and T_0=q(q-1)/2+t(t-1)/2.

(1-3) Group of projectiveties

(1-3-1) Definition [3]

Let V be a vector space over the field k. The general linear group GL (V) is the group of all linear automorphisms of V. The subgroup consisting of those linear automorphisms with determinant 1 is called the special linear group SL(V).

Note : when V=V(n ,q),we use the notations GL(n ,q) and SL(n ,q).

(1-3-2) Theorem [3]

The order of GL(V) and SL(V) are given by:

$$|GL(n,q)|=(q^n-q^{n-1})(q^n-q^{n-2})\ldots(q^{n-1}) = q^{n(n-1)/2}\cdot\prod_{i=1}^{n}(q^i-1).$$

$$|SL(n,q)|= q^{n(n-1)/2}\cdot\prod_{i=2}^{n}(q^i-1).$$

(1-3-3) Definition [3]

The projective general linear group PGL(V),and the projective special linear group PSL(V),are defined as follows:

PGL(V)= GL(V)/Z(GL(V)),where Z(GL(V)) is the centre of GL(V).

PSL(V)= SL(V)/ Z(SL(V)).

When V=V(n, q) it is customary to write the projective groups just defined as

PGL(n ,q)and PSL(n,q).

(1-3-4) Theorem: [3]

The set PGL(n ,q) forms a group which is called a projective group.

(1-3-5) Definition: [1]

The projective group denoted byPGL(n ,q) is the group of all projective transformations in projective space PG(n-1,q).

2. Construction and Classification of k-arcs in PG(2,37) over the Galois field GF(37)

In this paper, the construction and classification of the k-arcs which 4≤k≤7 has been obtained and also the group of projectvities of the projectively distinct k-arcs are found.

(2-1) The projective plane PG (2,37)

Let f(x)= x^3-3x^2-x-2 be an irreducible monic polynomial over GF(37) then the companion matrix T of f(x)

$$T=\begin{bmatrix}0&1&0\\1&0&0\\2&1&3\end{bmatrix}$$

is a cyclic projectivity on PG(2,37).

Let p_0 be the point U_0=(1,0,0) then $p_i=p_0T_i$,i=0,…,1406,are the 1407 points of PG(2,37).(see Table(1))

Table(1) Points of PG (2,37)

i	P_i
0	2 1 1
1	1 2 1
2	1 1 2
3	2 20 21
…	…
1405	2 37 16
1406	2 4 37

Let L_1 be the line at infinity (z=0) which contains the points:-

0,1,9,29,152,156,182,193,262,300,323,401,404,419,425,489,539,605,621,679,689,714, 754,851,923,945,972,1034,1195,1229,1231,1248,1262,1308,1321,1353,1360and 1365

then $L_i=L_1T_{i-1}$, i=1,…,1407,are the lines of PG(2,37), the 1407 lines L_i are given by the rows in (Table(2)).

Table(2) Lines of PG(2,37)

Lines	Points for each line
Line1	0,1,9,29,152,156,182,193,262,193,262,300,323,401,404,419,425,489,539,605,621 1248,1262,1308,1321,679,689,714,754,851,923,945,972,1034,1195,1229,1231،, 1353,1360,1365
Line2	1,2,10,30,153,157,183,194,263,301,324,402,405,420,426,490,540,606,622,680,690,715,755,852,924, 946,973,1035,1196,1230,1232,1249,1263,1309,1322,1354,1361 , 1366
…	…
Line1407	14060,8,28,151,155,181,192,261,299,322,400,403,418,424,488,538,604، ،620,678,688,713,753,850,922,944,971,1033,1194,1228,1230,1247,1261,1307,1320 1352,1359,1364

3.Construction k-arcs for 4≤k≤7

Let P_0,P_1,P_2,P_3 are $P_0=(1,0,0), P_1=(0,1,0), P_2=(0,0,1), P_3=(1,1,1)$ which form 4-arcs points respect to fundamental theorem of projective geometry(1-3) then 4-arc is the equivalent arc $\{P_0, P_1, P_2, P_3\}$ construct arcs from k=5 into k=7 respect to 4-arc which includes points $\{P_0, P_1, P_2, P_3\}$a according to these in the following algorithm work used computer program .

(3-1)Algorithm work used for the construction and classification of the k-arcs which 4≤k≤7 in PG(2,37).

1-Determine lines which are 2- secants for arc k=$\{P_0, P_1, P_2, P_3\}$

2-Find the points which don't lie in lines 2-secants .

3-Adding these point odd into k-arc and getting 5-arc .

4- Finding projectively distinct k-arcs.

5- Finding groups [9] which are fixing projectively distinct k-arcs.

6-Repeating the steps from 2 into 5 to finding 5-arcs,6-arcs and7-arcs.

7-Finding complete arcs for all steps if they exist .

(3-2)Projectively Distinct 5-arcs

(3-2-1) The Construction of the Projectively Distinct 5-arcs

(3-2-2)Definition [2]

Let k be a k-arc. the points of PG(2,q)\k which don't belong to any bisecant of k ,will be called the points of index zero of k .

Consider the set k={0,1,2,1109} of four points of PG(2,37)no three of them are collinear. These four points form 4-arcs,to construct Projectively distinct 5-arcs through the Projectively distinct 4-arcs respect to definition (3-2-2) we mention four points over plane lines and simulation points lines which are bisecants from points of plane PG(2,37) then add these points odd into references four points and getting 5-arcs , by using the computer program we have 24 Projectively distinct 5-arcs and classify it respect to [9] as the show in table below .

Table(3) Projectively Distinct 5-arcs

LINES	p_1	p_2	p_3	p_4	p_5	G	|G|	T_0	T_1	T_2
X1	0	1	2	1109	3	C_2	2	1227	170	10
X2	0	1	2	1109	4	I	1	1227	170	10
X3	0	1	2	1109	5	C_2	2	1227	170	10
X4	0	1	2	1109	6	C_2	2	1227	170	10
X5	0	1	2	1109	7	I	1	1227	170	10
X6	0	1	2	1109	8	I	1	1227	170	10
X7	0	1	2	1109	11	I	1	1227	170	10
X8	0	1	2	1109	14	I	1	1227	170	10
X9	0	1	2	1109	16	I	1	1227	170	10
X10	0	1	2	1109	17	I	1	1227	170	10
X11	0	1	2	1109	27	C_2	2	1227	170	10
X12	0	1	2	1109	31	I	1	1227	170	10
X13	0	1	2	1109	35	C_2	2	1227	170	10
X14	0	1	2	1109	43	C_2	2	1227	170	10
X15	0	1	2	1109	44	C_2	2	1227	170	10
X16	0	1	2	1109	50	C_5	5	1227	170	10
X17	0	1	2	1109	55	C_2	2	1227	170	10
X18	0	1	2	1109	94	C_3	3	1227	170	10
X19	0	1	2	1109	213	I	1	1227	170	10
X20	0	1	2	1109	232	I	1	1227	170	10
X21	0	1	2	1109	251	I	1	1227	170	10
X22	0	1	2	1109	292	C_2	2	1227	170	10
X23	0	1	2	1109	294	I	1	1227	170	10
X24	0	1	2	1109	599	C_2	2	1227	170	10

All the projectively distinct 5-arcs $x_i(i=1,2,\ldots,24)$, are incomplete 24-arcs.

(3-3) Projectively Distinct 6-arcs

(3-3-1) The Construction of the Projectively Distinct 6-arcs

To construct all 6-arcs through projectively distinct 5-arcs,use the same way of constructing 5-arcs.So we have the projective plane PG(2,37) consisting of 3691 arcs from projectivily distant 6-arcs and there exist 3375 arcs type of identity group I, also there exist 12 arcs that have a group of type S_4 as the following table below shows .

Table(4) Projectively Distinct 6-arcs

LINES	P_1	P_2	P_3	P_4	P_5	$_6P$	T_0	T_1	T_2
Y1	0	1	2	1109	5	350	1194	198	15
Y2	0	1	2	1109	6	1007	1194	198	15
Y3	0	1	2	1109	6	1173	1194	198	15
Y4	0	1	2	1109	14	768	1194	198	15
Y5	0	1	2	1109	35	215	1194	198	15
Y6	0	1	2	1109	35	865	1194	198	15
Y7	0	1	2	1109	35	1131	1194	198	15
Y8	0	1	2	1109	43	506	1194	198	15
Y10	0	1	2	1109	43	999	1194	198	15
Y11	0	1	2	1109	55	1339	1194	198	15
Y12	0	1	2	1109	292	1005	1194	198	15

Also there exist 12 arcs that have a group of type C_2XC_2 as the following table below shows.

All the projectively distinct 6-arcs $Y_i(i=1,2,\ldots,12)$, are incomplete12-arcs .

Table (5) Projectively Distinct 6-arcs

LINES	P_1	P_2	P_3	P_4	P_5	P_6	T_0	T_1	T_2
Z1	0	1	2	1109	5	1372	1194	198	15
Z2	0	1	2	1109	5	629	1194	198	15
Z3	0	1	2	1109	6	866	1194	198	15
Z4	0	1	2	1109	6	1223	1194	198	15
Z5	0	1	2	1109	8	1214	1194	198	15
Z6	0	1	2	1109	11	273	1194	198	15
Z7	0	1	2	1109	14	1227	1194	198	15
Z8	0	1	2	1109	27	35	1194	198	15
Z9	0	1	2	1109	27	1343	1194	198	15
Z10	0	1	2	1109	35	186	1194	198	15
Z11	0	1	2	1109	44	485	1194	198	15
Z12	0	1	2	1109	55	1175	1194	198	15

All the projectively distinct 6-arcs Zi(i=1,2,…,12), are incomplete12-arcs.

Also there exists one group of type C_7xC_3 ,one group of type C_2XC_4,one group of type D_{15} and exists one group of type D_5 , one group of type D_4 ,two groups of type $C_2XC_2XC_2$, and two groups of type C_3XC_3 as the following table below shows .

Table(6) Projectively Distinct 6-arcs

LINES	P1	P2	P3	P4	P5	P6	G	T_0	T_1	T_2
F1	0	1	2	1109	94	820	C_7XC_3	1194	198	15
F2	0	1	2	1109	6	637	C_2XC_4	1194	198	15
F3	0	1	2	1109	50	317	D_{15}	1194	198	15
F4	0	1	2	1109	27	373	D_5	1194	198	15
F5	0	1	2	1109	3	104	D_4	1194	198	15
F6	0	1	2	1109	3	417	$C_2XC_2XC_2$	1194	198	15
F7	0	1	2	1109	5	837	$C_2XC_2XC_2$	1194	198	15
F8	0	1	2	1109	44	430	C_3XC_3	1194	198	15
F9	0	1	2	1109	50	211	C_3XC_3	1194	198	15

All the projectively distinct 6-arcs F_i (i=1,2,…,9), are incomplete 9-arcs . Also it is found 34 arcs that have a group of type C_3 as the following table below shows.

Table(7) Projectively Distinct 6-arcs

LINES	P_1	P_2	P_3	P_4	P_5	P_6	T_0	T_1	
H1	0	1	2	1109	3	148	1194	198	15
H2	0	1	2	1109	3	252	1194	198	15
H3	0	1	2	1109	7	1091	1194	198	15
H4	0	1	2	1109	12	40	1194	198	15
H5	0	1	2	1109	4	34	1194	198	15
H6	0	1	2	1109	4	173	1194	198	15
H7	0	1	2	1109	4	195	1194	198	15
H8	0	1	2	1109	4	529	1194	198	15
H9	0	1	2	1109	4	611	1194	198	15
H10	0	1	2	1109	4	701	1194	198	15
H11	0	1	2	1109	4	1122	1194	198	15
H12	0	1	2	1109	4	1323	1194	198	15
H13	0	1	2	1109	7	14	1194	198	15
H14	0	1	2	1109	7	291	1194	198	15
H15	0	1	2	1109	7	428	1194	198	15
H16	0	1	2	1109	7	470	1194	198	15
H17	0	1	2	1109	7	984	1194	198	15
H18	0	1	2	1109	7	1045	1194	198	15
H19	0	1	2	1109	7	1203	1194	198	15
H20	0	1	2	1109	8	466	1194	198	15
H21	0	1	2	1109	8	483	1194	198	15
H22	0	1	2	1109	8	550	1194	198	15

H23	0	1	2	1109	14	775	1194	198	15
H24	0	1	2	1109	14	253	1194	198	15
H25	0	1	2	1109	16	393	1194	198	15
H26	0	1	2	1109	17	937	1194	198	15
H27	0	1	2	1109	27	944	1194	198	15
H28	0	1	2	1109	31	562	1194	198	15
H29	0	1	2	1109	35	1063	1194	198	15
H30	0	1	2	1109	55	228	1194	198	15
H31	0	1	2	1109	213	899	1194	198	15
H32	0	1	2	1109	232	655	1194	198	15
H33	0	1	2	1109	3	308	1194	198	15
H34	0	1	2	1109	50	1152	1194	198	15

All the projectively distinct 6-arcs $H_i(i=1,2,\ldots,34)$, are incomplete 34-arcs . Also it is found 4 arcs that have a group of type C_5 as the following table below shows.

Table(8) Projectively Distinct 6-arcs

LINES	P1	P2	P3	P4	P5	P6	T_0	T_1	T_2
J_1	0	1	2	1109	43	506	1194	198	15
J_2	0	1	2	1109	16	772	1194	198	15
J_3	0	1	2	1109	27	244	1194	198	15
J_4	0	1	2	1109	27	576	1194	198	15

All the projectively distinct 6-arcs $J_i(i=1,2,3,4)$, are incomplete 4-arcs . Also it is found 245 arcs that have a group of type C_2 as the following table below shows.

Table(9) Projectively Distinct 6-arcs

	P1 P2 P3 P4 P5 P6	T_0	T_1	T_2
W1	0 1 2 1109 3 26	1194	198	15
W2	0 1 2 1109 3 60	1194	198	15
W3	0 1 2 1109 3 61	1194	198	15
W4	0 1 2 1109 3 107	1194	198	15
W5	0 1 2 1109 3 128	1194	198	15
W6	0 1 2 1109 3 140	1194	198	15
W7	0 1 2 1109 3 192	1194	198	15
W8	0 1 2 1109 3 242	1194	198	15
W9	0 1 2 1109 3 382	1194	198	15
W10	0 1 2 1109 3 387	1194	198	15
W11	0 1 2 1109 3 392	1194	198	15
W12	0 1 2 1109 3 400	1194	198	15
W13	0 1 2 1109 3 414	1194	198	15
W14	0 1 2 1109 3 439	1194	198	15
W15	0 1 2 1109 3 457	1194	198	15
W16	0 1 2 1109 3 551	1194	198	15
W17	0 1 2 1109 3 554	1194	198	15
W18	0 1 2 1109 3 626	1194	198	15
W19	0 1 2 1109 3 648	1194	198	15
W20	0 1 2 1109 3 675	1194	198	15
W21	0 1 2 1109 3 730	1194	198	15
W22	0 1 2 1109 3 737	1194	198	15
W23	0 1 2 1109 3 898	1194	198	15
W24	0 1 2 1109 3 934	1194	198	15
W25	0 1 2 1109 3 951	1194	198	15
W26	0 1 2 1109 3 1003	1194	198	15
W27	0 1 2 1109 3 1011	1194	198	15
W28	0 1 2 1109 3 1024	1194	198	15
W29	0 1 2 1109 3 1056	1194	198	15
W30	0 1 2 1109 3 1063	1194	198	15
W31	0 1 2 1109 3 1068	1194	198	15
W32	0 1 2 1109 3 1069	1194	198	15
W33	0 1 2 1109 3 1266	1194	198	15

W34	0 1 2 1109 3 1303	1194	198	15
W35	0 1 2 1109 4 158	1194	198	15
W36	0 1 2 1109 4 165	1194	198	15
W37	0 1 2 1109 4 197	1194	198	15
W38	0 1 2 1109 4 213	1194	198	15
W39	0 1 2 1109 4 214	1194	198	15
W40	0 1 2 1109 4 406	1194	198	15
W41	0 1 2 1109 4 475	1194	198	15
W42	0 1 2 1109 4 559	1194	198	15
W43	0 1 2 1109 4 632	1194	198	15
W44	0 1 2 1109 4 774	1194	198	15
W45	0 1 2 1109 4 777	1194	198	15
W46	0 1 2 1109 4 802	1194	198	15
W47	0 1 2 1109 4 853	1194	198	15
W48	0 1 2 1109 4 865	1194	198	15
W49	0 1 2 1109 4 877	1194	198	15
W50	0 1 2 1109 4 900	1194	198	15
W51	0 1 2 1109 4 912	1194	198	15
W52	0 1 2 1109 4 939	1194	198	15
W53	0 1 2 1109 4 980	1194	198	15
W54	0 1 2 1109 4 1015	1194	198	15
W55	0 1 2 1109 4 1087	1194	198	15
W56	0 1 2 1109 4 1131	1194	198	15
W57	0 1 2 1109 4 1136	1194	198	15
W58	0 1 2 1109 4 1157	1194	198	15
W59	0 1 2 1109 4 1186	1194	198	15
W60	0 1 2 1109 4 1236	1194	198	15
W61	0 1 2 1109 4 1286	1194	198	15
W62	0 1 2 1109 4 1318	1194	198	15
W63	0 1 2 1109 4 1319	1194	198	15
W64	0 1 2 1109 4 1381	1194	198	15
W65	0 1 2 1109 4 1397	1194	198	15
W66	0 1 2 1109 4 1405	1194	198	15
W67	0 1 2 1109 5 64	1194	198	15
W68	0 1 2 1109 5 202	1194	198	15
W69	0 1 2 1109 5 205	1194	198	15
W70	0 1 2 1109 5 224	1194	198	15
W71	0 1 2 1109 5 238	1194	198	15
W72	0 1 2 1109 5 258	1194	198	15
W73	0 1 2 1109 5 260	1194	198	15
W74	0 1 2 1109 5 277	1194	198	15
W75	0 1 2 1109 5 289	1194	198	15
W76	0 1 2 1109 5 291	1194	198	15
W77	0 1 2 1109 5 389	1194	198	15
W78	0 1 2 1109 5 403	1194	198	15
W79	0 1 2 1109 5 436	1194	198	15
W80	0 1 2 1109 5 437	1194	198	15
W81	0 1 2 1109 5 445	1194	198	15
W82	0 1 2 1109 5 461	1194	198	15
W83	0 1 2 1109 5 534	1194	198	15
W84	0 1 2 1109 5 588	1194	198	15
W85	0 1 2 1109 5 759	1194	198	15
W86	0 1 2 1109 5 861	1194	198	15
W87	0 1 2 1109 5 917	1194	198	15
W88	0 1 2 1109 5 1125	1194	198	15
W89	0 1 2 1109 5 1059	1194	198	15
W90	0 1 2 1109 5 1287	1194	198	15

W91	0 1 2 1109 5 1381	1194	198	15
W92	0 1 2 1109 6 22	1194	198	15
W93	0 1 2 1109 6 31	1194	198	15
W94	0 1 2 1109 6 36	1194	198	15
W95	0 1 2 1109 6 76	1194	198	15
W96	0 1 2 1109 6 83	1194	198	15
W97	0 1 2 1109 6 95	1194	198	15
W98	0 1 2 1109 6 212	1194	198	15
W99	0 1 2 1109 6 249	1194	198	15
W100	0 1 2 1109 6 289	1194	198	15
W101	0 1 2 1109 6 309	1194	198	15
W102	0 1 2 1109 6 311	1194	198	15
W103	0 1 2 1109 6 326	1194	198	15
W104	0 1 2 1109 6 472	1194	198	15
W105	0 1 2 1109 6 485	1194	198	15
W106	0 1 2 1109 6 506	1194	198	15
W107	0 1 2 1109 6 508	1194	198	15
W108	0 1 2 1109 6 525	1194	198	15
W109	0 1 2 1109 6 585	1194	198	15
W110	0 1 2 1109 6 643	1194	198	15
W111	0 1 2 1109 6 685	1194	198	15
W112	0 1 2 1109 6 693	1194	198	15
W113	0 1 2 1109 6 746	1194	198	15
W114	0 1 2 1109 6 768	1194	198	15
W115	0 1 2 1109 6 836	1194	198	15
W116	0 1 2 1109 6 885	1194	198	15
W117	0 1 2 1109 6 984	1194	198	15
W118	0 1 2 1109 6 1252	1194	198	15
W119	0 1 2 1109 6 1305	1194	198	15
W120	0 1 2 1109 6 1398	1194	198	15
W121	0 1 2 1109 7 21	1194	198	15
W122	0 1 2 1109 7 144	1194	198	15
W123	0 1 2 1109 7 211	1194	198	15
W124	0 1 2 1109 7 254	1194	198	15
W125	0 1 2 1109 7 299	1194	198	15
W126	0 1 2 1109 7 344	1194	198	15
W127	0 1 2 1109 7 411	1194	198	15
W128	0 1 2 1109 7 471	1194	198	15
W129	0 1 2 1109 7 486	1194	198	15
W130	0 1 2 1109 7 590	1194	198	15
W131	0 1 2 1109 7 656	1194	198	15
W132	0 1 2 1109 7 773	1194	198	15
W133	0 1 2 1109 7 779	1194	198	15
W134	0 1 2 1109 7 879	1194	198	15
W135	0 1 2 1109 7 937	1194	198	15
W136	0 1 2 1109 7 958	1194	198	15
W137	0 1 2 1109 7 1013	1194	198	15
W138	0 1 2 1109 7 1060	1194	198	15
W139	0 1 2 1109 7 1090	1194	198	15
W140	0 1 2 1109 7 1092	1194	198	15
W141	0 1 2 1109 7 1131	1194	198	15
W142	0 1 2 1109 7 1161	1194	198	15
W143	0 1 2 1109 7 1215	1194	198	15
W144	0 1 2 1109 7 1223	1194	198	15
W145	0 1 2 1109 7 1284	1194	198	15
W146	0 1 2 1109 7 1324	1194	198	15
W147	0 1 2 1109 8 42	1194	198	15

W148	0 1 2 1109 8 113			1194	198	15
W149	0 1 2 1109 8 244			1194	198	15
W150	0 1 2 1109 8 265			1194	198	15
W151	0 1 2 1109 8 267			1194	198	15
W152	0 1 2 1109 8 362			1194	198	15
W153	0 1 2 1109 8 568			1194	198	15
W154	0 1 2 1109 8 844			1194	198	15
W155	0 1 2 1109 8 1090			1194	198	15
W156	0 1 2 1109 8 1141			1194	198	15
W157	0 1 2 1109 8 1153			1194	198	15
W158	0 1 2 1109 8 1182			1194	198	15
W159	0 1 2 1109 11 99			1194	198	15
W160	0 1 2 1109 11 233			1194	198	15
W161	0 1 2 1109 11 299			1194	198	15
W162	0 1 2 1109 11 369			1194	198	15
W163	0 1 2 1109 11 385			1194	198	15
W164	0 1 2 1109 11 635			1194	198	15
W165	0 1 2 1109 11 685			1194	198	15
W166	0 1 2 1109 11 748			1194	198	15
W167	0 1 2 1109 11 822			1194	198	15
W168	0 1 2 1109 11 1170			1194	198	15
W169	0 1 2 1109 11 1294			1194	198	15
W170	0 1 2 1109 14 35			1194	198	15
W171	0 1 2 1109 14 61			1194	198	15
W172	0 1 2 1109 14 299			1194	198	15
W173	0 1 2 1109 14 559			1194	198	15
W174	0 1 2 1109 14 885			1194	198	15
W175	0 1 2 1109 14 954			1194	198	15
W176	0 1 2 1109 14 1105			1194	198	15
W177	0 1 2 1109 14 1240			1194	198	15
W178	0 1 2 1109 14 1256			1194	198	15
W179	0 1 2 1109 14 1346			1194	198	15
W180	0 1 2 1109 14 1357			1194	198	15
W181	0 1 2 1109 14 1378			1194	198	15
W182	0 1 2 1109 14 1395			1194	198	15
W183	0 1 2 1109 16 52			1194	198	15
W184	0 1 2 1109 16 213			1194	198	15
W185	0 1 2 1109 16 355			1194	198	15
W186	0 1 2 1109 16 388			1194	198	15
W187	0	1	2 1109 16 436	1194	198	15
W188	0	1	2 1109 16 444	1194	198	15
W189	0	1	2 1109 16 588	1194	198	15
W190	0	1	2 1109 16 706	1194	198	15
W191	0	1	2 1109 16 741	1194	198	15
W192	0	1	2 1109 16 838	1194	198	15
W193	0	1	2 1109 16 1242	1194	198	15
W194	0	1	2 1109 17 506	1194	198	15
W195	0	1	2 1109 17 970	1194	198	15
W196	0	1	2 1109 17 1042	1194	198	15
W197	0	1	2 1109 17 1083	1194	198	15
W198	0	1	2 1109 17 1143	1194	198	15
W199	0	1	2 1109 27 151	1194	198	15
W200	0	1	2 1109 27 208	1194	198	15
W201	0	1	2 1109 27 326	1194	198	15
W202	0	1	2 1109 27 366	1194	198	15
W203	0	1	2 1109 27 424	1194	198	15
W204	0	1	2 1109 27 686	1194	198	15

W205	0	1	2	1109	31 282	1194	198	15
W206	0	1	2	1109	31 535	1194	198	15
W207	0	1	2	1109	31 565	1194	198	15
W208	0	1	2	1109	31 734	1194	198	15
W209	0	1	2	1109	31 751	1194	198	15
W210	0	1	2	1109	31 768	1194	198	15
W211	0	1	2	1109	31 1098	1194	198	15
W212	0	1	2	1109	35 833	1194	198	15
W213	0	1	2	1109	35 1220	1194	198	15
W214	0	1	2	1109	44 803	1194	198	15
W215	0	1	2	1109	44 1392	1194	198	15
W216	0	1	2	1109	50 1127	1194	198	15
W217	0	1	2	1109	55 228	1194	198	15
W218	0	1	2	1109	55 278	1194	198	15
W219	0	1	2	1109	55 453	1194	198	15
W220	0	1	2	1109	55 471	1194	198	15
W221	0	1	2	1109	55 590	1194	198	15
W222	0	1	2	1109	55 1047	1194	198	15
W223	0	1	2	1109	55 1092	1194	198	15
W224	0	1	2	1109	55 1099	1194	198	15
W225	0	1	2	1109	55 1104	1194	198	15
W226	0	1	2	1109	55 1259	1194	198	15
W227	0	1	2	1109	94 87	1194	198	15
W228	0	1	2	1109	94 396	1194	198	15
W229	0	1	2	1109	213 533	1194	198	15
W230	0	1	2	1109	213 650	1194	198	15
W231	0	1	2	1109	213 1274	1194	198	15
W232	0	1	2	1109	232 443	1194	198	15
W233	0	1	2	1109	232 807	1194	198	15
W234	0	1	2	1109	251 1392	1194	198	15
W235	0	1	2	1109	292 569	1194	198	15
W236	0	1	2	1109	292 725	1194	198	15
W237	0	1	2	1109	292 1156	1194	198	15
W238	0	1	2	1109	599 51	1194	198	15
W239	0	1	2	1109	599 143	1194	198	15
W240	0	1	2	1109	599 266	1194	198	15
W241	0	1	2	1109	599 872	1194	198	15
W242	0	1	2	1109	599 877	1194	198	15
W243	0	1	2	1109	599 1181	1194	198	15
W244	0	1	2	1109	599 1242	1194	198	15
W245	0	1	2	1109	599 1344	1194	198	15

All the projectively distinct 6-arcs $W_i(i=1,2,\ldots,245)$, are incomplete 245-arcs .

(3-4) Projectively Distinct 7-arcs:

(3-4-1) The Construction of the Projectively Distinct 7-arcs

The number of projectively distinct 7-arcs in PG(2,37) has (292786) arcs and it is big number which is found by computer program ,we cannot classify it because there are many arcs and it takes more than (150) hours on computer and it is not yet stopping .therefore we used more than 6 computers to run it and get results of complete arcs but the computers continue working in the same method , for this reasons we stop working on this step.

Existence of Complete (k,3)-arcs in PG(2,37)

In previos sections we constructed all the projectively distinct k-arcs for k=4,5,6,7,and we see that ,k-arcs with $4\leq k\leq 7$ are incomplete. these results are obtained by computer research. We shall now prove that in theorem(4-5-1) that complete (k,3)-arcs do not exist by geometric method.

(4-1)Definitions [7,8]

1-A (k ,n)-arc K is a set of k points ,such that there is some n but no (n+1)are collinear where $n\geq 2$.

2-A (k ,n)-arc is complete if ,there is no (k+1,n)-arc containing it.

3- the maximum value in which (k ,n)-arc exists in the projective plane PG(2,q) is denoted by $m(n)_{2,q}$.

(4-2) Lemma

For a (k ,n)-arc K ,the following equation holds :[7,8]

1- $\sum_{i=0}^{r} T_i = q^2+q+1$

2- $\sum_{i=1}^{r} iT_i = n(q+1)$

3- $\sum_{i=2}^{r} i(i-1)T_i = n(n-1)$

4- $\sum_{i=1}^{r} R_i = q+1$

5- $\sum_{i=2}^{r} (i-1)R_i = n-1$

6- $\sum_{i=0}^{r} S_i = q+1$

7- $\sum_{i=1}^{r} iS_i = n$

8- $\sum_{p} R_i = iT_i$

9- $\sum_{Q} S_i = (q+1-i)T_i$

Where the summation in the equation (8) is taken over all p ∈ K, and is taken over all Q∈PG(2,q)\K in the equation (9).

(4-3) Lemma

For a (k ,n)-arc K ,the following equation holds:[6,7]

$\sum_{j=1}^{k} b_j R_{ij} = ir_i$ ....(1)

$\sum_{j=1}^{k} b_j = k$ ....(2)

$\sum_{j=1}^{1} m_j S_{ij} = (q+1-i)r_i$ ....(3)

$\sum_{j=1}^{1} m_j = q^2+q+1-k$ ....(4)

(4-4) Lemma [7]

If K is a complete (k ,n)-arc ,then:

$(q+1-n)\ r_n \geq q^2+q+1-k$,with equality iff $S_n=1$ for all Q in PG(2,q)\K

(4-5) Conclusion

The maximum value $m(3)_{2,37}$ for which (k,3)-arcs does not exist.

(4-5-1)Theorem

In PG(2,37), a complete (k,3)-arc does not exist for 3≤k≤37.

Proof

For3≤k≤37. the equations (4) and (5) of lemma (4.2) become:

R1 + R2 + R3 = 38

R2 + 2 R3= k-1

Let m = [(k-1) / 2] , where [(k-1) / 2] is the integral part of (k-1) / 2 .

So the maximum value of R_3 can accrue is m. Assume that r_i= [(k-1-2i)] , i=0,1,…,m . It is clear that m is positive for k≥3 .The possible types of (k,3)-arc; 3≤k≤37 are given in the following in the table below :

TYPE OF POINT	R_1	R_2	R_3
(m)	37-3m+(k-1)	2m-(k-1)	m
(m-1)	37-3m+(k+1)	2m-(k+1)	m-1
.	.	.	.
(m-i)	37-3m+(k+2i-1)	2m-(k+2i-1)	m-i
.	.	.	.
(0)	37-(k-1)	(k-1)	0

Suppose α_{m-i} denoted the number of points of PG (2,37) of type

(37-3m+(k+2i-1), 2m-(k+2i-1), m-i) , i=0,1,…,m. According to equation (1) and (2) of lemma (4.3) we have,

$$m\alpha_m + (m-1)\alpha_{m-1} + ... + \alpha_1 = 3r_3 ...(*),$$

where r3 is the total number of 3-secants of (k,3)-arc in PG(2,37) , with 3≤k≤37.

Since m≥0,for k≥3, we obtain

$$m(\alpha_m + \alpha_{m-1} + ... + \alpha_1) = m(\sum_{k=0}^{m} \alpha_k)...(**),\text{is}$$

bigger than; $m\alpha_m + (m-1)\alpha_{m-1} + ... + \alpha_1 = \sum_{k=0}^{m} k\alpha_k$.

Therefore, $m(\sum_{k=0}^{m} k\alpha_k) = mk > (\sum_{k=0}^{m} k\alpha_k) = 3r_3$.

This implies that mk > 3r3 or, r3 < mk /3 . Furthermore, Since m ≤ (k-1) / 3.

Since m ≤ (k-1) / 2 , then we have

r3 < k(k-1) / 6 ……………..(1)

On the other hand if the (k,3)-arc K is complete for3≤k≤37 , then

according to lemma (4.4), we have 6r3≥1407-k or $r_3 \geq (1407-k)/6$……..(2)

Now, for k=3 we obtain from the equations (1) and (2)r3<1 and r3>234,which is impossible .So a complete (3,3)-arc does not exist in PG(2,37).

For k=37 , we obtain from equations (1) and (2) r3 < 222 and r3 > 228 which is impossible , so a complete (37,3)-arc does not exist in PG(2,37).

Finally ,if for 3<n<37the (n,3)-arcs is complete ,then we have from equation (1)

r3<n(n-1)/6<(37*36)/6 <222 , also from equation (2)we have r3≥(1407-n)/6>

(1407-37)/6>228.This is impossible, therefore, a complete (37,3)-arc does not exist in PG(2,37) for 3≤k≤37.

The Results

(5-1) Projectively Distance k-arcs

Table(10)represents the classification of projectively distance k-arcs for k=5,6 which is N_k

represents the number of projectively distinct arcs .G_s representing groups k-arcs and describtion then all the projectively distinct k-arcs are incomplete arcs.

this is taken from a computer work to get these results, (250)computer hours.

Table(10) Projectively Distance k-arcs

K=5 N_k=24		K=6 N_k=3691	
G_s	#	G_s	#
I	12	I	3375
C_2	10	S_4	12
C_3	1	C_2x C_2	12
C_5	1	C_7x C_3	1
		C_2x C_4	1
		D_{15}	1
		D_5	1
		D_4	1
		C_2x C_2 xC_2	2
		C_3x C_3	2
		C_3	34
		C_5	4

		C_2	245

(5-2) The Maximum Value $m(n)_{2q}$ in the projective plane PG(2,q)

The maximum value $m(3)_{2,37}$ for which (k,3)-arcs does not exist for $3 \le k \le 37$.

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الأقواس-k المختلفة اسقاطيا في المستوي الاسقاطي ذي الرتبة 37

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الملخص

القوس –k في المستوي PG(2,q) هو مجموعة من النقاط بحيث إن كل خط في المستوي يقطعه بما لا يزيد عن نقطتين ويوجد خط يقطعه بنقطتين بالضبط، ويسمى القوس–k بأنه تام إذا لم يكن بالإمكان وجود قوس–k+1 يحتويهالغرض الأساسي من هذا البحث هو دراسة وا يجاد الأقواس–k المختلفة اسقاطيا ،k=4,5,6,7 في المستوي الاسقاطي PG(2,37) إذ قمنا بتصنيف وبناء هذه لأقواس، وا يجاد الزمر لكل قوس –k مختلف اسقاطيا ووصفها. وأيضا برهنا إن PG(2,37)ليس لديه قوس أعظمي.