Study α $T_{1/2}$ space

Adnan Zarif , Asaad Shaker Hameed AL-Khafaji
Department of mathematics , Faculty of Science , Tishreen University, Lattakia , Syria.
(Received: 13 / 3 / 2012 ---- Accepted: 26 / 10 / 2011)

Abstract
In this paper we introduce a definition of a new space is called $\alpha - T_{1/2}$ by a new concept of generalized closed sets of the type α, we studied the relationship between this space and the spaces $\alpha - T_0$ and $\alpha - T_1$ and note that space $\alpha - T_{1/2}$ is located between $\alpha - T_0$ and $\alpha - T_1$.
Keywords: generalized closed set, $\alpha - open$ set, $\alpha - closed$ set , α - generalized closed set , $\alpha - T_{1/2}$, $\alpha - T_0$, $\alpha - T_1$.

Introduction
The concept of g-closed sets in topological spaces was introduced in 1970 by N. Levine [1] , a subset A of (X,τ) to be g- closed set cl(B) $\subseteq$ U, whenever B $\subseteq$ U and U is open set . After the work of N. Levine on g-closed sets, various mathematicians turned their attention to the generalizations of various concepts in topology .

Later, in 1994, H.Maki, R.Devi and K.Balachandran .[2] , generalized the concept of g-closed sets to α - generalized closed sets By definition a subset of A of (X,τ) is said to be α - generalized closed set

$if\ cl_\alpha(B) \subseteq U\ ;\ B \subseteq U\ and\ U\ is\ open\ set$.

By using this concept we study and discuss a new topological spaces which is called $\alpha - T_{1/2}$ space . A space (X,τ) is $\alpha - T_{1/2}$ space if every α - generalized closed subset of (X,τ) it is $\alpha - closed$ set .

And we have clarify the relation between $\alpha - T_{1/2}$ space and $\alpha - T_0$ space [3] , and $\alpha - T_1$ space [3] .

Objective of the research
By using the concept α -generalized closed set we study and discuss a new topological spaces which is called $\alpha - T_{1/2}$ space .

Research methods
The search is located in the area of mathematical analysis and it serves the public topology, depending on the concept of α -generalized closed sets to obtain new results related to $\alpha - T_{1/2}$ space .

Definitions and concepts
Definition 4.1
Let X be a topological space. A subset B of X is called :

1- $\alpha - open\ set$ [4] $if\ B \subseteq int(cl(int(B)))$

2- $\alpha - closed\ set$ [4] $if\ cl(int(cl(B))) \subseteq B$

3 - The intersection of all $\alpha - closed\ sets$ containing a subset B of X is called $\alpha - closure$ of B and denoted by $cl_\alpha(B)$.

4 – The α - interior of a subset B of X is the largest $\alpha - open$ set contained in B , and denoted by $int_\alpha(B)$.

We denote the family of α-open sets of (X,τ) by $O_\alpha(X)$, and denote the family of all $\alpha - closed$ of (X,τ) by $C_\alpha(X)$.

Definition 4.2
A space (X,τ) is said to be :

1- $\alpha - T_0$ [3] if, for $x,y \in X, x \neq y$, there exists $\alpha - open$ set containing ($x\ but\ not\ y$) or ($y\ but\ not\ x$) .

2- $\alpha - T_1$ [3] if, for $x,y \in X, x \neq y$, there exists U_1, U_2 are $\alpha - open$ sets such that :

($x \in U_1\ and\ y \notin U_1$) or

($y \in U_2\ and\ x \notin U_2$) .

Results and discussions
Proposition 5.1
Every closed subset of a topological space (X,τ) is g –closed . (the converse is not true)

Proof

Let $A \subseteq X$ be closed set, and let $A \subseteq U$, where U is open set, since A is closed set then $cl(A) = A$, hence $cl(A) \subseteq U$, i.e. A is g – closed .

Example 5.1
Let $X = \{a,b,c\}, \tau = \{X,\phi,\{a\},\{c\},\{a,c\}\}$, so , $\tau^c = \{\phi, X,\{b,c\},\{a,b\},\{b\}\}$,

let $A = \{a\}$, $U = \{a,b,c\}$ open set ,

Now, since $cl(A) = \{a,b\} \subseteq U$, i.e. $A = \{a\}$ is g - closed set , but it is not closed set .

Proposition 5.2
Every g - closed subset of a topological space (X,τ) is αg –closed . (the converse is not true) .

Proof

Let $A \subseteq X$ be g -closed set, and let $A \subseteq U$, where U is open set, since A is g - closed set , then , $cl(A) \subseteq U$, and hence $int(cl(A)) \subseteq int(U)$, but U is open set so , $int(cl(A)) \subseteq U$.

Since $cl_\alpha(A)$ is the smallest α - closed set containing A, so, $cl_\alpha(A) = A \cup cl(int(cl(A))) \subseteq A \cup cl(U) \subseteq U$, i.e. A is αg –closed .

Example 5.2
Let $X = \{a,b,c\}, \tau = \{X,\phi,\{a\},\{c\},\{a,c\}\}$, so , $\tau^c = \{\phi, X,\{b,c\},\{a,b\},\{b\}\}$,

let $A = \{c\}$, $U = X$ open set .

Now, since $cl(A) = \{b,c\} \subseteq U$, and $int(cl(A)) = \{c\}$, $cl(int(cl(A))) = \{b,c\} \subseteq U$ i.e .

$A = \{c\}$ is αg -closed set, but it is not g -closed set, Since if we take $U = \{a, c\}$, $cl(A) = \{b, c\} \not\subset U$.

Proposition 5.3

Every closed subset of a topological space (X,τ) is α -closed . (the converse is not true) .

Proof

Let $A \subseteq X$ be closed set, then $cl(A) = A$, hence $\text{int}(cl(A)) = \text{int}(A)$, but $\text{int}(A) \subseteq A$, so $\text{int}(cl(A)) \subseteq A$, and $cl(\text{int}(cl(A))) \subseteq cl(A)$. Then $cl(\text{int}(cl(A))) \subseteq A$, i.e. A is α -closed .

Example 5.3

Let $X = \{a,b,c,d\}, \tau = \{X,\phi,\{a\},\{c\},\{a,c\},\{a,b,d\}\}$,

So, $\tau^c = \{\phi,X,\{b,c,d\},\{a,b,d\},\{b,d\},\{c\}\}$,Let $A = \{b,c\}$.

Now, since, $cl(A) = \{b,c,d\} \subseteq U$, and $\text{int}(cl(A)) = \{c\}$, $cl(\text{int}(cl(A))) = \{c\} \subseteq A$, i.e. $A = \{b,c\}$ is α -closed set , but it is not closed set .

Theorem 5.1

(X,τ) , the following are equivalent : For a space

1 - (X,τ) is $\alpha - T_{1/2}$.

2 - For each singleton $\{x\}$ of X , $\{x\}$ is $\alpha - open$ set or $\alpha - closed$ set .

Theorem 5.2

(X,τ) , the following are equivalent : For a space

1 - (X,τ) is $\alpha - T_{1/2}$.

2- Every subset of X is the intersection of all $\alpha - open$ sets and all $\alpha - closed$ sets containing it .

Proof

$(1) \Rightarrow (2)$, if (X,τ) is $\alpha - T_{1/2}$ with $B \subseteq X$,therefore by Theorem 5.1 then for each singleton $\{x\}$ of X , $\{x\}$ is $\alpha - open$ set or $\alpha - closed$ set .

$\Rightarrow B = \bigcap\{X \setminus \{x\}; x \notin B\}$ is the intersection of all $\alpha - open$ sets and all $\alpha - closed$ sets containing it .

$(2) \Rightarrow (1)$, for each $x \in X$, *then* $X \setminus \{x\}$ is the intersection of all $\alpha - open$ sets and all $\alpha - closed$ sets containing it , hence $X \setminus \{x\}$ is either $\alpha - open$ set or $\alpha - closed$ set .

Therefore by Theorem 5.1 , (X,τ) is $\alpha - T_{1/2}$.

Lemma 5.1

(X,τ) , the following are equivalent : For a space

1 - Every subset of X is α - generalized closed set .

2 - $O_\alpha(X) = C_\alpha(X)$.

Proof

$1) \Rightarrow (2)$, Let is $U \in O_\alpha(X)$, Then by hypothesis , U is α -generalized closed set) Which implies that $cl_\alpha(U) \subset U$, so, $cl_\alpha(U) = U$, therefore $U \in C_\alpha(X)$

$\Rightarrow O_\alpha(X) \subseteq C_\alpha(X)$ .... (1)

Let $V \in C_\alpha(X) \Rightarrow X \setminus V \in O_\alpha(X)$, Then by hypothesis $X \setminus V$ is α -generalized closed set , and then $X \setminus V \in C_\alpha(X)$

$\Rightarrow V \in O_\alpha(X) \Rightarrow C_\alpha(X) \subseteq O_\alpha(X)$ ..... (2)

From (1) and (2) $\Rightarrow O_\alpha(X) = C_\alpha(X)$.

$(2) \Rightarrow (1)$, If B is subset of X such that $B \subseteq U$ where $U \in O_\alpha(X)$

Then $U \in C_\alpha(X) \Rightarrow cl_\alpha(U) = U$

Now , $B \subseteq U \Rightarrow cl_\alpha(B) \subseteq cl_\alpha(U) = U \Rightarrow cl_\alpha(B) \subseteq U$

$\Rightarrow$ B is α -generalized closed set .

Proposition 5.4

The property of being a $\alpha - T_{1/2}$ space is hereditary.

Proof

If Y is a subspace of a $\alpha - T_{1/2}$ space X , and $y \in Y \subseteq X$, then $\{y\}$ is $\alpha - open$ set or $\alpha - closed$ set in X (by Theorem 5.1) . Therefore $\{y\}$ is either $\alpha - open$ set or $\alpha - closed$ set in Y . Hence Y is a $\alpha - T_{1/2}$ space .

Theorem 5.3

A space X is -T$_1$ space if and only if { x } is -closed , x X .

Proof

Let X be -T$_1$ space .

Let p X , to prove {p} is -closed set .

x p}^{c} X \{p} x p in X ,

Hence there exists an -open set G such that x G, p G or x G, p G .

If x G, pG x G {p}^{c} {p}^{c} is an-open set{p} is -closed set .

Let {p} be an -closed set, p X , to prove X is -T$_1$ space .

Let x y in X ,

Hence { x }, { y } are -closed sets { x }^{c}, { y }^{c} are -open sets and y {x}^{c}, x {x}^{c}, x {y}^{c}, y { y }^{c}

Therefore X is -T$_1$ space.

Theorem 5.4

Every $\alpha - T_1$ space is $\alpha - T_{1/2}$ space. (the converse is not true) .

Proof

Since X is $\alpha - T_1$ by using theorem 5.3 , then { x } is -closed , x X . And by using theorem 5.1 we will get X is $\alpha - T_{1/2}$ space .

Example 5.4

Let $X = \{a,b,c\}$, $\tau = \{\varnothing, X, \{a\}\}$

$\tau_\alpha(X) = \{\varnothing, X, \{a\}, \{a,b\}, \{a,c\}\}$,

$\tau_\alpha^c(X) = \{\varnothing, X, \{b,c\}, \{c\}, \{b\}\}$

(X,τ) *is* $\alpha - T_{1/2}$ space But not $\alpha - T_1$ space .

Theorem 5.5

Every $\alpha - T_{1/2}$ *space is* $\alpha - T_0$ *space* . (the converse is not true) .

Proof

Let $x, y \in X ; x \neq y$

Since X is $\alpha - T_{1/2}$ space by using theorem 5.1, Therefore $\{x\}$ is either $\alpha - open$ set or $\alpha - closed$ set , x X . (1) If $\{x\}$ is $\alpha - open$ set , x X .

Since $x \neq y$, *Therefore* $x \in \{x\}$ *and* $y \notin \{x\}$

$\Rightarrow X$ *is* $\alpha - T_0$ *space* .

(2) If $\{x\}$ *is* α *– closed set*, $\forall x \in X$,

Then $X \setminus \{x\}$ *is* α *– open set* $\Rightarrow X$ *is* $\alpha - T_0$ *space*

Therefore $x \notin X \setminus \{x\}$ *and* $y \in X \setminus \{x\}$

Example 5.5

Let $X = \{1,2,3\}$, $\tau(X) = \{\varnothing, X, \{1\}, \{1,2\}\}$

$\tau_\alpha(X) = \tau(X)$ $\tau_\alpha^c(X) = \{\varnothing, X, \{2,3\}, \{3\}\}$

(X, τ) *is* $\alpha - T_0$, *But is not* $\alpha - T_{1/2}$,*because* ,

$\{2\}$ *is not* α *– open and is not* α *– closed set*

Conclusions and recommendations

1 - development of new concepts in general topology.

2 - The emergence of new spaces Topologic different from the known spaces .

3- *To find the relationsh ip between* $\alpha - T_{1/2}$ *space and* $\alpha - T_0$ *and* $\alpha - T_1$ *space* .

4- The property of being a $\alpha - T_{1/2}$ space is hereditary. The study of properties of space $\alpha - T_{1/2}$ and the possibility of find spaces other than the space $\alpha - T_{1/2}$ is located between $\alpha - T_0$ and $\alpha - T_1$ remain an issue and need to be resolved in the future .

References

[1] N. Levine, (1970), Generalized closed sets in topological spaces, Rend. Circ. Math. Palermo,19 , 89-96 .

[2] H. Maki, R. Devi and K. Balachandran (1994), Associated topologies of generalized *α – closed sets and α – generalized closed sets* , Mem. Fac. Sci. Kochi Univ. Ser. A Math.15, 51-63 .

[3] H.Maki,R. Devi and K. Balachandran, (1993) , Generalazied *α – closed sets* in topology, Bull. Fukuoka Univ. Ed. Part III 42, 13-21 .

[4] O . Njastad , (1965), on some classes of nearly open sets, pacific J. Math. 15 961-970.

دراسة الفضاء $T_{1/2}$ α

عدنان ظريف ، أسعد شاكر حميد الخفاجي

قسم الرياضيات ، كلية العلوم ، جامعة تشرين ، اللاذقية ، سورية

(تاريخ الاستلام: 13 / 3 / 2012 ---- تاريخ القبول: 26 / 10 / 2011)

الملخص

في هذا البحث قدمناتعريفاً لفضاء جديد يسمى $\alpha - T_{1/2}$ حيث عرفناه بواسطة مفهوم جديد من المجموعات المغلقة المعممة من النوع α ودرسنا العلاقة بين هذا الفضاء والفضائيين $\alpha - T_0$ و $\alpha - T_1$ علماً أن الفضاء يقع بين الفضائيين $\alpha - T_0$ و $\alpha - T_1$.

الكلمات المفتاحية : المجموعة المغلقة المعممة , المجموعة المفتوحة من النوع α , المجموعة المغلقة من النوع α , الفضاء $\alpha - T_{1/2}$ الفضاء , $\alpha - T_0$ الفضاء. $\alpha - T_1$.