On Composition of Some New Kinds of Homeomorphisms in Intiuitionstic Topological Space

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Abstract
In this paper we define the new kinds of homeomorphisms in intiuitionstic topological space say (almost homeomorphism , almost k- homeomorphism , almost S-k- homeomorphism , almost S*-k- homeomorphism , almost contra homeomorphism , almost contra k- homeomorphism , almost contra S-k- homeomorphism , almost contra S*-k- homeomorphism , regular- closed homeomorphism, completely homeomorphism , perfectly homeomorphism and slightly homeomorphism) where k= {semi, α , pre , β}, and we study the composition of this homeomorphisms with (homeomorphism, k- homeomorphism, S-k- homeomorphism S*-K-homeomorphism, contra homeomorphism ,contra k- homeomorphism , contra S-k- homeomorphism , contra S*-k- homeomorphism) in intiuitionstic topological space .

Introduction
The notion of contra continuous was introduced by (Dontchev, 1996) in [3], and the notion of almost contra continuous was introduced by (Joseph,J.E. and Kwack ,1980) in [7] ,these notions are generalized to Intiuitionstic Topological spaces by (Ali.M.Jasem and Yunis J.Yaseen 2009) in [1] . The definition of regular-closed continuous , completely continuous , slightly continuous and perfectiy continuous functions in general topology was introduced by (Al-hawez ,Z .T.2008) in [2], (Kilicman, A. and Salleh, Z. 2006) in [8] and (Dontchev,J.and Noiri,T 1998) in [4] . generalized these to intuitionistic topological space by (Ali. M. Jasem and Yunis J.Yaseen 2009) in [1] .
Also the notions of Intiuitionstic homeomorphism (Intiuitionstic k- homeomorphism , Intiuitionstic S-k-homeomorphism and Intiuitionstic S*-k-homeomorphism) functions was introduced by (Hanna H.Alwan and Yunis J. Yaseen 2007) in [5] and the notions of Intiuitionstic contra homeomorphism (Intiuitionstic contra k- homeomorphism, Intiuitionstic contra S-k- homeomorphism and Intiuitionstic contra S*-k- homeomorphism) functions was introduced by (Yunns J. Yaseen , Suham M. Ali and Amall A.F 2011) in [9] .In this paper, we define the regular- closed open, completely open, slightly open , perfectly open , almost open, almost k-open , almost contra open and almost contra k-open functions in intuitionistic topological space. And we define the regular -closed homeomorphism, completely homeomorphism, slightly homeomorphism, perfectly homeomorphism , almost homeomorphism, almost k- homeomorphism , almost contra homeomorphism and almost contra k-homeomorphism functions in intuitionistic topological space . Also in this papre we study the composition of homeomorphisms in Intiuitionstic Topological spaces ; say the composition of (contra homeomorphism and contra homeomorphism), (contra homeomorphism and homeomorphism , (S-k-homeomorphism and S*-k- homeomorphism) , (contra S-k-homeomorphism and contra S*- k-homeomorphism) ,(S-k-homeomorphism and contra S*-k-homeomorphism), (contra S-k-homeomorphism and S*-k-homeomorphism), (S-k-homeomorphism and homeomorphism) , (homeomorphism and S*-k-homeomorphism) , (almost homeomorphism and regular- closed homeomorphism) , (almost homeomorphism and completely homeomorphism), (completely homeomorphism and regular-closed homeomorphism), (slightly homeomorphism and perfectly homeomorphism) . where k= {semi, α , pre, β} .

Preliminaries

Let X be a non-empty set, an intuitionistic set A (IS, for short) is an object having the form $A=\langle x, A_1, A_2\rangle$ where A_1 and A_2 are disjoint subset of X. the set A_1 is called a member of A, while A_2 is called non- member of A [1] . An intuitionistic topology (IT, for short) on a non-empty set X, is a family T of IS in X containing $\widetilde{\emptyset}, \widetilde{X}$ and closed under arbitrary unions and finitely intersections. In this case the pair (X,T) is called intuitionistic topological space (ITS, for short), any IS in T is known as an intuitionistic open set (IOS, for short) in X ,and the complement of IOS is intuitionistic closed set (ICS, for short) [5] .
The definition of intuitionistic semi open set, intuitionistic α open set, intuitionistic pre open set and intuitionistic regular open set (IOS,ISOS, IαOS, IPOS,I βOS, and IROS for short) also the definition of intuitionistic semi closed set, intuitionistic α closed set , intuitionistic pre closed set, intuitionistic β closed set and intuitionistic regular closed set (ICS, ISCS, IαCS, IPCS, IβCS and IRCS for short) are in [5] .
Every IROS(IRCS) is IOS(ICS) and every IOS(ICS) is ISOS, IαOS, IPOS and IβOS (ISCS, IαCS, IPCS and I βCS for short) [5] .

Let(X,T) , (Y,Ψ) and (Z,σ) are ITS's and let $f: X \to Y$ and $g: Y \to Z$ then
$gof: X \to Z$ is defined by $gof(x)=g(f(A))$,where A is IS in X and $(gof)^{-1}(B)=f^{1}(g^{-1}(B))$ where B is IS in X .[6]

Definition 2.1

[1] Let (X,T) and (Y,σ) be two ITS's and let $f: X \to Y$ be a function and let k = {semi, α , per , β } , then f is said to be :

1. An intuitionistic continuous (I cont. , for short) function if the inverse image of each IOS in Y is IOS in X .
2. An intuitionistic k-continuous (I k-cont. ,for short) function if the inverse image of each IOS in Y is IKOS in X .
3. An intuitionistic contra continuous (I contra cont., for short) function if the inverse image of each IOS in Y is ICS in X .
4. An intuitionistic contra k-continuous (I contra k-cont. ,for short) function if the inverse image of each IOS in Y is IKCS in X .
5. An intuitionistic almast continuous (I almost cont. ,for short) function if the inverse image of each IROS in Y is IOS in X .
6. An intuitionistic almast k-continuous (I almost K-cont. ,for short) function if the inverse image of each IROS in Y is IKOS in X .
7. An intuitionistic almast contra continuous (I almost contra cont., for short) function if the inverse image of each IROS in Y is ICS in X .
8. An intuitionistic almast contra k-continuous (I almost contra K- cont. ,for short) function if the inverse image of each IROS in Y is IKCS in X .
9. An intuitionistic regular -closed continuous (I regular- closed cont., for short) function if the inverse image of each IOS in Y is IRCS in X .
10. An intuitionistic completely continuous (I completely cont., for short) function if the inverse image of each IOS in Y is IROS in X .
11. Anintuitionistic slightly continuous (I slightly cont., for short) function if the inverse image of each I clopen set (IOS and ICS) in Y is ICS in X .
12. Anintuitionistic perfectly continuous (I perfectly cont., for short) function if the inverse image of each IOS in Y is I clopen set (IOS and ICS) in X .

Definition 2.2

Let (X,T) and (Y,σ) be two ITS's and let f: X $\rightarrow$ Y be a function and let k = {semi, α , per , β , then f is said to be:

1. An intuitionistic open (I open , for short) function if the image of each IOS in X is IOS in Y .
2. An intuitionistic k- open (I k-open , for short) function if the image of each IOS in X is IKOS in Y.
3. An intuitionistic contra open (I contra open , for short) function if the image of each IOS in X is ICS in Y.
4. An intuitionistic contra K-open (I contra k-open , for short) function if the image of each IOS in X is IKCS in Y.
5. an intuitionistic closed (I contra closed , for short) function if the image of each ICS in X is ICS in Y.
6. An intuitionistic k-closed (I contra k-closed , for short) function if the image of each ICS in X is IKCS in Y.
7. an intuitionistic contra closed (I contra closed , for short) function if the image of each ICS in X is IOS in Y.
8. An intuitionistic contra k-closed (I contra k-closed , for short) function if the image of each ICS in X is IKOS in Y.

Definition 2.3

Let (X,T) and (Y,σ) be two ITS's and let f: X $\rightarrow$ Y be a bijective function and let k = {semi, α , per , β} , then f is said to be:

1. An intuitionistic homeomorphism (I hom. , for short) function if f is I continuous and I open function .
2. An intuitionistic k- homeomorphism (I k- hom. , for short) function if f is I k- continuous and I k-open function .
3. An intuitionistic strong k-homeomorphism (I S-k-hom., for short) function if f is I k-continuous and I open function .
4. An intuitionistic S*- k-homeomorphism (I S*-k-hom. , for short) function if f is I continuous and I k-open function .

Definition 2.4

Let (X,T) and (Y,σ) be two ITS's and let f: X $\rightarrow$ Y be a bijctive function and let k = {semi, α , per , β} , then f is said to be:

1. An intuitionistic contra homeomorphism (I contra hom. , for short) function if f is I contra continuous and I contra open function .
2. An intuitionistic contra k- homeomorphism (I contra k- hom., for short) function if f is I contra k-continuous and I contra k- open function .
3. An intuitionistic contra strong k-homeomorphism (I contra S-k-hom., for short) function if f is I contra k-continuous and I contra open function .
4. An intuitionistic contra S*- k-homeomorphism (I contra S*-k-hom. , for short) function if f is I contra continuous and I contra k-open function .

Remark 2.5

since every IROS is IOS , every IOS is IKOS where k={semi ,α , pre, β}and every clopen is closed and open set ; then :

1.Every I cont. function is I almost cont. function .
2.Every I k-cont. function is I almost k- cont. function .
3.Every I contra cont. function is I almost contra cont. function .
4. Every I contra k-cont. function is I almost contra k-cont. function .
5.Every I regular-closed cont. function is I contra cont. function .
6.Every I regular- closed cont. function is I contra K-cont. function .
7.Every I contra cont. function is I almost contra cont. function .
8. Every I contra cont. function is I almost contra cont. function .
9. Every I contra cont. function is I almost contra cont. function .

10. Every I contra cont. function is I slightly cont. function .
11. Every I perfectly cont. function is I contra cont. function .
12. Every I perfectly cont. function is I almost contra cont. function .

The composition of some new kinds of homeomorphisms

Before we study the composition of some kinds of homeomorphisms, we define some new kinds of homeomorphisms :

Definition 3.1

Let (X,T) and (Y,σ) be two ITS's and et f: X $\rightarrow$ Y be a function and let k = {semi, α , per , β} , then f is said to be:

1. An intuitionistic almost open (I almost open, for short) function if the image of each IROS in X is IOS in Y.
2. An intuitionistic almost k- open (I almost k-open, for short) function if the image of each IROS in X is IKOS in Y.
3. An intuitionistic almost contra open (I almost contra open , for short) function if the image of each IROS in X is ICS in Y.
4. An intuitionistic almost contra k-open(I almost contra K- open , for short) function if the image of each IROS in X is IKCS in Y.
5. An intuitionistic regular-closed open (I regular closed open., for short) function if the image of each IOS in X is IRCS in Y.
6. An intuitionistic completely open (I completely open., for short) function if the image of each IOS in X is IROS in Y.
7. An intuitionistic slightly open (I slightly open., for short) function if the image of each I clopen set (IOS and ICS) in X is ICS in Y.
8. An intuitionistic perfectly open(I perfectly open., for short)function if the inverse image of each IOS in X is I clopen set (IOS and ICS) in Y.
9. an intuitionistic almost closed(I almost contra closed , for short) function if the image of each IRCS in X is ICS in Y.
10. An intuitionistic almost K-closed (I contra k-closed.,for short) function if the image of each IRCS in X is IKCS in Y.
11. An intuitionistic almost contra closed(I almost contra closed , for short) function if the image of each IRCS in X is IOS in Y.
12. An intuitionistic almost contra K-closed (I contra k-closed.,for short) function if the image of each IRCS in X is IKOS in Y.
13. An intuitionistic completely closed (I completely closed , for short) function if the image of each ICS in X is IRCS in Y.
14. An intuitionistic slightly closed (I slightly closed, for short) function if the image of each I clopen set (IOS and ICS) in X is IOS in Y.
15. An intuitionistic perfectly closed (I perfectly open., for short) function if the inverse image of each ICS in X is I clopen set (IOS and ICS) in Y.

Remark 3.2

If f is bijective function then : the concept of I almost open (resp. almost k- open , almost contra open , almost contra K-open ,slightly open ,completely open and
perfectly open)function and the concept of I almost closed (resp. almost k- closed , almost contra closed, almost contra k- closed , slightly closed , completely closed and perfectly closed)function is equivalent.

Definition 3.3

Let (X,T) and (Y,σ) be two ITS's and let f: X $\rightarrow$ Y be a bijective function then f is said to be:

1. An intuitionistic almost homeomorphism (I almost hom. , for short) function if f is I almost continuous and I almost open function .
2. An intuitionistic almost k- homeomorphism (I almost k- hom. , for short) function if f is I almost k- continuous and I almost k- open function .
3. An intuitionistic almost strong k-homeomorphism (I almost S-k-hom. , for short) function if f is I almost k-continuous and I almost open function .
4. An intuitionistic almost S*- k-homeomorphism (I almost S*-k-hom. , for short) function if f is I almost continuous and I almost k-open function .
5. An intuitionistic almost contra homeomorphism (I almost contra hom. , for short) function if f is I almost contra continuous and I almost contra open function .
6. An intuitionistic almost contra k- homeomorphism (I almost contra k- hom. , for short) function if f is I almost contra k- continuous and I almost contra k- open function .
7. An intuitionistic almost contra strong k-homeomorphism (I almost contra S-k-hom. , for short) function if f is I almost contra k-continuous and I almost contra open function .
8. An intuitionistic almost contra S*- k-homeomorphism (I almost contra S*-k-hom. , for short) function if f is I almost contra continuous and I almost contra k-open function .
9. An intuitionistic regular -closed homeomorphism (I regular closed hom. , for short) function if f is I regular closed continuous and I regular closed open function.
10. An intuitionistic completely homeomorphism (I completely hom. , for short) function if f is I completely continuous and I completely open function.
11. An intuitionistic slightly homeomorphism (I slightly hom. , for short) function if f is I slightly continuous and I slightly open function
12. An intuitionistic perfectly homeomorphism (I perfectly hom. , for short) function if f is I perfectly continuous and I perfectly open function .

Proposition 3.4

1. Every I open function is I almost open function.
2. Every I k- open function is I almost k- open function.

3. Every I contra open function is I almost contra open function.
4. Every I contra k- open function is I almost contra k- open function.
5. Every I regular- closed open function is I contra open function.
6. Every I regular- closed open function is I contra K- open function.
7. Every I contra open function is I slightly open function.
8. Every I perfectly open function is I contra open function.
9. Every I perfectly open function is I almost contra open function.

Proof

Let f :X → Y is I open function and let A is I ROS in X then A is IOS in X then f(A) is IOS in Y [since f is I open function] then f is I almost open function .

1. Let f :X → Y is I k-open function and let A is I ROS in X then A is IOS in X then f(A) is I kOS in Y [since f is I k-open function] then f is I almost k-open function .
2. Let f :X → Y is I contra open function and let A is I ROS in X then A is IOS in X then f(A) is ICS in Y[since f is I contra open function] then f is I almost contra open function .
3. Let f :X → Y is I contra k-open function and let A is I ROS in X then A is IOS in X then f(A) is IKCS in Y [since f is I contra k-open function] then f is I almost contra k- open function .
4. Let f :X → Y is I regular- closed open function and let A is IOS in X then f(A) is IRCS in Y [since f is I regular closed open function],but IRCS is ICS, then f(A) is ICS in Y. f is I contra open function .
5. Let f:X→Y is I regular- closed open function and let A is IOS then f(A) is IRCS in Y [since f is I open function] but IRCS is ICS and ICS is IKCS then f(A) is IKCS in Y then f is I almost contra k- open function.
6. Let f :X → Y is I contra open function and let A is clopen set in X then A is IOS in X then f(A) is ICS in Y[since f is I contra open function] then f is I slightly open function .
7. Let f :X → Y is I perfectly open function and let A is IOS in X then f(A) is clopen set in Y[since f is I perfectly open function] but clopen is ICS then f is I contra open function .
8. By 8 and 3 in this proposition .

Remark 3.5

By (Remark 2.5) and (proposition 3.4) we get

1. Every I homeomorphism [resp. I k- hom., I contra hom. and I contra k- hom.] function is I almost homeomorphism[resp. I almost k-hom. ,I almost contra hom. and I almost contra k- hom.] function.
2. Every I regular-closed hom. function is I contra hom. function.
3. Every I regular-closed hom. function is I contra k- hom. function.
4. Every I contra hom. function is I slightly hom. function.
5. Every I perfectly hom. function is I contra hom. Function.
6. Every I perfectly hom. function is I almost contra hom. function.

Next , results of composition of some kinds of homeomorphism functions :

Proposition 3.6

Let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y and g :Y → Z two functions then :

1. If f and g are I hom. functions then gof is I hom. function .
2. If f and g are I contra hom. functions then gof is I hom. function .
3. If f is I contra hom. function and g is I hom. function then gof is I contra hom. function .
4. If f is I hom. function and g is I contra hom. function then gof is I contra hom. function .

Proof

1. Let A is IOS in X then f (A) is IOS in Y (since f is I open function) then g(f(A)) is IOS in Z (since g is I open function) then gof is I open function .Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y(since g is I cont. function) then $f^{1}(g^{-1}(B))$ is IOS in X (since f is I cont.s function) then gof is I cont. function. Hence gof is I hom. function.
2. Let A is IOS in X then f (A) is ICS in Y (since f is I contra open function) then g(f(A)) is IOS in Z (since g is I contra open function) then gof is I open function . Now let B is IOS in Z then $g^{-1}(B)$ is ICS in Y(since g is I contra cont. function) then $f^{1}(g^{-1}(B))$ is IOS in X (since f is I contra cont. function) then gof is I cont. function. Hence gof is I hom. function .
3. Let A is IOS in X then f (A) is ICS in Y (since f is I contra open function) then g(f(A)) is ICS in Z (since g is I open function) then gof is I contra open function . Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y (since g is I cont. function) then $f^{1}(g^{-1}(B))$ is ICS in X (since f is I contra cont. function) then gof is I contra cont. function. Hence gof is I contra hom. function .
4. Let A is IOS in X then f (A) is IOS in Y (since f is I open function) then g(f(A)) is ICS in Z (since g is I contra function) then gof is I contra open function . Now let B is IOS in Z then $g^{-1}(B)$ is ICS in Y (since g is I contra cont. function) then $f^{1}(g^{-1}(B))$ is ICS in X (since f is I cont. function) then gof is I contra cont. function . Hence gof is I contra hom. function .

Proposition 3.7

let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y and g :Y → Z two functions then :

1. If f is I S-K- hom. function and g is I S*-K-hom. function then gof is I k-hom. Function .
2. If f is I contra S-K- hom. function and g is I contra S*-K-hom. function then gof is I k-hom. Function .
3. If f is I contra S-K- hom. function and g is I S*-K-hom. function then gof is I contra k-hom. function.
4. If f is I S-K- hom. function and g is I contra S*-K-hom. function then gof is I contra k-hom. function.
5. If f is I hom. function and g is I S*-K-hom. function then gof is I S*-k-hom. Function .

6. If f is I S-K- hom. function and g is I hom. function then gof is I S- k-hom. Function .

Proof

Let A is IOS in X then f(A) is IOS in Y (since f is I open function) then g(f(A)) is IKOS in Z (since g is I k-open function) hence gof is I k- open function . Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y(since g is I cont. function) then $f^{1}(g^{-1}(B))$ is IKOS in X (since f is I k-cont. function) hence gof is I k- cont. function . Then gof is I k- hom. Function .

1. Let A is IOS in X then f(A) is ICS in Y [since f is I contra open function] then g(f(A)) is I KOS in Z [since g is I contra k-open function] hence gof is I k- open function . Now let B is IOS in Z then $g^{-1}(B)$ is ICS in Y [since g is I contra cont. function] then $f^{1}(g^{-1}(B))$ is I KOS in X [since f is I contra k-cont. function] hence gof is I k-cont. function. Then gof is I k- hom. function.
2. Let A is IOS in X then f(A) is ICS in Y (since f is I contra open function) then g(f(A)) is I KCS in Z (since g is I k-open function then it is I k-closed function) hence gof is I contra k- open function . Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y (since g is I cont. function) then $f^{1}(g^{-1}(B))$ is I KCS in X (since f is I contra k-cont. function) hence gof is I contra k-cont. function. Then gof is I contra k- hom. function.
3. Let A is IOS in X then f(A) is IOS in Y (since f is I open function) then g(f(A)) is IKCS in Z (since G is I contra k-open function) hence gof is I contra k-open function . Now let B is IOS in Z then $g^{-1}(B)$ is ICS in Y[since g is I contra cont. function] then $f^{1}(g^{-1}(B))$ is IKCS in X (since f is I k-cont. function) hence gof is I contra k-cont. function .Then gof is I contra k- hom. Function .
4. Let A is IOS in X then f(A) is IOS in Y (since f is I open function) then g(f(A)) is IKOS in Z (since g is I k-open function) hence gof is I k- open function . Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y (since g is I cont. function) then $f^{1}(g^{-1}(B))$ is IOS in X (since f is I cont. function) hence gof is I cont. function Then gof is I S*-k- hom. Function .
5. Let A is IOS in X then f(A) is IOS in Y (since f is I open function) then g(f(A)) is IOS in Z [since g is I open function],hence gof is I open function . Now let B is IOS in Z then $g^{-1}(B)$ is IOS in Y (since g is I cont. function) then $f^{1}(g^{-1}(B))$ is IKOS in X (since f is I k-cont. function) hence gof is I k- cont. function . Then gof is I S-k- hom. Function .

Proposition 3.8

let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y and g :Y → Z two functions then :

1. If f and g are I contra hom. functions then gof is I almost hom. function .
2. If f and g are I hom. functions then gof is I almost hom. function .
3. If f is I contra hom. function and g is I hom. function then gof is I almost contra hom. function.
4. If f is I hom. function and g is I contra hom. function then gof is I almost contra hom. function
5. If f is I S-K- hom. function and g is I S*-K-hom. function then gof is I almost k-hom. function.
6. If f is I contra S-K- hom. function and g is I contra S*-K-hom. function then gof is I almost k-hom. function .
7. If f is I contra S-K- hom. function and g is I S*-K-hom.function then gof is I almost contra k-hom. function.
8. If f is I S-K- hom. function and g is I contra S*-K-hom. function then gof is I almost contra k-hom. function.

Proof

by (proposition 3.6 and proposition 3.7) and (Remark 3.5) we can prove this proposition .

Proposition 3. 9

let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y and g :Y → Z two functions then :

1. If f is I almost hom. function and g is I regular-closed hom. function then gof is I almost contra hom. function .
2. If f is I regular -closed hom. function and g is I almost hom. function then gof is I almost contra hom. function .
3. f is I completely hom. function and g is I regular closed hom. function then gof is I regular- closed hom. function .
4. f is I regular- closed hom. function and g is I completely hom. function then gof is I regular- closed hom. function .
5. f is I almost hom. function and g is I completely hom. function then gof is I almost hom. function .
6. f is I completely hom. function and g is I almost hom. function then gof is I hom. function .

Proof

1. Let A is IROS in X then f (A) is IOS in Y (since f is I almost open function) then g(f(A)) is IRCS in Z (since g is I regular-closed open function) then g(f(A)) is ICS in Z(since every IRCS is ICS) then gof is I almost contra open function . Now let B is IOS in Z then $g^{-1}(B)$ is IRCS in Y(since g is I regular- closed cont. function) then $f^{1}(g^{-1}(B))$ is ICS in X (since f is I almost cont. function) then gof is I contra cont. function then it is I almost contra cont. function . Hence gof is I almost contra hom. function .

2. Let A is IROS in X , but every IROS is IOS, then A is IOS in X ,then f(A) is IRCS in Y (since f is I regular-closed open function) then g(f(A)) is ICS in Z (since g is I almost open function) then gof is I almost contra open function . Now let B is IROS in Z then $g^{-1}(B)$ is IOS in Y(since g is I almost contra cont. function) then $f^{1}(g^{-1}(B))$ is IRCS in X (since f is I regular -closed cont. function), but every IRCS is ICS ,then $f^{1}(g^{-1}(B))$ is ICS in X, then gof is I almost contra cont. function . Hence gof is I almost contra hom. function .

3. Let A is IOS in X then f (A) is IROS in Y (since f is I completely open function),but every IROS is IOS then g(f(A)) is IRCS in Z (since g is I regular -closed open function) then gof is I regular closed open function .Now let B is IOS in Z then $g^{-1}(B)$ is IRCS in Y (since g is I regular- closed cont. function) every I RCS is ICS then $f^{1}(g^{-1}(B))$ is IRCS in X (since f is I completely cont. function) then gof is I regular-closed cont. function. Hence gof is I regular- closed hom. function .

4. Let A is IOS in X , then f(A) is IRCS in Y (since f is I regular-closed open function), but every IRCS is ICS, then f(A) is ICS in Y ,then g(f(A)) is IRCS in Z (since g is I completely closed function) then gof is I regular-closed open function. Now let B is IOS in Z then $g^{-1}(B)$ is IROS in Y(since g is I completely cont. function) , but every IROS is IOS then $g^{-1}(B)$ is IOS in Y then $f^{1}(g^{-1}(B))$ is IROS in X (since f is I regular -closed cont. function) , then gof is I regular -closed cont. function . Hence gof is I regular-closed hom. function

5. Let A is IROS in X then f (A) is IOS in Y (since f is I almost open function) then g(f(A)) is IROS in Z (since g is I completely open function) then g(f(A)) is IOS in Z; then gof is I almost open function . Now let B is IOS in Z then $g^{-1}(B)$ is IROS in Y(since g is I completely cont. function) then $f^{1}(g^{-1}(B))$ is IOS in X (since f is I almost cont. function) then gof is I cont. function then it is I almost cont. function . Hence gof is I almost hom. function .

6. Let A is IOS in X then f (A) is IROS in Y (since f is I completely open function), then g(f(A)) is IOS in Z (since g is I almost open function) then gof is I open function .Now let B is IROS in Z then $g^{-1}(B)$ is IOS in Y(since g is I almost cont. function)then $f^{1}(g^{-1}(B))$ is IROS in X (since f is I completely cont. function) , but every IROS is IOS then gof is I cont. function. Hence gof is I hom. function .

Remark 3.10

The composition of I completely hom. function and I regular-closed hom. function is :

1. I contra hom.function , since every I regular-closed hom. function is contra hom. function .see (Remark 3.5) .
2. I contra k-hom. function, since every I regular closed hom. function is I contra k-hom. function. see (Remark 3.5) .

Remark 3.11

The composition of I completely hom. function and I almost hom. function is I almost hom.function , since every I hom. function is almost hom. function .

Proposition 3.12

let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y is I contra hom. function and g :Y → Z is I hom. function then gof is I slightly hom. function .

Proof

we can prove that by (Remark 3.5) and (Proposition 3.6) .

Proposition 3.13

let (X,T) , (Y,Ψ) and (Z,σ) are ITS's and let f : X→Y is I slightly hom. function and g :Y → Z is I perfectly hom. function then gof is I slightly hom. function.

Proof

let A is I clopen set in X then f(A) is ICS in Y (since f is I slightly open function) then g(f(A)) is I clopen in Z (since g is perfectly open function) then g(f(A)) is ICS , then gof is slightly open function . Now let B is IOS in Z then $g^{-1}(B)$ is I clopen set in Y (since g is I perfectly cont. function) then $f^{1}(g^{-1}(B))$ ICS in X (since f is I slightly cont. function) hence gof is I contra cont. function then it is slightly cont. function .Then gof is I slightly hom. function.

Remark 3.14

The composition of I contra hom. function does not I contra hom. function .

The following example shows that :

Example : Let $X = \{a,b,c\}$, $T_1 = \{\tilde{\emptyset},\tilde{X},A\}$ where $A = \langle x,\{b\},\{a\}\rangle$ and $Y = \{1,2,3\}$, $T_2 = \{\tilde{\emptyset},\tilde{Y},B\}$ where $= \langle y,\{1\},\{2\}\rangle$, and $Z = \{\sigma,\gamma,\delta\}$, $T_3 = \{\tilde{\emptyset},\tilde{Z},C\}$ where $C = \langle z,\{\gamma\},\{\sigma\}\rangle$.Define a function $f:(X,T_1) \to (Y,T_2)$ by $f(a) = 1, f(b) = 2$ and $f(c) = 3$, and a function $g:(Y,T_2) \to (Z,T_3)$ define by $g(1) = \sigma$, $g(2) = \gamma$, and $g(3) = \delta$.

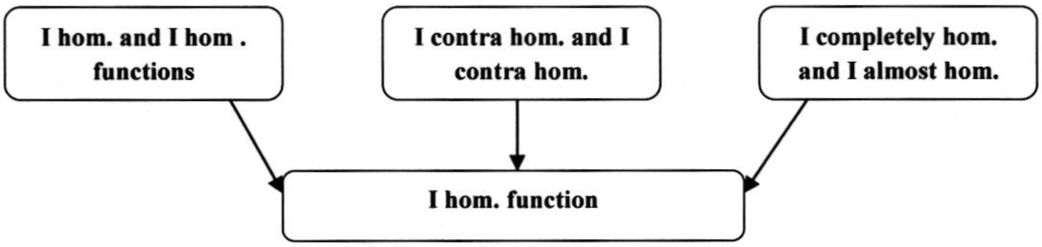
f is I contra cont. function since : the inverse image of IOS in T_2 is ICS in T_1 , $f^{-1}(B) = (\langle x,\{a\},\{b\}\rangle$ is ICS in T_1 since $((f^{-1}(B))^c = A \in T_1)$, and f is I contra copen function since : the image of IOS in T_1 is ICS in T_2 $(f(A) = \langle x,\{2\},\{1\}\rangle$ is ICS in T_2 since $((f(A))^c = B \in T_2)$, hence f is I contra hom . function .

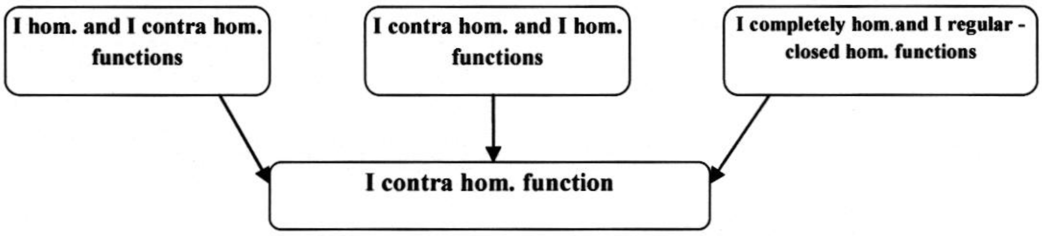
Also g is I contra hom. function since: g is I contra cont function

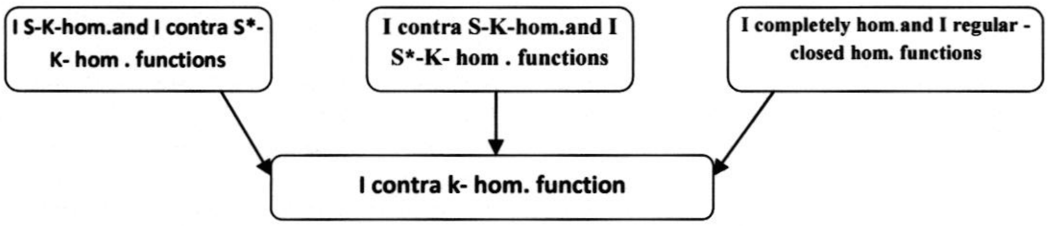
$(g^{-1}(C) = \langle y,\{2\},\{1\}\rangle$ is ICS in T_2 since $(g^{-1}(C))^c = B \in T_2$) and g is I contra open unction$(g(B) = \langle z,\{\sigma\},\{\gamma\}\rangle$ is ICS in T_3 since $(g(B))^c = C \in T_3$).

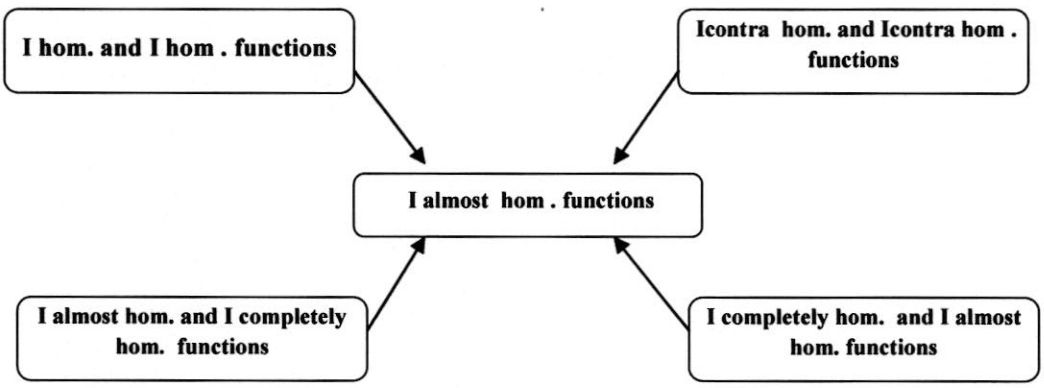
Now gof is not I contra hom. function since: $(gof)(A) = g(f(A)) = g(\langle y,\{2\},\{1\}\rangle$ $=\langle z,\{\gamma\},\{\sigma\}\rangle = C$ then $\forall A \in T_1$ then $(gof)(A) \in T_3$, hence gof is not contra open function , also gof is not contra cont. function $((gof)^{-1}(C) = f^{-1}(g^{-1}(C)) = f^{-1}(\langle y,\{1\},\{2\}\rangle) = \langle x,\{b\},\{a\}\rangle =$ A , then $\forall$ C $\in T_3$ then $(gof)^{-1}(C) \in T_1$).

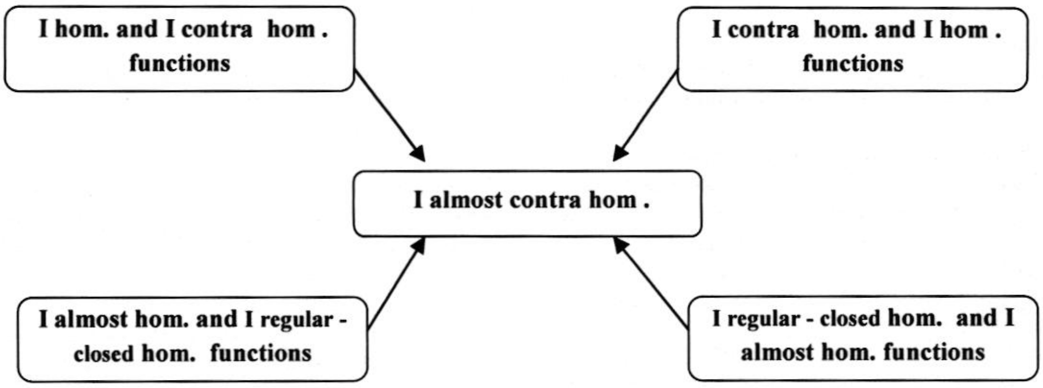
We summarized the above result by the following diagram :

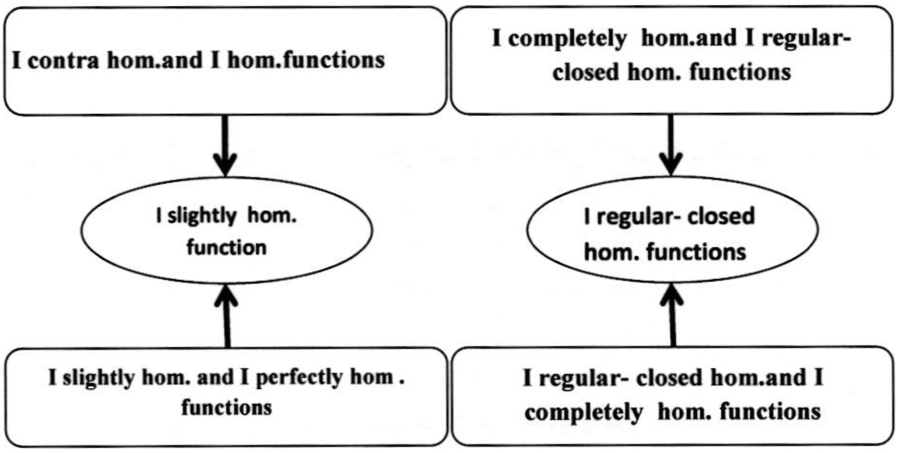

I hom. and I hom . functions
I contra hom. and I contra hom.
I completely hom. and I almost hom.
I hom. function

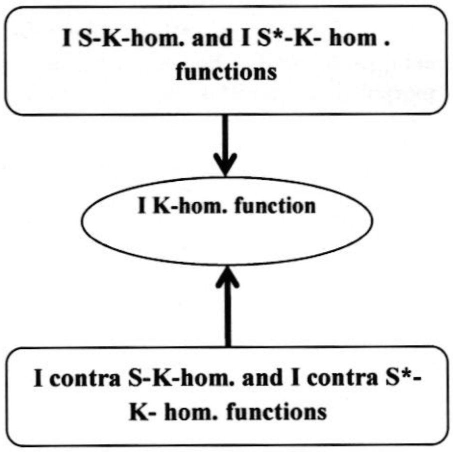

I hom. and I contra hom. functions
I contra hom. and I hom. functions
I completely hom.and I regular - closed hom. functions
I contra hom. function


I S-K-hom.and I contra S*-K- hom . functions
I contra S-K-hom.and I S*-K- hom . functions
I completely hom.and I regular - closed hom. functions
I contra k- hom. function


I hom. and I hom . functions
Icontra hom. and Icontra hom . functions
I almost hom . functions
I almost hom. and I completely hom. functions
I completely hom. and I almost hom. functions


I hom. and I contra hom . functions
I contra hom. and I hom . functions
I almost contra hom .
I almost hom. and I regular - closed hom. functions
I regular - closed hom. and I almost hom. functions


I contra hom.and I hom.functions
I completely hom.and I regular- closed hom. functions
I slightly hom. function
I regular- closed hom. functions
I slightly hom. and I perfectly hom . functions
I regular- closed hom.and I completely hom. functions


I S-K-hom. and I S*-K- hom . functions
I K-hom. function
I contra S-K-hom. and I contra S*- K- hom. functions

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حول تركيب بعض انواع جديدة من الدوال المتشاكلة في الفضاءات التبولوجية الحدسية

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الملخص

في هذا البحث تم تعريف انواع جديدة من الدوال المتشاكلة في الفضاءات التبولوجية الحدسية مثل :

(almost homeomorphism, almost k- homeomorphism, almost S-k- homeomorphism, almost S*-k- homeomorphism, almost contra homeomorphism, almost contra k- homeomorphism, almost contra S-k- homeomorphism, almost contra S*-k- homeomorphism, regular- closed homeomorphism, completely homeomorphism , perfectly homeomorphism and slightly homeomorphism) .

حيث k= {semi, α , per , β } ، ثم قدمنا دراسة لتركيب هذه الدوال مع

(k- homeomorphism, S-k- homeomorphism, S*-K- homeomorphism, contra homeomorphism, contra k- homeomorphism, contra S-k- homeomorphism, contra S*-k- homeomorphism) .

في الفضاءات التبولوجية الحدسية .