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Free Vibration of Simply Supported Beam on Elastic Foundations

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Abstract

Fourier series and finite element analysis are utilized to obtain the solution of simply supported beams resting on elastic foundations with different loadings in order to arrive at a free vibration. The equation of the free vibration of beam on elastic foundation is derived and solved. Good agreement has been obtained between the results of the present study Fourier and finite element analysis and other previous solutions. The effect of modulus of subgrade reaction, axial tension force (foundation modulus) and beam depth on the behavior is studied.

Keywords: Beams, elastic foundation, free vibration, simply supported.

1. Introduction

The study of large amplitude of simply supported beams were given in the work of Kreiger [1] wherein the governing partial differential equations were modified to ordinary differential equations, and the obtained solution is given in terms of elliptic functions using a one-term approximation. Similarly, Butgreen[2] showed the solution for the large amplitude vibration problems of simply supported beams based on the classical continuum approach. Srinivasan employed the Ritz-Galerkin technique to solve the governing nonlinear differential equation of dynamic equilibrium for free of simply supported beams and plates [3, 4]. Evenesen[5] extended the earlier studies for various boundary conditions using the perturbation method. Ray & Bert[6] carried out experimental analysis to verify the analytical solutions for the nonlinear vibrations of hinged beams and compared the solution schemes such as the assumed space mode, assumed time mode and Ritz-Galerkin methods and concluded that the latter two are better than the former.

Pandalai & Sathyamoorthy [7] developed model equations for the nonlinear free vibrations of beams and plates and shells using Lagranges equations and highlighted the difference in the nature of the model equations for beams and plates, rings and shells.

Lou & Sikarskie[8] employed form-function approximations to study the nonlinear forced vibrations of buckled beams.

Rehfield [9] used an approximate method of nonlinear vibration problems with material nonlinear effects for various boundary conditions.

Mustafa [10] used Laplace transformation method to solve the free vibration of simply supported beams.

The process of the formulation of the equations of dynamic equilibrium and of the respective equations of natural vibration of beams on three-parameter elastic foundation within the framework of the second order theory was presented by Morfidis [11]

2. Differential Equation

The partial differential equation for the free undamped transverse vibration of thin beams resting on two parameter elastic foundation is given by [11]:

$$EI\,\frac{\partial^4 w}{\partial x^4} + K.w - T\frac{\partial^2 w}{\partial x^2} = -\mu\frac{\partial^2 w}{\partial t^2} \qquad (1)$$

where,
EI is the flexural rigidity of the beam,
w is the deflection,
x is the distance along the beam
K is the modulus of subgrade reaction
T is the membrane force
μ is the beam mass per unit length
t is the time

Writing the deflection (w) in Fourier half range sine series ,then

$$w(x,t) = \sum_{n=1}^{\infty} w_n(t) \sin\frac{n\pi x}{L} \qquad (2)$$

where L is the beam length

The above expression satisfies the boundary condition

$$\left.\begin{matrix} w \\ \frac{\partial^2 w}{\partial x^2}\end{matrix}\right\} = 0 \qquad \text{at } x = \begin{cases}0\\ L\end{cases}$$

Substituting equation (2) into the basic equation (1) , and simplifying, then

$$\left(\frac{EIn^4\pi^4}{L^4} + K + T\frac{n^2\pi^2}{L^2}\right) w_n(t) = -\mu \ddot{w_n}(t) \qquad (3)$$

Solving and applying initial conditions, then

$$w(x,t) = \sum_{n=1}^{\infty} B_n \cos(\alpha_n t) \sin(\frac{n\pi x}{L})$$

where,

$$\alpha_n = \sqrt{\frac{1}{\mu}\left(\frac{n^4\pi^4 EI}{L^4} + K + \frac{n^2\pi^2}{L^2}T\right)}$$

is the natural frequency and

$$B_n = \frac{2PL^3}{\pi^4 EI}\frac{1}{n^4} sin\frac{n\pi a}{L}$$

for concentrated load Figure (1)

or

$$B_n = \frac{2q_o L^4}{\pi^5 EI}\frac{1}{n^5}\left(1 - cos\frac{n\pi a}{L}\right)$$

for partial uniform load Figure (1)

The period of vibration is given by

$$T_p = \frac{2\pi}{\alpha_n}$$

The frequency of vibration is given by

$$f = \frac{1}{T_p}$$

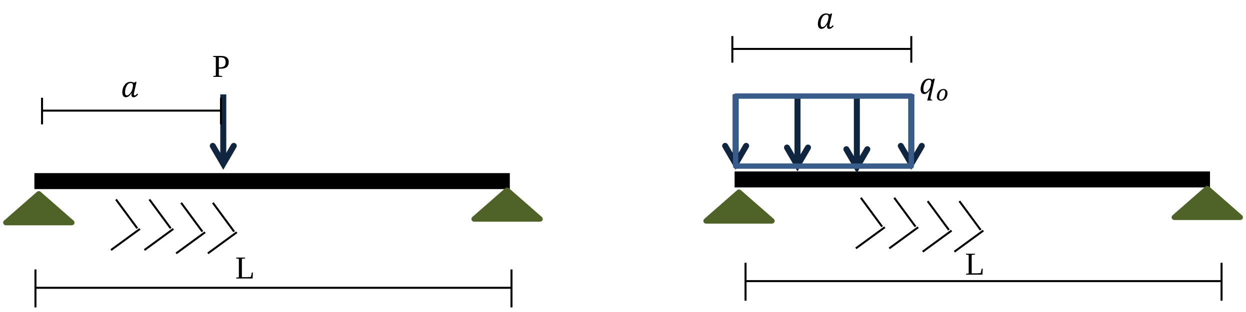

Figure 1: Beam under concentrated and uniform partial load.

3. Finite Element Analysis

The frame element is used to model the beam using SeismoStruct Software and link element to model the spring. SeismoStruct is Finite Element package capable of predicting the large displacement behavior of space frames under static or dynamic loading, taking into account both geometric nonlinearities and material inelasticity. Concrete, steel and frp material models are available, together with a large library of 3D elements that may be used with a wide variety of pre-defined steel, concrete and composite section configurations.

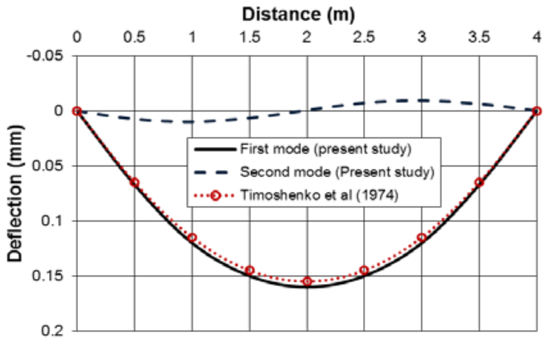

Figure 2: Deflection profile for the first and second modes.

4. Application

4.1 Simply Supported Beam under Partial Uniform Load

The following properties of a simply supported beam under uniformly distributed load will be considered to draw the mode shape of deflection: L=4m, E=20000 MPa, I=0.0054 m^4, q_o=10 kN/m, a=2 m, μ=450kg/m , K=0.0, T=0.0

The relationship between deflection and the distance for the simply supported beam with no foundation are shown in Figure (2). The results are identical to that obtained by Timoshenko[12].

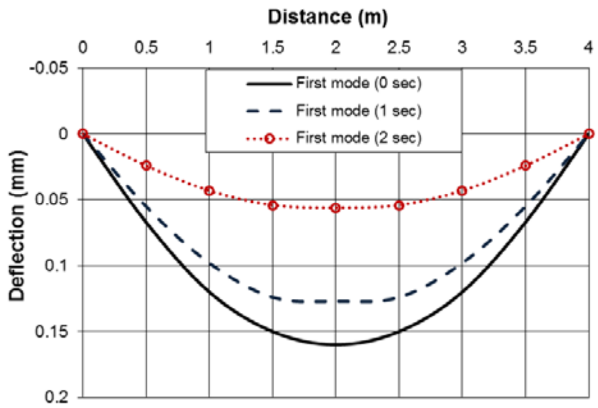

Figure 3: Deflection profile for the first mode.

4.2 Simply Supported Beam under Partial Uniform Load and on Two Parameter Elastic Foundations

The following properties of a simply supported beam under uniformly distributed load will be considered to draw the mode shape of deflection: L=4m, E=20000 MPa, I=0.0054 m^4, q_o=10 kN/m, m=450kg/m , K=10000 kN/m^3, T=600kN (assumed).

The relationship between deflection and the distance for beam on one parameter foundation is shown in Figure (4) for both present study Fourier and finite element methods. Good agreements are obtained between these methods. The effect of foundation on natural frequency is shown in Figure (5). The maximum effect is found to be 22 % for used values of the selected parameters.

The effect of beam depth on natural frequency of beam on two parameter foundation is shown in Figure (6). The effect is found to be significant for beam depth increases from 0.4m to 0.8 m the natural frequency is increased by 150% for the first mode.

The effect of modulus of subgrade reaction on natural frequency of beam on one parameter foundation is found to be significant for modulus increases from 0.0 to 100000 kN/m^3 (very dense sand) the natural frequency is increased by 85% for the first mode as shown in Figure (7).

The effect of membrane tension on natural frequency of beam on two parameter foundation is found to be insignificant for modulus increases from 0.0 to 6000 kN the natural frequency is increased by 5% for the first mode as shown in Figure(8).

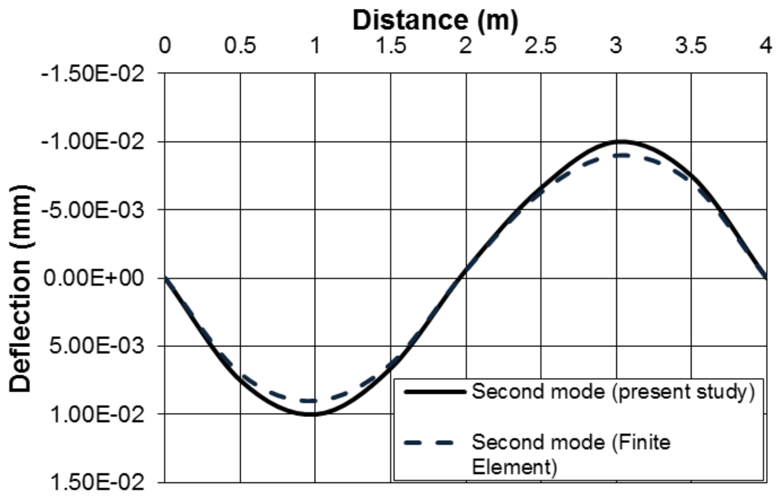

Figure 4: Deflection profile for beam on one parameter foundation

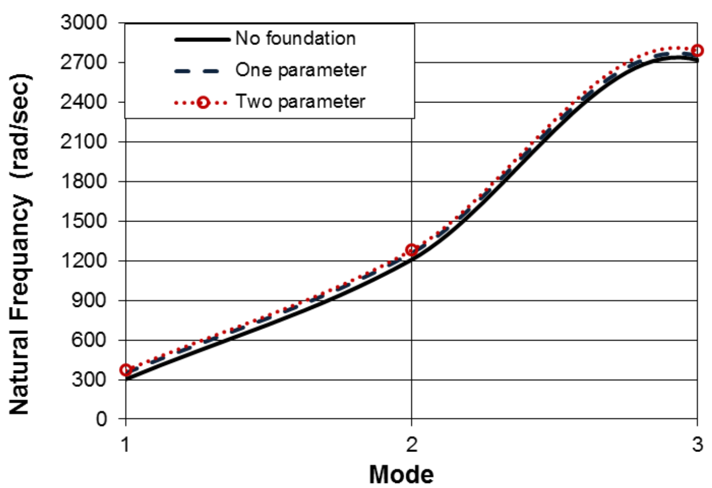

Figure 5: Effect of foundation on the natural frequency of beam

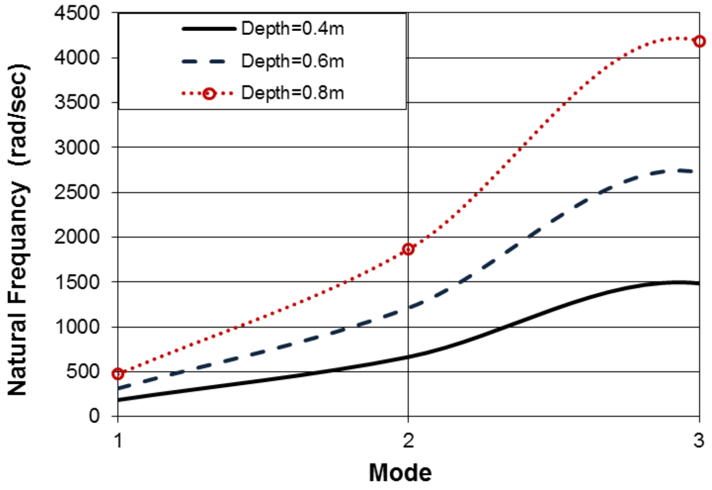

Figure 6: Effect of beam depth on the natural frequency of beam

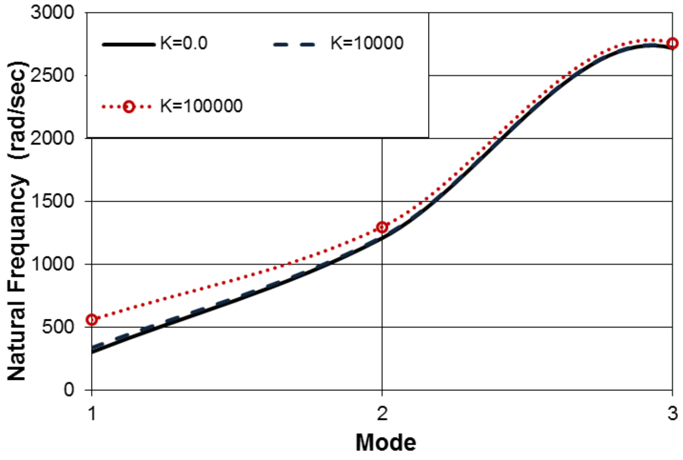

Figure 7: Effect of modulus of subgrade reaction on the natural frequency of beam

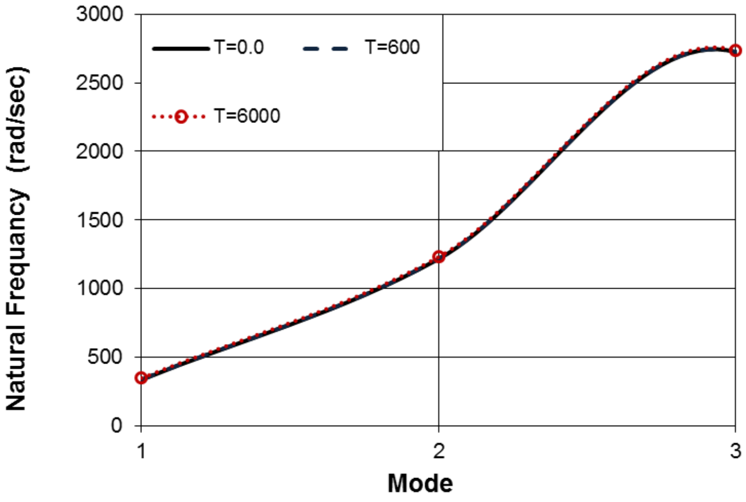

Figure 8: Effect of membrane tension on the natural frequency of beam (k=10000 kN/m^3)

5. Conclusions

The present study Fourier series method is suitable to be used for the solution of free vibration of beams. The natural frequency equation was derived to take into account the two parameters of foundations. The deflection modes were obtained for different loading and foundation parameters. The obtained results are in good agreement with the solution of Timoshenko and finite elements.

The effect of foundation on natural frequency of simply supported beam is found to be 22% for the used values of the selected parameters.

The effect of beam depth on natural frequency of beam on two parameter foundation is found to be significant for beam depth increases from 0.4m to 0.8 m the natural frequency is increased by 150% for the first mode.

The effect of modulus of subgrade reaction on natural frequency of beam on one parameter foundation is found to be significant for modulus increases from 0.0 to 100000 kN/m^3 the natural frequency is increased by 85% for the first mode.

The effect of membrane tension on natural frequency of beam on two parameter foundation is found to be insignificant for modulus increases from 0.0 to 6000 kN the natural frequency is increased by 5% for the first mode as shown in Fig.(8).

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الاهتزاز الحر للعتبات ذات الاسناد البسيط والمسندة على اسس مرنة

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الخلاصة

تم استخدام متوالية فورير وطريقة العناصر المحددة في تحليل التبات بسيطة الاسناد والمسيندة على اسس مرنة وباحمال متنوعة للوصول الى حالة الاهتزاز الحر. تم اشتقاق معادلة الاهتزاز الحر للعتبات المسندة على اسس مرنة وبمعاملين. تم الحصول على توافق جيد بين النتائج المستحصلة باستخدام الطريقتين المستخدمتين ومع نتائج باحثين اخريين. تم دراسة تأثير معامل ضغط التربة ومعامل الشد الغشائي وعمق العتبة على تصرف العتبات المسندة على اسس مرنة.